4 Bipolar Junction Transistors

INTRODUCTION

A bipolar junction transistor (BJT) consists of two p-regions separated by an n-region as shown in Fig. 4.1 (a) or two n-regions separated by a p-region. The former is called p-n-p transistor and the latter n-p-n transistor. The middle region is designated as the base of the transistor and the regions at the ends as emitter and collector. A BJT consists of two p-n junctions (emitter-base junction and collector-base junction) and three terminals (emitter, base and collector).

A two diode equivalent of p-n-p transistor is shown in Fig. 4.1(b). Diode D_1 is forward-biased and D_2 is reverse-biased. Large current flows through D_1 and the current through D_2 is very small (reverse saturation current). Therefore, this structure does not function as a transistor. The same situation arises if the width of the middle region (base) is large. But if the width of the middle region is very small, most of the charge carriers reach the collector-base junction without recombination. (The current through a reverse-biased p-n junction can be increased by increasing the minority carrier concentration within the depletion layer or within a diffusion length from the edges of the depletion layer). These charge carriers increase the current through the reverse-biased collector-base junction and the collector current is nearly equal to the emitter current. Here, the emitter acts as the injector or emitter of minority carriers and collector collects the minority carriers emitted from the emitter to base.

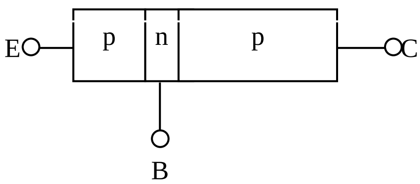
The current is transferred from a low resistance region (forward-biased emitter-base junction) to a high resistance region (reverse-biased collector-base junction). The name transistor is derived from the terms **trans**fer of res**istor**. A change in current at the emitter causes almost same change in current at the collector. The same current is transferred from low resistance region to high resistance region.

We assume the structure in Fig. 4.1 (a) only for the simplicity of analysis. The other simplifying assumptions are:

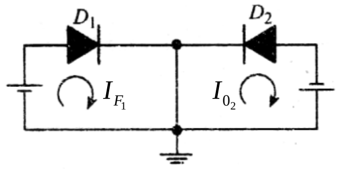
(1) The cross-sectional area of emitter, base and collector regions are equal,

(2) All the regions are uniformly doped and

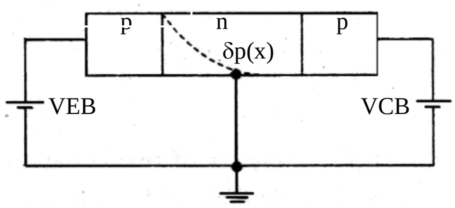
(3) Junctions are abrupt.



(a) A Simplified schematic of a p-n-p transistor



(b) Representation of p-n-p transistor by two p-n junctions



(c) A long base transistor

Fig. 4.1

4.1 FABRICATION OF MONOLITHIC BJT

The actual structure of the transistor is entirely different from that shown in Fig. 4.1(a). The structure and fabrication procedure of a discrete transistor is given in section 2.3.2. The cross-sectional view and fabrication process of a monolithic transistor is shown in Fig. 4.2.

The starting material for an n-p-n transistor is a p-substrate which acts as a mechanical support to the device. Substrate has a resistivity 3-10 Ω cm and thickness 250-400 μm. The important steps involved are:

Step 1: Buried layer - Grow thermal oxide and etch window using mask 1 for n+ buried layer. Diffuse donor impurity through the window.

Step 2: Epitaxial layer - Strip off the oxide after step 1. Deposit n-epitaxial layer over the enitre surface of the wafer. Thickness of epi-layer vary from 1 to 20 μm.

Step 3: Isolation - The collector regions of adjacent transistors on the wafer are isolated by forming a p-region extending from the surface to the substrate through the epi-layer using mask 2. This forms islands or tubs of n-layers surrounded by p-region. Highest negative potential is applied to the substrate. Reverse-biased p-n junctions are formed between substrate and collector regions which isolate the collector regions of adjacent transistors.

Step 4: Base diffusion - Using mask 3 windows are cut for the base region and boron is diffused through it.

Step 5: Emitter and collector contact diffusion - Using mask 4 emitter and collector contact regions are defined and n+ regions are diffused, n+ diffusion is done in the collector region to form ohmic contact.

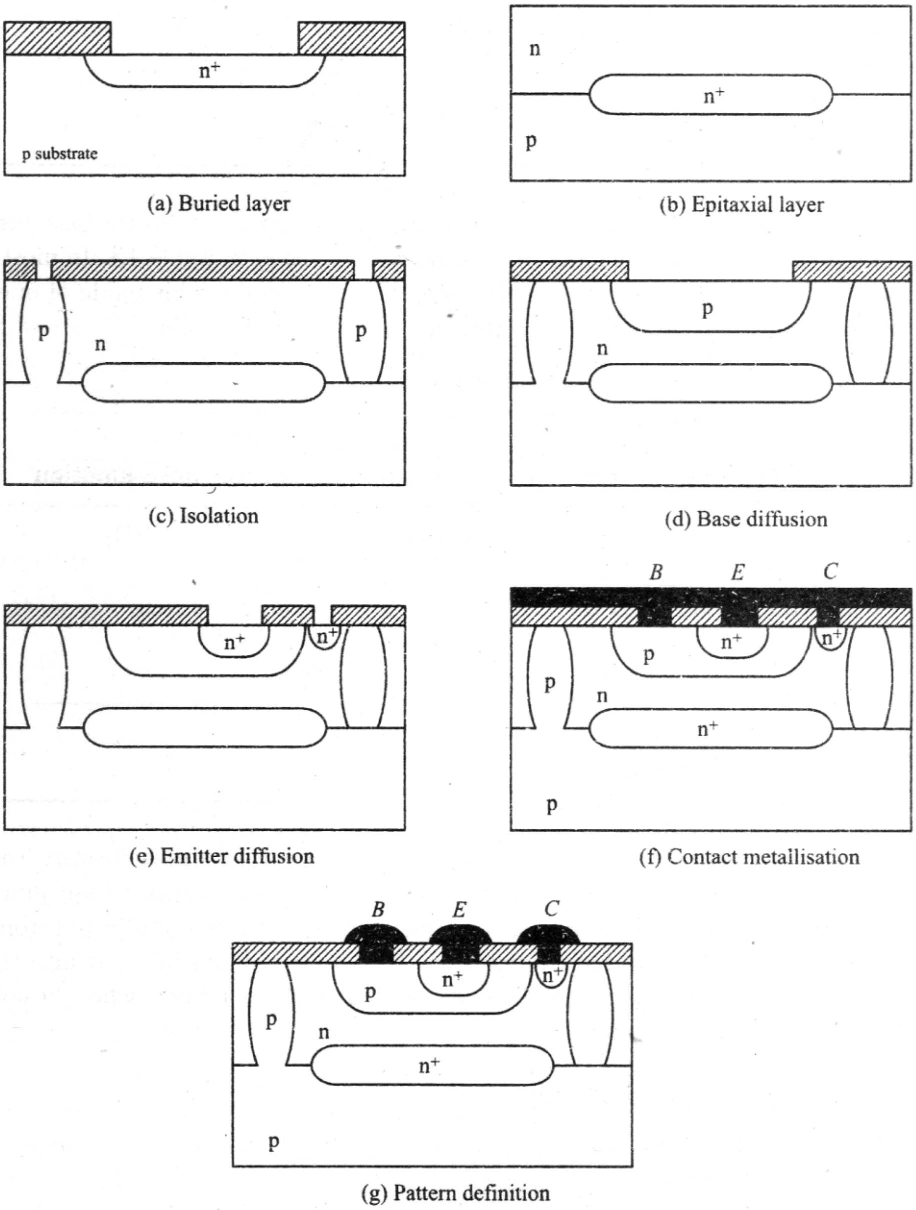


Fig. 4.2 Fabrication sequence of monolithic BJT

Step 6: Contact metallisation - Using mask 5 windows are opened for metallic contacts of the transistor terminals. Aluminium is deposited on the entire surface.

Step 7: Pattern defenition - Using mask 6 which defines the interconnection pattern, the metal formed by step 5 is etched away to form interconnecting metallisation.

Step 8: Bonding pads and packaging - Contact to the IC are formed on metallic pads located at the periphery of the chip. IC chip is bonded to package leads through bonding pads. Mask 7 is used to define bonding holes on the aluminium pads. Gold wire of diameter 25 μm is used to connect package leads to bonding pads.

4.2 MODES OF OPERATION

Depending on the bias conditions of the emitter-base junction and collector-base junction, there are different modes of operations for a transistor as listed in Table 4.1. In most of the applications transistors are operated in the forward active mode. This mode of operation is also referred to as normal mode or normal active mode of operation.

Table 4.1

Mode of operation	Bias condition	
	Emitter-base junction	Collector-base junction
Forward active	Forward-bias (FB)	Reverse-bias (RB)
Inverse active	RB	FB
Forward saturation	FB	FB; $\lvert V_{EB}\rvert > \lvert V_{CB}\rvert$
Inverse saturation	FB	FB; $\lvert V_{EB}\rvert < \lvert V_{CB}\rvert$
Cutoff	RB	RB

4.3 CURRENT COMPONENTS

Fig. 4.3 shows the different current components in a p-n-p BJT under forward (normal) active mode of operation. In normal active mode of operation, emitter-base junction is .forward-biased, and collector-base junction is reverse-biased. Holes are injected from emitter to base and electrons from base to emitter. A portion of holes injected into the base recombine with electrons in the base region and the remaining portion reaches the collector. Minority carrier current I_{CBO} flows across the base collector junction.

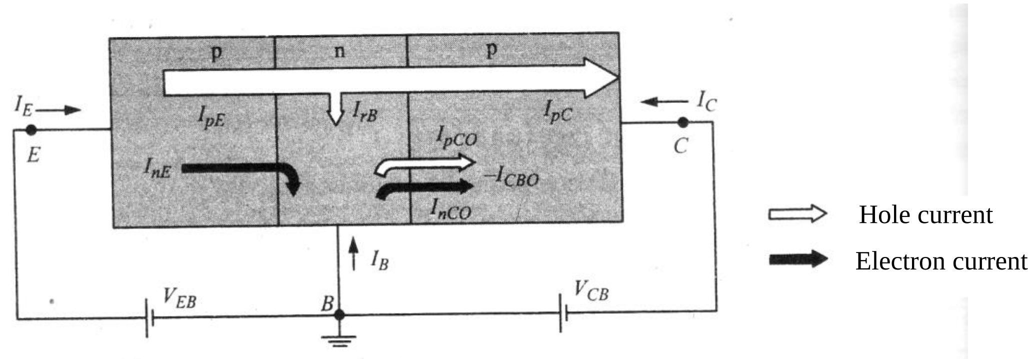


Fig. 4.3 Current components in a p-n-p BJT in forward active mode of operation

The following are the major components of current:

I_{pE} - Emitter current due to holes injected from emitter to base.
I_{nE} - Emitter current due to electrons injected form base to emitter.
I_{rB} - Base current due to recombination in the base region.
I_{pC} - Collector current due to holes reaching the collector which are injected from the emitter.

I_{CBO} - Reverse saturation current of collector-base junction with emitter open. This current is constituted by the minority carriers crossing the junction. It is also known as leakage current of collector-base junction.

4.4 TERMINAL CURRENTS

Emitter current (I_E), collector current (I_C) and base current (I_s) are the terminal currents of a BJT. The terminal currents of a p-n-p transistor can be expressed in terms of the current components as

$$I_E = I_{pE} + I_{nE} .$$
$$I_C = -(I_{pC} - I_{CBO})$$
$$I_B = -(I_{rB} + I_{nE} + I_{CBO})$$

Emitter current (I_E) is constituted by the holes injected from emitter to base and electrons injected from base to emitter.

Collector current (I_C) is the negative of difference between (I_{pC}) and (I_{CBO}). Base current (I_B) consists of three components - current due to carriers injected from base to emitter (I_{nE}), current due to electrons supplied for recombination in the base (I_{rB}) and the reverse saturation current of base collector junction (I_{CBO}). The major component of base current is I_{rB}.

For a transistor, the currents flowing into the device is taken as positive. Therefore for a p-n-p transistor, emitter current is positive, collector and base currents are negative. For a n-p-n transistor, emitter current is negative and collector and base currents are positive,

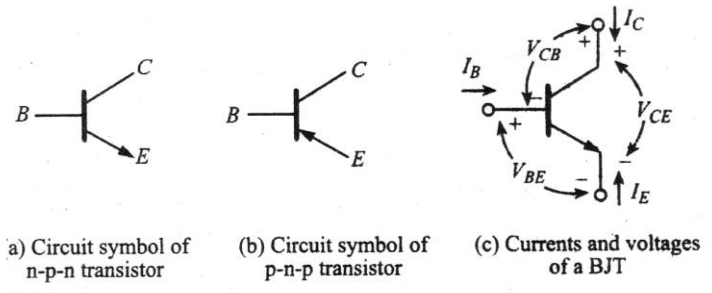


Fig. 4.4 Circuit symbol of BJT

The circuit symbol for transistors are shown in Fig. 4.4(a) and (b). The arrow on emitter terminal represents the actual direction of emitter current. The designation of different currents and voltages in a BJT are shown in Fig. 4.4(c).

4.5 BASIC PERFORMANCE PARAMETERS

The most important parameters of a transistor are its emitter injection efficiency (injection efficiency) and base transport factor (transport factor). As far as a circuit designer is concerned, short-circuit common-base current gain (α) and common-emitter current gain (β) are the basic parameters of a transistor. But these parameters are decided by the injection efficiency and transport factor.

Injection Efficiency

Emitter injection efficiency (γ) is the effectiveness in injecting charge carriers from emitter to base rather than that from base to emitter. It is the ratio of emitter current due to holes injected from emitter to base to the total emitter current, for a p-n-p transistor. Emitter injection efficiency (γ) for a p-n-p transistor may be expressed as

$$\gamma = \frac{I_{pE}}{I_E} = \frac{I_{pE}}{I_{pE} + I_{nE}}$$

$$= \frac{1}{1 + \frac{I_{nE}}{I_{pE}}} \qquad (4.1)$$

Equation (4.1) shows that injection efficiency can be maximized by minimizing I_{nE} / I_{pE}. To achieve this, ratio of emitter doping to base doping must be maximum. (See also the expression derived for injection efficiency of a p-n junction in Problem 3.10). Thus, injection efficiency of a. transistor is decided by the ratio of emitter doping to base doping.

Base Transport Factor

The base transport factor (α_T) of a BJT is the effectiveness of the base in transporting charge carriers injected from the emitter to the collector through the base. During the transit through the base some of the charge carriers are lost due to recombination. A better transport factor means less loss of charge carriers in the base due to recombination.

$$\text{Base transport factor } \alpha_T = \frac{I_{pC}}{I_{pE}}$$

$$= \frac{I_{pC}}{I_{pC} + I_{rB}}$$

$$= \frac{1}{1 + \frac{I_{rE}}{I_{pC}}} \qquad (4.2)$$

Equation (4.2) shows that transport factor can be maximised by minimising I_{rB} . I_{rB} can be minimised by reducing doping in the base region and by reducing the width of the base. If the width of the base is comparable to the minority carrier diffusion length (L_B), most of the charge carriers that are injected from emitter to base recombines in the base region and those reaching the collector will be very small. Therefore, the width of the base must be much less than the minority carrier diffusion length ($W_B << L_B$).

Short-circuit common-base current gain (α) of the transistor is defined as

$$\alpha_{dc} = \frac{I_{pC}}{I_E} = \frac{-(I_C - I_{CBO})}{I_E} \cong \frac{I_C}{I_E} \qquad (4.3)$$

$$\alpha_{ac} = \frac{\partial}{\partial I_E}[-(I_C - I_{CBO})] = -\frac{\partial I_C}{\partial I_E} \qquad (\because I_{CBO} \text{ is independent of } I_E) \qquad (4.4)$$

For a good transistor $\alpha_{(dc)}$ and $\alpha_{((ac)}$ must be close to unity.

In the reverse-biased collector-base junction, carrier multiplication occurs with increase in V_{CB} and multiplication factor (M) increases with increase in V_{CB}. The avalanche multiplication in the collector region is given by

$$M = \frac{\text{Current leaving collector region}}{\text{Currententering collector region}}$$

$$= \frac{(I_C - I_{CBO})}{I_{pC}} \tag{4.5}$$

The common base current gain (α) is related to injection efficiency (γ) and transport factor (α_T) as

$$\alpha = \gamma\alpha_T M \tag{4.6}$$

$$= \frac{I_{pE}}{I_E}\frac{I_{pC}}{I_{pE}}\frac{-(I_C - I_{CBO})}{I_{pC}}$$

$$= \frac{-(I_C - I_{CBO})}{I_E} \tag{4.7}$$

Common-emitter current gain (β) is given by

$$\beta = \left.\frac{I_C}{I_B}\right|_{V_{CEconstant}} \tag{4.8}$$

It is related to α as

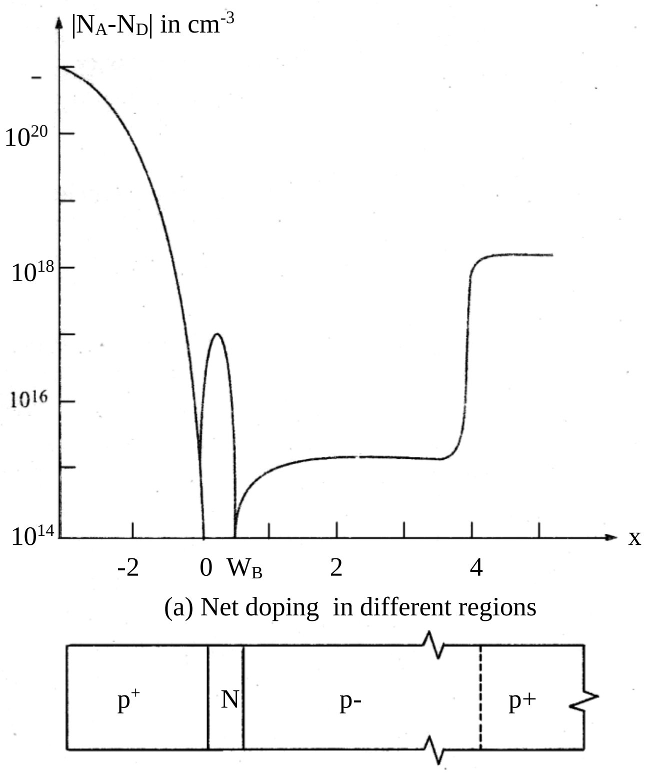
$$\beta = \frac{\alpha}{1-\alpha} \tag{4.9}$$

β is maximum when α is maximum (close to unity), α and β of a BJT can be maximised by maximising injection efficiency and transport factor. For that emitter doping must be maximum, base doping must be minimum and base width must be minimum.

Collector Region

Our discussion so far shows that the performance of a transistor are decided by parameters in the base and emitter regions. To improve switching speed and to reduce parasitic capacitance, the collector resistance must be as low as possible. For this collector doping must be as high as possible. But this poses a serious problem known as punch through.

The base region is very narrow, its width cannot be increased at any cost. As the reverse bias at the collector-base junction increases, the depletion layer extends more and more into the base region. If the base doping is less compared to collector doping, depletion layer penetrates more into the base reducing the effective base width. The base gets completely depleted at a low reverse voltage itself. The complete penetration of depletion layer into the base region is called punch through, which is a form of breakdown. The collector-base voltage loses control over collector current. To avoid premature punch-through (punch through at low V_{CB}) the collector doping must be less than the base doping so that depletion layer penetrates more into collector region. The low collector doping increases the collector resistance, reduces switching speed and increases power dissipation.



(a) Net doping in different regions

(b) Physical position of emitter base and collector

Fig. 4.5 Practical doping profile of BJT

This poses two conflicting requirements for collector doping. To overcome this to some extent, the collector doping near the base is kept lower than tlie base doping and collector doping away from the base is kept higher than the base doping. A practical doping profile is shown in Fig. 4.5.

The doping in the collector near the base is much less than that in the base which avoid premature breakdown by punch through. At the same time, collector doping is high away from the base collector junction which helps to reduce the effective resistance of the collector to some extent.

Example 4.1 The following parameters are given for a p-n-p transistor. I_{pE} = 10 mA, I_{nE} = 0.02 mA, I_{pC} = 9.99 mA, I_{nC} = 0.002 mA. Determine γ, α_T, α, β, I_B, I_C, I_E and I_{CBO} (Neglect avalanche multiplication in collector-base junction).

Solution

$$\alpha_T = \frac{I_{pC}}{I_E} = \frac{9.99}{10} = 0.999$$

$$\gamma = \frac{I_{pE}}{I_E} = \frac{I_{pE}}{I_{pE} + I_{nE}} = \frac{10}{10+0.02} = 0.998$$

$$\alpha = \alpha_T . \gamma = 0.999 \times 0.998 = 0.997$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.997}{1-0.997} = 332$$

$$I_C = -(I_{pC} - I_{nC})$$
$$= (9.99 + 0.002)$$
$$= -9.992 \text{ mA}$$

$$I_E = (I_{pE} + I_{nE}) = 10 + 0.02$$
$$= 10.02 \text{ mA}$$

$$I_B = -(I_C + I_E)$$
$$= -(-9.992 + 10.02)$$
$$= -0.028 \text{ mA}$$

$$\therefore \quad I_{CBO} = \frac{I_C - \beta I_B}{1+\beta}$$
$$= \frac{-9.992 + 332 \times 0.028}{1+332} = 2\mu A$$

4.6 ENERGY BAND DIAGRAM

The energy band diagrams of p-n-p transistor under thermal equilibrium, forward active region and saturation region are shown in Fig. 4.6(a) to (c). Some tips to draw energy band diagram of BJT are given below.

(1) To draw the energy band diagram in any mode of operation, start with equilibrium energy band diagram. $\left(\frac{dE_F}{dx} = 0\right)$.

(2) Draw the Fermi level and leave space for depletion regions of emitter-base and collector-base junctions.

(3) Complete energy band diagram in the neutral regions, knowing the dopings.

(4) Connect E_C and E_V in different regions continuously.

Biased Transistor

(1) For a biased transistor take energy band diagram in the base as reference. Draw the energy band diagram in the base region exactly same as that in equilibrium condition.

(2) Leave space for depletion layers. Also notice the change in depletion layer width with bias.

(3) The Fermi level position in the collector and emitter regions shift up or down with respect to that in base by qVa. The Fermi levels move up in these regions if the potential is negative with respect to that in base and shift down if the potential is positive with respect to potential at base.

(4) Knowing Fermi level position draw the other energy levels, whose relative positions with respect to E_F is same as that in the equilibrium energy band diagram.

(5) Complete the diagram by connecting E_C and E_V in different regions by smooth continuous lines.

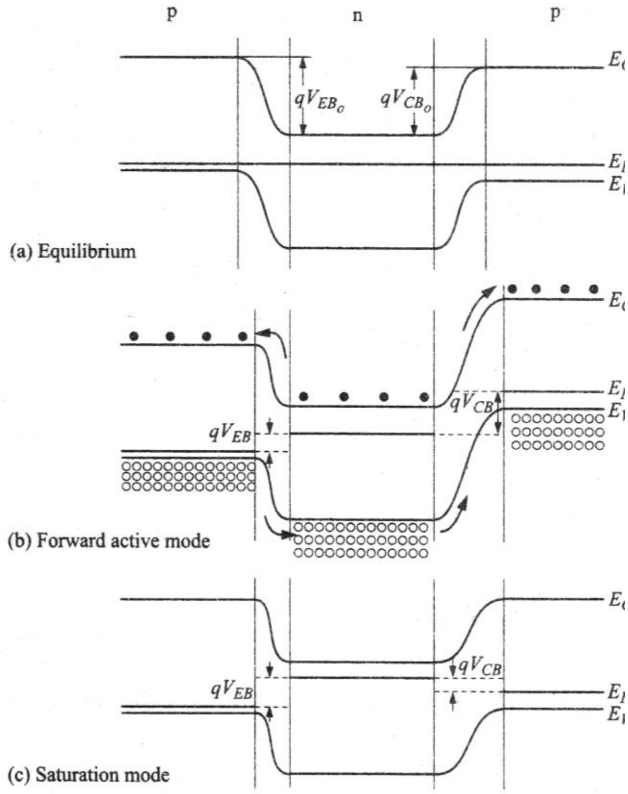


Fig. 4.6 Energy band diagrams of p-n-p BJT

4.7 MINORITY CARRIER DISTRIBUTION

The terminal currents in a transistor are evaluated from the slope of the minority carrier distribution as in the case of a p-n junction, Fig. 4.7 shows the minority carrier distribution in a p-n-p transistor in saturation mode of operation.

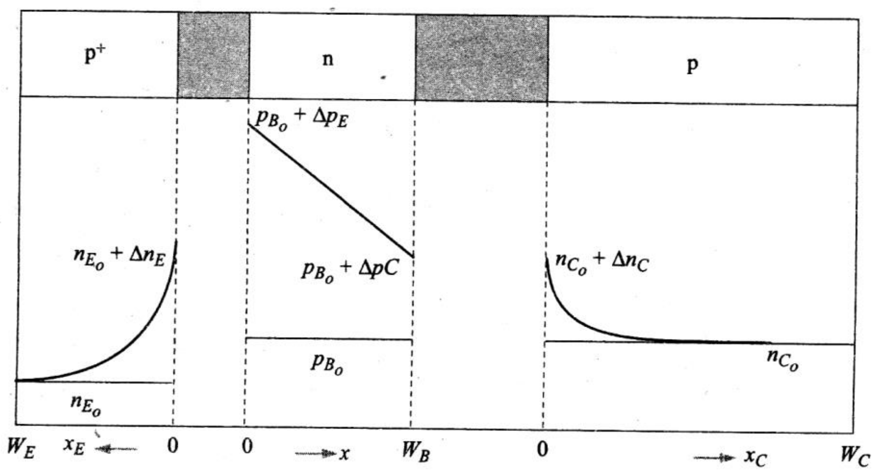


Fig. 4.7 Minority carrier distribution in a p-n-p transistor in saturation mode of operation

The meanings of different symbols used are as follows:

$n_{E_o} - \dfrac{n_i^2}{N_{AE}}$, equilibrium electron concentration in emitter.

$p_{B_o} - \dfrac{n_i^2}{N_D}$, equilibrium hole concentration in base region.

$n_{C_o} - \dfrac{n_i^2}{N_{AC}}$, equilibrium electron concentration in collector region.

Δ_{pE} - concentration of holes injected from emitter to base at emitter end of base (x = 0).

Δ_{nE} - concentration of electrons injected from base to emitter at the edge of depletion layer (x_E =0).

Δ_{pC} - concentration of hole injected from collector to base at the collector end of the base (x = W_B).

Δ_{nC} - concentration of electrons injected from base to collector at the edge of depletion layer (x_C = 0).

Similar to equations (3.26) and (3.27),

$$\Delta_{pE} = p_{Bo}\left(e^{V_{EB}/V_T}-1\right)=\frac{n_i^2}{N_D}\left(e^{V_{EB}/V_T}-1\right) \tag{4.10a}$$

$$\Delta_{pE} = p_{Bo}\left(e^{V_{CB}/V_T}-1\right)=\frac{n_i^2}{N_D}\left(e^{V_{CB}/V_T}-1\right) \tag{4.10b}$$

$$\Delta_{nE} = n_{Eo}\left(e^{V_{EB}/V_T}-1\right)=\frac{n_i^2}{N_{AE}}\left(e^{V_{EB}/V_T}-1\right) \tag{4.10c}$$

$$\Delta_{nC} = n_{Co}\left(e^{V_{CB}/V_T}-1\right)=\frac{n_i^2}{N_{AC}}\left(e^{V_{CB}/V_T}-1\right) \tag{4.10d}$$

The minority carrier distribution in the base region is almost linear. This is because the current flow through the base is by diffusion only and recombination in the base is negligible. Since, recombination is negligible, by continuity equation the current inflow and outflow must be same throughout the base region, i.e. the diffusion current remains constant throughout the base region. Thus the slope is constant and distribution of minority carrier is linear.

4.8 DERIVATION OF TERMINAL CURRENTS

In this section, we derive expressions for terminal currents of a p-n-p transistor. These expressions are applicable to n-p-n transistors also, with suitable modifications. Terminal currents of a transistor are derived with the following approximations.

(1) The area of cross-section is same for emitter, base and collector regions.
(2) The junctions are abrupt.
(3) Doping is uniform in all regions.
(4) The minority carrier currents in the neutral regions are by diffusion only (By depletion approximation). This type of transistor is called diffusion transistor.
(5) Low-level injection condition exists in all regions.

(6) The transistor is in steady-state condition.
(7) Current flow is one dimensional.
(8) No generation or recombination in depletion regions.

The major components of current in a BJT are obtained from the slope of minority carrier distribution in the base region. Under steady-state condition, with current by diffusion only the continuity equation for holes in the base region reduces to

$$\frac{d^2 \delta p(x)}{dx^2} = \frac{\delta p(x)}{L_p^2} \tag{4.11}$$

Solution to this equation is of the form

$$\delta p(x) = C_1 e^{-x/Lp} + C_2\, e^{x/Lp} \tag{4.12}$$

Referring to Fig. 4.7, the boundary conditions are

$$\begin{aligned} \text{at} \quad x &= 0; \quad \delta_p(x) = \Delta_{pE} \\ x &= W_B; \quad \delta_p(x) = \Delta_{pC} \end{aligned}$$

Applying boundary conditions to equation (4.12)

$$\Delta_{pE} = C_1 + C_2 \tag{4.13}$$

$$\Delta_{pC} = C_1\left(e^{-W_B/L_p}\right) + C_2\left(e^{W_B/L_p}\right) \tag{4.14}$$

[Equation (4.13) $\times\, e^{-W_B/L_p}$] - [equation (4.14)] gives

$$\Delta_{pE}\, e^{-W_B/L_p} - \Delta_{pC} = C_2\left(e^{-W_B/L_p} - e^{W_B/L_p}\right)$$

$$\therefore \qquad C_2 = \frac{\Delta_{pE}\left(e^{-W_B/L_p}\right) - \Delta_{pC}}{-\left(e^{W_B/L_p} - e^{-W_B/L_p}\right)} \tag{4.15}$$

$$= \frac{\Delta_{pC} - \Delta_{pE}\left(e^{-W_B/L_p}\right)}{e^{W_B/L_p} - e^{-W_B/L_p}}$$

$$C_1 = \Delta_{pE} - C_2$$

$$= \Delta_{pE} - \left(= \frac{\Delta_{pC} - \Delta_{pE} e^{-W_B/L_p}}{e^{W_B/L_p} - e^{-W_B/L_p}} \right)$$

$$= \frac{\Delta_{pE} e^{-W_B/L_p} - \Delta_{pC}}{e^{W_B/L_p} - e^{-W_B/L_p}} \tag{4.16}$$

Substituting the values of C_1 and C_2 in equation (4.12)

$$\delta p(x) = \frac{\Delta_{pE} e^{-W_B/L_p} - \Delta_{pC}}{e^{W_B/L_p} - e^{-W_B/L_p}} \times e^{-x/L_p} + \frac{\Delta_{pC} - \Delta_{pE} e^{-W_B/L_p}}{e^{W_B/L_p} - e^{-W_B/L_p}} \times e^{-x/L_p}$$

$$= \frac{\left(\Delta_{pE} e^{-W_B/L_p} - \Delta_{pC}\right) e^{-x/L_p} + \left(\Delta_{pC} - \Delta_{pE} e^{-W_B/L_p}\right) e^{-x/L_p}}{e^{W_B/L_p} - e^{-W_B/L_p}}$$

$$= \frac{\Delta_{pE}\left[e^{\left(\frac{W_B - x}{L_p}\right)} - e^{-\left(\frac{W_B - x}{L_p}\right)}\right] + \Delta_{pC}\left(e^{x/L_p} - e^{-x/L_p}\right)}{e^{W_B/L_p} - e^{-W_B/L_p}}$$

$$= \frac{\Delta_{pE} \sinh\left(\frac{W_B - x}{L_p}\right)}{\sinh\left(\frac{W_B}{L_p}\right)} + \frac{\Delta_{pC} \sinh\left(\frac{x}{L_p}\right)}{\sinh\left(\frac{W_B}{L_p}\right)} \tag{4.17}$$

The hole diffusion current in the base

$$I_p(x) = -qAD_p \cdot \frac{d}{dx}\delta p(x)$$

$$= qA\frac{D_p}{L_p}\left(C_1 e^{-x/L_p} - C_2 e^{x/L_p}\right) \tag{4.18}$$

$$I_{pE} = I_p(x) \quad \text{at} \quad x=0$$

$$\therefore \quad I_{pE} = qA\frac{D_p}{L_p}\left(C_1 - C_2\right)$$

$$= qA\frac{D_p}{L_p}\left[\frac{\Delta_{pE} e^{W_B/L_p} - \Delta_{pC} - \Delta_{pC} + \Delta_{pE} e^{-W_B/L_p}}{e^{W_B/L_p} - e^{-W_B/L_p}}\right]$$

$$= qA\frac{D_p}{L_p}\left[\frac{2\Delta_{pE}\cosh\left(\frac{W_B}{L_p}\right) - 2\Delta_{pC}}{2\sinh\left(\frac{W_B}{L_p}\right)}\right]$$

$$= qA\frac{D_p}{L_p}\left[\Delta_{pE}\coth\left(\frac{W_B}{L_p}\right) - \Delta_{pC}\,\mathrm{cosech}\left(\frac{W_B}{L_p}\right)\right] \tag{4.19}$$

$$I_{pC} = I_p(x) \text{ at } x = W_B$$

$$= qA\frac{D_p}{L_p}\left(C_1 e^{-x/L_p} - C_2 e^{x/L_p}\right)$$

$$= qA\frac{D_p}{L_p}\left[\frac{\Delta_{pE} - \Delta_{pC} e^{-W_B/L_p} - \Delta_{pC} e^{W_B/L_p}}{e^{W_B/L_p} - e^{-W_B/L_p}}\right]$$

$$= qA\frac{D_p}{L_p}\left[\frac{\Delta_{pE} - \Delta_{pC}\cosh\left(\frac{W_B}{L_p}\right)}{\sinh\left(\frac{W_B}{L_p}\right)}\right]$$

$$= qA\frac{D_p}{L_p}\left[\Delta_{pE}\,\mathrm{cosech}\left(\frac{W_B}{L_p}\right) - \Delta_{pC}\coth\left(\frac{W_B}{L_p}\right)\right] \tag{4.20}$$

Similarly by solving continuity equation in the emitter region for minority carrier distribution, the current due to electrons injected from base to emitter may be evaluated as

$$I_{nE} = qA\,\frac{D_{nE}}{L_{nE}}\Delta_{nE}\coth\left(\frac{W_E}{L_{nE}}\right) \tag{4.21}$$

The current due to electron injected from base to collector is given by

$$I_{nC} = -qA\,\frac{D_{nC}}{L_{nC}}.\Delta_{nC}\coth\left(\frac{W_C}{L_{nC}}\right) \tag{4.22}$$

$$\begin{aligned} I_E &= I_{pE} + I_{nE} \\ &= qA\,\frac{D_p}{L_p}\left[\Delta_{pE}\coth\left(\tfrac{W_B}{L_p}\right) - \Delta_{pC}\cos ech\left(\tfrac{W_B}{L_p}\right)\right] \\ &\quad + qA\,\frac{D_{nE}}{L_{nE}}\Delta_{nE}\coth\left(\frac{W_E}{L_{nE}}\right) \end{aligned} \tag{4.23}$$

Substituting for Δ_{pE}, Δ_{pC} and Δ_{nE} from equation (4.10)

$$\begin{aligned} I_E &= qAn_i^2\left[\frac{D_p}{L_p}\frac{1}{N_D}\coth\left(\tfrac{W_B}{L_p}\right) + \frac{D_{nE}}{L_{nE}}\frac{1}{N_{AE}}\coth\left(\frac{W_E}{L_{nE}}\right)\right]\left(e^{V_{EB}/V_T} - 1\right) \\ &\quad + qA\frac{n_i^2}{N_D}\frac{D_p}{L_p}\cos ech\left(\tfrac{W_B}{L_p}\right)\left(e^{V_{CB}/V_T} - 1\right) \end{aligned} \tag{4.24}$$

$$\begin{aligned} I_C &= (I_{pC} + I_{nC}) \\ &= qA\,\frac{D_p}{L_p}\left[\Delta_{pE}\cos ech\left(\tfrac{W_B}{L_p}\right) - \Delta_{pC}\coth\left(\tfrac{W_B}{L_p}\right)\right] \\ &\quad + qA\,\frac{D_{nC}}{L_{nC}}\Delta_{nC}\coth\left(\frac{W_C}{L_{nC}}\right) \end{aligned} \tag{4.25}$$

$$\begin{aligned} &= qAn_i^2\left[\frac{D_{nC}}{L_{nC}}\times\frac{1}{N_{AC}}\coth\left(\frac{W_C}{L_{nC}}\right) + \frac{D_p}{L_p}\times\frac{1}{N_D}\coth\left(\tfrac{W_B}{L_p}\right)\right]\left(e^{V_{CB}/V_T} - 1\right) \\ &\quad - qA\frac{D_p}{L_p}\frac{n_i^2}{N_D}\cos ech\left(\tfrac{W_B}{L_p}\right)\left(e^{V_{EB}/V_T} - 1\right) \end{aligned} \tag{4.26}$$

$$I_B = -(I_C + I_E) \tag{4.27}$$

4.8.1 DC Parameters

From equations (4.19) and (4.21), for a pnp BJT in the active region, neglecting Δ_{pC}

$$\frac{I_{nE}}{I_{pE}} = \frac{qA\frac{D_{nE}}{L_{nE}}.\Delta_{nE}.\coth\left(\frac{W_E}{L_{nE}}\right)}{qA\frac{D_p}{L_p}.\Delta_{pE}.\coth\left(\frac{W_E}{L_p}\right)}$$

if $W_B << L_p$ $\quad \coth\left(\tfrac{W_B}{L_p}\right) \cong \dfrac{L_p}{W_B}$ $\quad$ [if x << 1 coth x $= \frac{1}{x}$

and $W_E << L_E$ $\quad \coth\left(\tfrac{W_E}{L_{nE}}\right) \cong \dfrac{L_{nE}}{W_E}$

$$\therefore \quad \frac{I_{nE}}{I_{pE}} = \frac{\frac{D_{nE}}{L_{nB}}\Delta_{nE}\frac{L_{nB}}{W_E}}{\frac{D_p}{L_p}\Delta_{pE}\frac{L_p}{W_E}} = \frac{\frac{D_{nE}}{L_{nB}}}{\frac{D_p}{L_p}}\frac{\Delta_{nE}}{\Delta_{pE}}\frac{W_B}{W_E} \tag{4.28}$$

Substituting for $\Delta_n E$ and $\Delta_p E$ from equation (4.10)

$$\frac{I_{nE}}{I_{pE}} = \frac{D_{nE}}{D_p} \times \frac{N_D}{N_{AE}} \times \frac{W_E}{W_E} \tag{4.29}$$

Injection efficiency $\gamma = \dfrac{1}{1+\dfrac{I_{nE}}{I_{pE}}}$

$$= \frac{1}{1+\dfrac{D_{nE}}{D_p} \times \dfrac{N_D}{N_{AE}} \times \dfrac{W_B}{W_E}} \tag{4.30}$$

Transport factor $\alpha_T = \dfrac{I_{pC}}{I_{pE}} = \dfrac{\left.\frac{d\delta p(x)}{dx}\right|_{x=W_B}}{\left.\frac{d\delta p(x)}{dx}\right|_{x=0}}$

In the active region of operation $\Delta_{pC} \cong 0$

$\therefore$ From equation (4.17)

$$\delta p(x) = \frac{\Delta_{pE} \sinh\left(\frac{W_B - x}{L_p}\right)}{\sinh\left(\frac{W_B}{L_p}\right)}$$

$$\therefore \alpha_T = \frac{\left.\cosh\left(\frac{W_B - x}{L_p}\right)\right|_{x=W_B}}{\left.\cosh\left(\frac{W_B - x}{L_p}\right)\right|_{x=0}}$$

$$= \frac{1}{\cosh\left(\frac{W_B}{L_p}\right)} \tag{4.31}$$

For $W_B << L_p$, equation (4.31) becomes

$$\alpha_T = \frac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_p}\right)^2} \qquad \left[\begin{array}{l}\cosh x = 1+\frac{x^2}{2} \\ for\ x << 1\end{array}\right. \tag{4.32}$$

The multiplication factor M is given by

$$M = \frac{1}{1-\left(\frac{V_{CB}}{V_{Br}}\right)^n} \tag{4.33}$$

where n varies between 2 and 7.

The common-base current gain (α) $\cong \alpha_T\, \gamma$ (neglecting avalanche multiplication in collector-base junction).

$$\alpha = \frac{1}{\left[1+\frac{1}{2}\left(\dfrac{W_B}{L_p}\right)^2\right]} \times \frac{1}{\left[1+\dfrac{D_{nE}}{D_p} \times \dfrac{N_D}{N_{AE}} \times \dfrac{W_B}{W_E}\right]}$$

$$\approx \frac{1}{1+\frac{D_{nE}}{D_p}\times\frac{N_D}{N_{AE}}\times\frac{W_B}{W_E}+\frac{1}{2}\left(\frac{W_B}{L_p}\right)^2} \quad (4.34)$$

The common-emitter current gain β can be expressed as

$$\beta = \frac{\alpha}{1-\alpha} = \frac{1}{\frac{1}{\alpha}-1}$$

Substituting for α we get

$$\beta = \frac{1}{\frac{D_{nE}}{D_p}\cdot\frac{N_D}{N_{AE}}\cdot\frac{W_B}{W_E}+\frac{1}{2}\left(\frac{W_B}{L_p}\right)^2} \simeq \frac{2L_p^2}{W_B^2} \quad (4.35)$$

(∵ $W_B << W_E$ and $N_D << N_{AE}$)

Example 4.2 The following parameters are given for an n-p-n transistor at 300 K.

	Emitter	**Base**	**Collector**
doping	10^{19} cm^{-3}	10^{16} cm^{-3}	10^{15} cm^{-3}
width	2μm	1μm	5μm
minority carrier life time	0.01μs	0.06μs	1μs
mobility	μp= 350 cm^2/Vs	μn = 1250 cm^2/Vs	μp= 450 cm^2/ Vs

Assume uniform area of cross-section 5 × 10^{-4} cm^2. If V_{BE} = 0.6 V and $|V_{CB}| > V_{BE}$, determine the following parameters at 300 K: I_{nE}, I_{pE}, I_{pC}, I_{nC} and I_B. Take $n_i = 1.5 \times 10^{10}$ cm^{-3}.

Solution

Emitter	**Base**	**Collector**
$N_{DE} = 10^{19}$ cm^{-3}	$N_A = 10^{16}$ cm^{-3}	$N_{DC} = 10^{15}$ cm^{-3}
$p_{Eo} = \frac{n_i^2}{N_{DE}}$ =22.5cm^{-3}	$n_{Bo} = \frac{n_i^2}{N_A}$ =22.5	$p_{Co} = \frac{n_i^2}{N_{DC}}$
	× 10^4cm^{-3}	= 22.5 × 10^5cm^{-3}
$D_{pE} = \mu_{pE} \times \frac{kT}{q}$	$D_n = \mu_n \times \frac{kT}{q}$	$D_{pC} = \mu_{pC} \times \frac{kT}{q}$
=9.1cm^2/s	=32.5cm^2/s	=11.7cm^2/s
$L_{pE} = \sqrt{D_{pE}\tau_{pE}}$	$L_n = \sqrt{D_n\tau_n}$	$L_{pC} = \sqrt{D_{pC}\tau_{pC}}$
= 3.02 × 10^{-4} cm	= 1.4 × 10^{-3} cm	= 3.42 × 10^{-3} cm

Δ_{nE} = $n_{Bo}\left(e^{V_{BE}/V_T}-1\right)$ = 2.25 × l0^4 ($e^{0.6/0.026}$ - l)

= 2.368 × 10^{14} cm^{-3}

Δ_{nC} = $n_{Bo}\left(e^{V_{CB}/V_T}-1\right)$ = $-n_{Bo}$ = -2.25 × 10^4 cm^{-3}

$\Delta_{pE} = p_{Eo}\left(e^{V_{BE}/V_T} - 1\right) = 22.5\ (e^{0.6/0.026} - l)$

$= 2.368 \times 10^{11}\ cm^{-3}$

$\Delta_{pC} = p_{Co}\left(e^{V_{CB}/V_T} - 1\right) = -p_{Co} = -2.25 \times 10^{5}\ cm^{-3}$

$$\coth\left(\frac{W_B}{L_n}\right) = \coth\left(\frac{1\times10^{-4}}{1.4\times10^{-3}}\right) = 14.02$$

$$\text{cosech}\left(\frac{W_B}{L_n}\right) = \text{cosech}\left(\frac{1\times10^{-4}}{1.4\times10^{-3}}\right) = 13.988$$

$$\coth\left(\frac{W_E}{L_{pE}}\right) = \coth\left(\frac{2\times10^{-4}}{3.02\times10^{-3}}\right) = 1.725$$

$$\coth\left(\frac{W_C}{L_{pC}}\right) = \coth\left(\frac{5\times10^{-4}}{3.42\times10^{-3}}\right) = 6.888$$

For an n-p-n transistor equation (4.15) modifies to

$$I_{nE} = -qA\frac{D_n}{L_n}\left[\Delta_{nE}\coth\left(\frac{W_B}{L_n}\right) - \Delta_{nC}\,\text{cosech}\left(\frac{W_B}{L_n}\right)\right]$$

$$= -1.6 \times 10^{-19} \times 5 \times 10^{-4} \times \frac{32.5}{1.4\times10^{-3}}$$

$$[2.368 \times 10^{14} \times 14.02 + 2.25 \times 10^{4} \times 13.988]$$

$= -6165.595\ \mu A$

$$I_{pE} = -qA\frac{D_{pE}}{L_{pE}}.\Delta_{pE}\coth\left(\frac{W_E}{L_{pE}}\right)$$

$$= -1.6 \times 10^{-19} \times 5 \times 10^{-4} \times \frac{9.1}{3.02\times10^{-4}} \times 2.368 \times 10^{11} \times 1.725$$

$= -0.9846\ \mu A$

$$I_{nC} = -qA\frac{D_n}{L_n}\left[\Delta_{nE}\,\text{cosech}\left(\frac{W_B}{L_n}\right) - \Delta_{nC}\coth\left(\frac{W_B}{L_n}\right)\right]$$

$$= -1.6 \times 10^{-19} \times 5 \times 10^{-4} \times \frac{32.5}{1.4\times10^{-3}}$$

$$[2.368 \times 10^{14} \times 13.988 + 2.25 \times 10^{4} \times 14.02]$$

$= -6151.52\ \mu A$

$$I_{pC} = qA\frac{D_{pC}}{L_{pC}}.\Delta_{pC}\coth\left(\frac{W_C}{L_{pC}}\right)$$

$$= 1.6 \times 10^{-19} \times 5 \times 10^{-4} \times \frac{11.7}{3.42\times10^{-3}} \times -2.25 \times 10^{5} \times 6.888$$

$= -4.24 \times 10^{-13}$ A

$I_E = I_{nE} + I_{pE} = -6165.595 - 0.9846 = -6166.5796\ \mu A$

$I_C = -(I_{nC} + I_{pC})$

$= -(-6.1515 \times 10^{-3} - 4.24 \times 10^{-13}) = 6.1515 \times l0^{-3}$A

$I_B = -(I_E + I_C)$

$= -(-6.1665 \times 10^{-3} + 6.1515 \times 10^{-3}) = 15\mu A.$

Example 4.3 For the transistor specified in Example 4.2 determine injection efficiency, transport factor, α and β.

Solution

$$\text{Injection efficiency } \gamma = \frac{1}{1+\dfrac{D_{pE}}{D_n}\times\dfrac{N_A}{N_{DE}}\times\dfrac{W_B}{W_E}}$$

$$= \frac{1}{1+\dfrac{9.1}{32.5}\times\dfrac{10^{16}}{10^{19}}\times\dfrac{1\times10^{-4}}{2\times10^{-4}}}$$

$$= 0.99986$$

$$\text{Transport factor } \alpha_T = \frac{1}{\cosh\left(\frac{W_B}{L_n}\right)}$$

$$= \frac{1}{\cosh\left(\frac{1\times10^{-4}}{1.4\times10^{-3}}\right)} = 0.99745$$

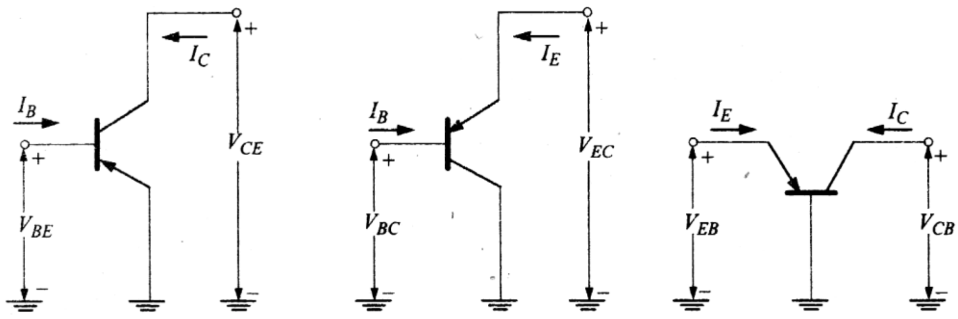
$$\text{Common-base current gain } \alpha = \alpha_T \times \gamma$$

$$= 0.99745 \times 0.99986 = 0.99731$$

$$\text{Common-emitter current gain } \beta = \frac{\alpha}{1-\alpha} = \frac{0.99731}{1-0.99731} = 370.$$

4.9 CIRCUIT ARRANGEMENTS OF TRANSISTOR

In transistor circuit arrangements one of the terminals acts as common terminal to input and output. Based on this, there are three different transistor arrangements or configurations namely common emitter, common base and common collector configurations. The symbolic representation of these configurations are shown in Fig. 4.8.



(a) Common emitter configuration b) Common collector configuration (c) Common base configuration

Fig. 4.8 Symbolic representation of transistor configurations

The input and output parameters of transistors in different configurations are listed Table 4.2.

Table 4.2 The input and output parameters of transistors

Parameters	Configuration		
	Common-emitter	Common-base	Common-collector
Input voltage	V_{BE}	V_{EB}	V_{BC}
Input current	I_B	I_E	I_B
Output voltage	V_{CE}	V_{CB}	V_{EC}
Output current	I_C	I_C	I_E

4.10 THE EBERS MOLL MODEL

Fig. 4.9 shows the Ebers Moll model of a p-n-p BJT. This is a large signal model representing BJT in any mode of operation. The dc terminal currents in any mode of operation can be evaluated using this model. The different symbols used are:

I_{ES} - Reverse saturation current of emitter-base diode with collector shorted to base.

I_{CS} - Reverse saturation current of collector-base diode with emitter shorted to base.

α_F - Forward alpha is α of the transistor when emitter-base junction is forward-biased and collector-base junction is reverse-biased.

α_I - Inverse alpha is α of the transistor when collector-base junction is forward-biased and emitter-base junction is reverse-biased. In this case collector acts as the emitter and emitter as the collector.

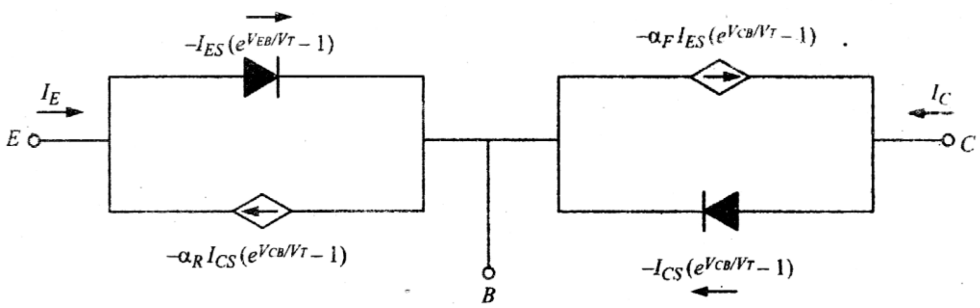


Fig. 4.9 Ebers Moll Model of a p-n-p BJT

I_{ES} and I_{CS} are negative for p-n-p transistor and positive for n-p-n transistor. In terms of these parameters the terminal currents are given by

$$I_E = \left[I_{ES}\left(e^{V_{EB}/V_T}-1\right)-\alpha_I I_{CS}\left(e^{V_{CB}/V_T}-1\right)\right] \tag{4.36}$$

$$I_C = \left[I_{CS}\left(e^{V_{CB}/V_T}-1\right)-\alpha_F I_{ES}\left(e^{V_{EB}/V_T}-1\right)\right] \tag{4.37}$$

Equations (4.36) and (4.37) are known as Ebers Moll equations.

By comparing equations (4.36) and (4.37) with equations (4.24) and (4.26) and equating coefficients of $\left(e^{V_{CB}/V_T}-1\right)$ and $\left(e^{V_{EB}/V_T}-1\right)$ respectively we get,

$$\alpha_F I_{ES} = \alpha_I I_{CS} = \text{qA}\,\frac{n_i^2}{N_D}.\frac{D_p}{L_p}\cos ech\left(\tfrac{W_B}{L_p}\right) \tag{4.38}$$

A setup to measure I_{ES} and α is shown in Fig. 4.10. Short-circuit the collector and base terminals of the transistor. Plot the forward characteristics of emitter-base junction (V_{EB} vs. I_E). The reverse saturation current of emitter-base junction under this condition represents I_{ES} The procedure to determine reverse saturation current is explained in Section 3.5.3.

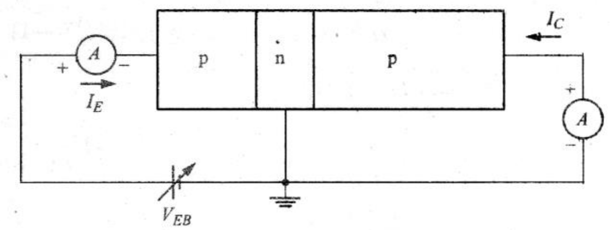


Fig. 4.10 Set-up to measure I_{ES} and α_F

Since, the collector and base are short-circuited, the ratio of I_C and I_E directly gives α_F.

$$\alpha_F = \left.\frac{I_C}{I_E}\right|_{V_{CB}=0}$$

I_{CS} and α can be obtained from the set-up shown in Fig. 4.11.

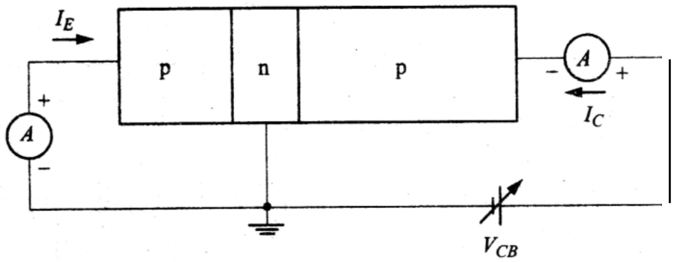


Fig. 4.11 Set-up to measure I_{CS} and α_I

I_{CS} is the reverse saturation current of collector-base junction.

$$\alpha_I = \left.\frac{I_E}{I_C}\right|_{V_{EB}=0}$$

The currents I_{ES} and I_{CS} may be expressed in terms of I_{EBO} and I_{CBO}.

where, I_{EBO} - reverse saturation current of emitter-base junction with collector-base junction open ($I_C = 0$).

I_{CBO} - reverse saturation current of collector-base junction with emitter-base junction open ($I_E = 0$).

Multiplying equation (4.36) by α_F

$$\alpha_F . I_E = -\alpha_F I_{ES}\left(e^{V_{EB}/V_T}-1\right)+\alpha_F\alpha_I I_{CS}\left(e^{V_{CB}/V_T}-1\right) \quad (4.39)$$

Adding equations (4.37) and (4.39), we get

$$I_C + \alpha_F . I_E = -I_{CS}(1-\alpha_F \alpha_I)\left(e^{V_{CB}/V_T}-1\right) \quad (4.40)$$

With emitter open, $I_E = 0$ and I_C is given by

$$I_C = -I_{CS}(1-\alpha_F \alpha_I)\left(e^{V_{CB}/V_T}-1\right)$$

$$= -I_{CBO}\left(e^{V_{CB}/V_T}-1\right)$$

where $I_{CBO} = I_{CS}(1-\alpha_F \alpha_I)$

or $\quad I_{CS} = \dfrac{I_{CBO}}{1-\alpha_F\alpha_I}$ (4.41)

Similarly, $\quad I_{ES} = \dfrac{I_{EBO}}{1-\alpha_F\alpha_I}$ (4.42)

I_{EBO} may be measured experimentally using the set-up shown in Fig. 4.12.

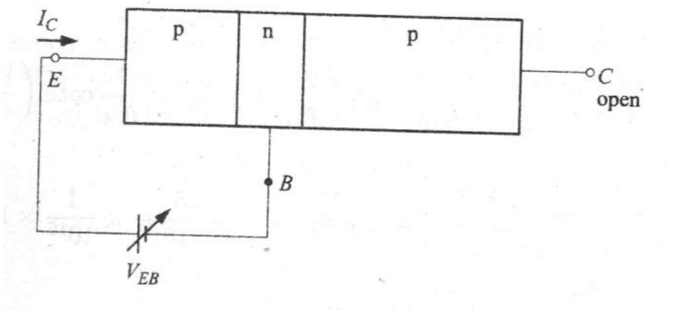


Fig. 4.12 Set-up to measure I_{EBO}

Plot the forward characteristics of emitter-base junction with collector open. Determine the reverse saturation current which represents I_{EBO}. I_{CBO} can be measured using a similar setup.

Example 4.4 The following parameters are given for an n-p-n transistor at 300 K.

	Emitter	**Base**	**Collector**
doping	10^{19} cm^{-3}	10^{16} cm^{-3}	10^{15} cm^{-3}
width	2μm	1μm	5μm
minority carrier lifetime	0.01μs	0.06μs	1μs
mobility	μp= 350 cm^2/Vs	μn = 1250 cm^2/Vs	μp= 450 cm^2/ Vs

Assume uniform area of cross-section of 5×10^{-4} cm^2 and n_i = = 1.5×10^{10} cm^{-3}. Determine I_{ES}, I_{CS}, α_F and α_I.

Solution

Emitter	**Base**	**Collector**
$N_{DE} = 10^{19}$ cm^{-3}	$N_A = 10^{16}$ cm^{-3}	$N_{DC} = 10^{15}$ cm^{-3}
D_{pE}=9.1cm^2/s	D_n=32.5cm^2/s	D_{pC}=11.7cm^2/s
$L_{pE} = 3.02 \times 10^{-4}$ cm	$L_n = 1.4 \times 10^{-3}$ cm	$L_{pC} = 3.42 \times 10^{-3}$ cm

$$\coth\left(\frac{W_B}{L_n}\right) = 14.02$$

$$\coth\left(\frac{W_E}{L_{pE}}\right) = 1.725$$

$$\coth\left(\frac{W_C}{L_{pC}}\right) = 6.888$$

Using equations (4.25) and (4.26)

$$I_{ES} = qAn_i^2\left[\frac{D_n}{L_n}\times\frac{1}{N_A}\coth\left(\frac{W_B}{L_n}\right)+\frac{D_{pE}}{L_{pE}}\times\frac{1}{N_{DE}}\coth\left(\frac{W_E}{L_{pE}}\right)\right]$$

$$= 1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 2.25 \times 10^{20}$$

$$= \left[\frac{32.5}{1.4\times10^{-3}}\times\frac{1}{10^{16}}\times14.02+\frac{9.1}{3.02\times10^{-4}}\times\frac{1}{10^{19}}\times1.725\right]$$

$$= 5.859 \times 10^{-13}\ \text{A}$$

$$I_{CS} = qAn_i^2\left[\frac{D_{pC}}{L_{pC}}\times\frac{1}{N_{DC}}\coth\left(\frac{W_C}{L_{pC}}\right)+\frac{D_n}{L_n}\times\frac{1}{N_A}\coth\left(\frac{W_B}{L_n}\right)\right]$$

$$= 1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 2.25 \times 10^{20}$$

$$= \left[\frac{11.7}{3.42\times10^{-3}}\times\frac{1}{10^{15}}\times6.888+\frac{32.5}{1.4\times10^{-3}}\times\frac{1}{10^{16}}\times14.02\right]$$

$$= 1.009 \times 10^{12}\ \text{A}$$

$$\alpha_F = \frac{1}{1+\frac{D_{pE}}{D_n}\times\frac{N_A}{N_{DE}}\times\frac{W_B}{W_E}+\frac{1}{2}\left(\frac{W_B}{L_n}\right)^2} \quad \text{(by equation 4.34)}$$

$$= \frac{1}{1+\frac{9.1}{32.5}\times\frac{10^{16}}{10^{19}}\times\frac{1\times10^{-4}}{2\times10^{-4}}+\frac{1}{2}\left(\frac{1\times10^{-4}}{1.4\times10^{-3}}\right)^2} = 0.9973$$

$$\alpha_I = \frac{1}{1+\frac{D_{pC}}{D_n}\times\frac{N_A}{N_{DC}}\times\frac{W_B}{W_C}+\frac{1}{2}\left(\frac{W_B}{L_n}\right)^2} \quad \text{(by equation 4.34)}$$

$$= \frac{1}{1+\frac{11.7}{32.5}\times\frac{10^{16}}{10^{15}}\times\frac{1\times10^{-4}}{5\times10^{-4}}+\frac{1}{2}\left(\frac{1\times10^{-4}}{1.4\times10^{-3}}\right)^2} = 0.5805.$$

4.11 REGIONS OF OPERATION AND MINORITY CARRIER DISTRIBUTION

The minority carrier distribution in the base region represents the major portion of currents through a transistor. The minority carrier distribution in a p-n-p transistor in different regions (modes) of operation are shown in Fig. 4.13.

Forward active mode: In this mode of operation emitter-base junction is forward-biased (V_{EB} positive) and collector-base junction is reverse-biased (V_{CB} negative). Therefore, $\Delta_{pC} = -p_{Bo}$, $\Delta_{nC} = -n_{Co}$. The common-base current gain in this mode is represented as α_F and its value is close to unity.

Inverse active mode: In this case collector-base junction is forward-biased and emitterbase junction is reverse-biased. Therefore $\Delta_{pE} = -p_{Bo}$, and $\Delta_{nE} = -n_{Eo}$. Collector acts as the emitter and the emitter as collector. Because of low doping in collector region, the injection efficiency is poor in this mode. Due to smaller emitter area, all the charge carriers that are emitted from the collector do not reach the emitter. They recombine in the remote region of base (See the actual structure of BJT in Fig. 4.2). As a result the transport factor is also poor. Thus α of the transistor is low in this mode.

Saturation mode: In this case both junctions are forward-biased and holes are injected from emitter and collector into the base region. This increases the recombination rate in the base, increasing I_{rB}. Therefore, the base current is large $\left(I_B >> \frac{I_C}{\beta}\right)$. The resistance of the transistor is very low in this mode, as two forward-biased junctions are connected in series opposition. The transistor acts as a closed switch and it is said to be in the ON condition. The voltage drop across the transistor is very low under this condition. ($V_{CE} = V_{BE} + V_{CB}$) (for p-n-p transistor in saturation region V_{BE} is negative and V_{CB} is positive).

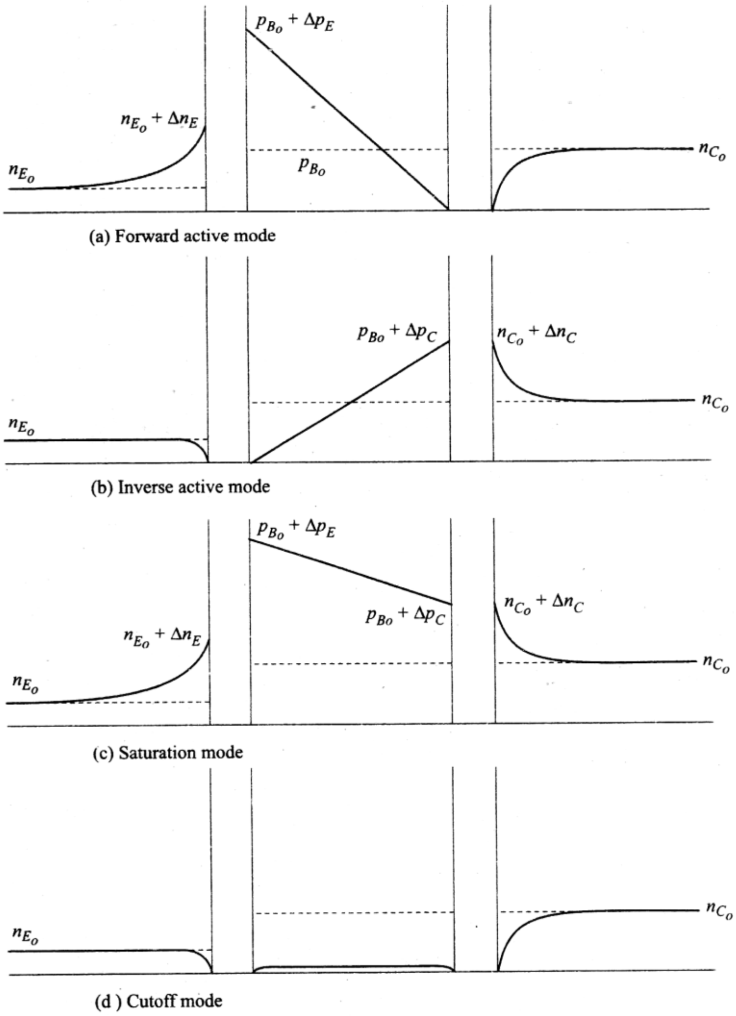


Fig. 4.13 Minority carrier distribution of a p-n-p transistor

Cutoff mode: In cutoff mode both junctions are reverse-biased. $\Delta_{nE} = -n_{Eo}$, $\Delta_{pE} = \Delta_{pC} = -p_{Bo}$ and $\Delta_{nC} = -n_{Co}$. The base region does not consist charge carriers and the resistance of the device is very high. It acts as an open switch and the transistor is said to be in the OFF condition.

4.12 REAL TRANSISTOR

In the discussion so far, we have assumed an ideal transistor with abrupt junctions, equal junction areas, uniform doping etc. In a real transistor, we have to account for the following non-idealities.

(1) Carrier recombination in emitter-base depletion layer.
(2) Drift in the base region (non-uniform doping).
(3) Effects of variation of V_{CB} on transistor currents (early effect).
(4) Avalanche multiplication in collector-base junction.
(5) Resistance of the base region.
(6) Non-ideal structure.
(7) Kirk effect.

4.12.1 Carrier Recombination in the Emitter-Base Depletion Layer

The emitter current consists of the recombination current in emitter-base depletion region given by

$$I_R = I_{R_o}\left(e^{V_{EB}/2V_T} - 1\right) \tag{4.43}$$

$$I_{R_o} = \frac{qAn_i X_{EB}}{2\tau_o} \tag{4.44}$$

where, X_{EB} - depletion layer width of emitter-base junction.
τ_o - carrier lifetime in the emitter-base depletion layer.
A - area of emitter-base junction.

∴ The expression for injection efficiency becomes

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En} + I_R}$$

I_R term reduces the injection efficiency, especially in silicon and gallium arsenide transistors at or below room temperature as I_R term is much larger than the other terms. The injection efficiency is small for small forward-bias on base emitter junction.

As V_{EB} and I_C increases, I_{R_o} term becomes negligible compared to other terms and injection efficiency increases with increase in I_C. Therefore α and β increases with increase in I_C. But at very high value of I_C where the injected hole concentration in the base become comparable to base doping, β starts to decrease with increase in I_C due to high injection effects. Variation of I_B and I_C with V_{EB} at $V_{CB} = 0$ is shown in Fig. 4.14(a). This plot is known as Gummel plot. An approximate plot of β as a function of I_C for a constant V_{CE} is shown in Fig. 4.14(b).

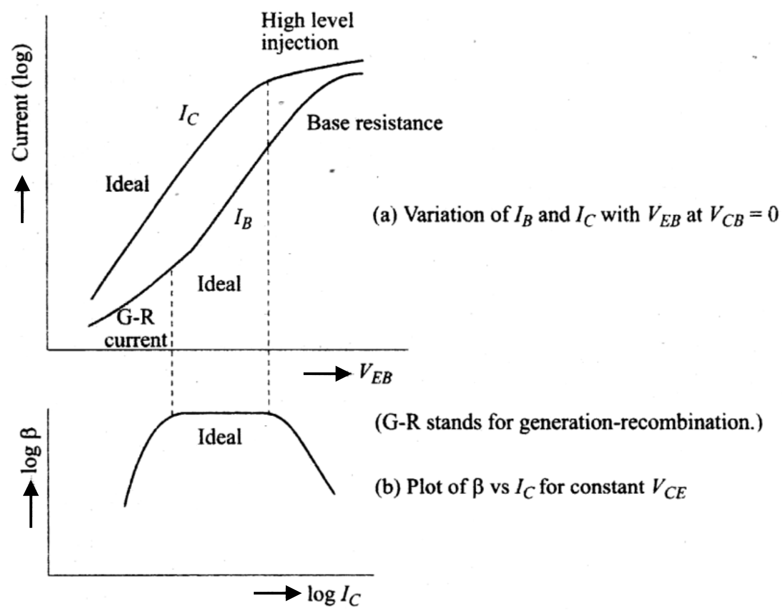


Fig. 4.14 Gummel plot

4.12.2 Drift in the Base Region

For implanted junction transistors the assumption of uniform doping in the base does not hold good as there is appreciable amount of impurity grading. $N_D(x)$ varies exponentially within the base region. In graded base region, a built-in electric field exists from emitter to collector (for p-n-p transistor). This adds a drift component to the transport of holes across the base. The drift and diffusion currents in the base must balance at equilibrium. If the net donor doping of the base is large enough that $n(x) \cong N_D(x)$, for the balance of electron drift and diffusion currents at equilibrium

$$I_n(x) = 0$$

i.e,

$$qA\mu_n N_D(x)E(x) + qAD_n \frac{dN_D(x)}{dx} = 0 \tag{4.45}$$

∴ Built-in electric field

$$E(x) = -\frac{D_n}{\mu_n} \cdot \frac{1}{N_D(x)} \cdot \frac{dN(x)}{dx} = \frac{-kT}{q} \frac{1}{N_D(x)} \frac{dN_D(x)}{dx} \tag{4.46}$$

For a donor doping profile that decreases in the positive x direction, the built-in electric field is positive directed from emitter to collector.

This electric field aids the transport of holes across the base region from emitter to collector. Therefore the transit time T(is reduced below that of a uniformly doped base transistor. Similarly electron transport in an n-p-n transistor is aided by the built-in field in the base. This reduction in transit time finds its application in high frequency devices.

4.12.3 Effect of Bias to Collector-Base Junction (Early Effect)

The effective width (W_B) of the base region is the difference between the total (metallurgical) base width (W_{Bo})and the depletion layer width of the collector-base junction into the base region (neglecting the depletion layer on the emitter base junction).

$$\text{i.e,} \quad W_B = W_{Bo} - X_{BC} \cdot \frac{N_{AC}}{N_D + N_{AC}}$$

$$= W_{Bo} - \sqrt{\frac{2\in}{q}(V_o - V_{BC} \times \frac{N_{AC}}{N_D(N_{AC}+N_D)}} \tag{4.47}$$

where X_{BC} - depletion width of collector base junction with bias.

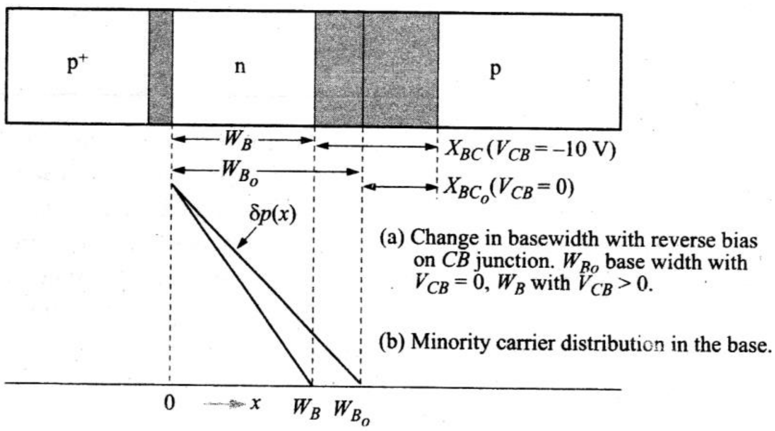


Fig. 4.15 Early effect

Equation (4.47) shows that the effective width of the base region decreases with increase in reverse-bias on the collector-base junction. This is called base width modulation or Early effect.

Due to base width modulation, as reverse-bias on the collector base junction increases I_C and I_E increases. Due to increase in V_{CB}, W_B decreases which increases the slope of minority carrier distribution as shown in Fig. 4.15. As the slope increases, I_{Ep} and I_{Cp} increases.

As the width of the base is reduced, the transport factor increases or the recombination in the base region reduces, reducing I_B and increasing α of the transistor. As α increases, β also increases. In short, due to base width modulation, an increase of reverse-bias across collector-base junction:

(1) increases I_C and I_E
(2) decreases I_B
(3) increases transport factor
(4) increases α and β

Punch Through

As V_{CB} increases further, the depletion layer penetrates more and more into the base and the effective width of base decreases and become zero at a collector-base reverse voltage called punch through voltage (V_{PT}) (if avalanche breakdown do not occur below this voltage). At punch through voltage width of base is zero.

$\therefore$ From equation (4.47)

$$V_{PT} = \frac{qN_D(N_{AC} + N_{DB})W_{Bo}^2}{2\epsilon N_{AC}} \tag{4.48}$$

Example 4.5 A p-n-p silicon BJT has $N_D = 8 \times 10^{15}$ cm^{-3} on the base and the collector is heavily doped. Given that W_{Bo} = 2 μm. Determine the breakdown voltage in the common-base mode if the critical field in silicon is 3×10^5 V/cm. Also calcualte the punch through voltage.

Solution

$$V_{BrCBO} = \frac{\epsilon}{2qN_D} E_{crit}^2 \qquad \text{(by equation (3.89))}$$

$$= \frac{8.854 \times 10^{-14} \times 11.8 \times \left(3 \times 10^5\right)^2}{2 \times 1.6 \times 10^{-19} \times 8 \times 10^{15}}$$

$$= 36.73 \text{ V}$$

$$V_{PT} = \frac{qN_D}{2\epsilon} W_{Bo}^2 \text{ (by equation (4.48) for } N_{AC} >> N_D\text{)}$$

$$= \frac{1.6 \times 10^{-19} \times 8 \times 10^{15} \times \left(2 \times 10^{-4}\right)^2}{2 \times 8.854 \times 10^{-14} \times 11.8}$$

$$= 24.50 \text{ V.}$$

4.12.4 Avalanche Multiplication in the Collector-Base Depletion Region

Avalanche multiplication takes place within the depletion layer of collector-base junction. At low V_{CB} multiplication will be negligible. But with increase in reverse-bias multiplication in the collector-base depletion layer increases. Due to this α of the transistor increases and the collector current also increases for a given I_E.

The current entering depletion layer of collector-base junction = I_{pC}

Current reaching the collector $= I_C$

$= -(I_{pC} - I_{CBO})$

Multiplication factor M
$$= \frac{-(I_{pC} - I_{CBO})}{I_{pC}} \tag{4.49}$$

Avalanche multiplication factor is also given by

$$M = \frac{1}{1 - \left(\frac{V_{CB}}{V_{Br}}\right)^n}$$

where n lies between 2 and 7.

$$\alpha = M\alpha_T\gamma$$

M is unity for low V_{CB}. When V_{CB} becomes closer to avalanche breakdown voltage, M increases and I_C increases sharply causing avalanche breakdown.

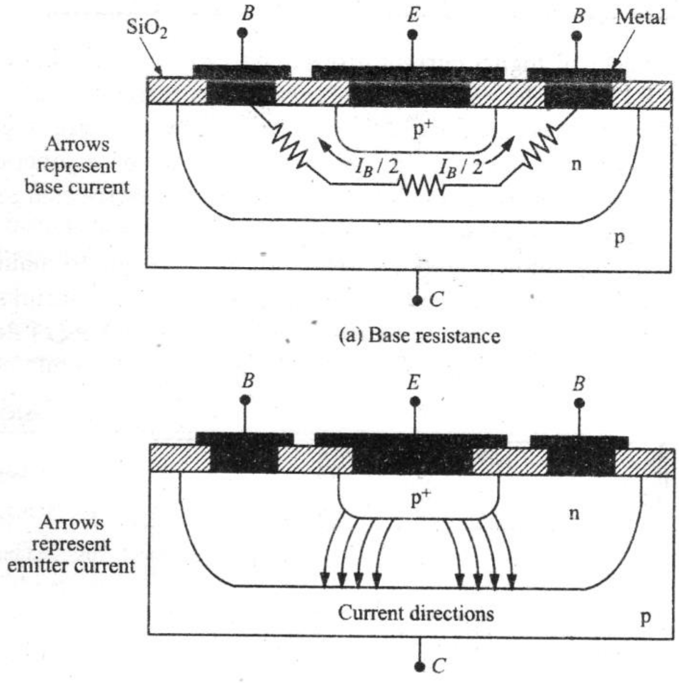
4.12.5 Resistance of the Base Region and Emitter Crowding

In the actual structure, base region has a large area compared to emitter and the resistance of the base is distributed over this region. The effective resistance of the base may be calculated from the doping of base region and its geometry. This resistance is called base spreading resistance r_B. Because of r_B the entire applied voltage will not drop across the junction. In the Ebers Moll model V_{EB} and V_{CB} must be replaced by V_{EB} - $I_B r_B$ and V_{CB} - $I_B r_B$ respectively. The base current is a lateral current flowing from the active base region to the base contact.

The base resistance increases with reverse-bias on collector-base junction due to increase in depletion layer width and reduced base width. The emitter-base forward bias is minimum at the centre of the emitter and increase towards the periphery due to reduced $I_B r_B$ drop. Therefore emitter injection is more near the periphery of the emitter than that at the centre. This is called emitter crowding and is shown in Fig. 4.16. Inter digitated emitter structure as shown in Fig. 4.17 reduces the problem of emitter crowding which may cause premature breakdown.

4.12.6 Non-Ideality of Structure

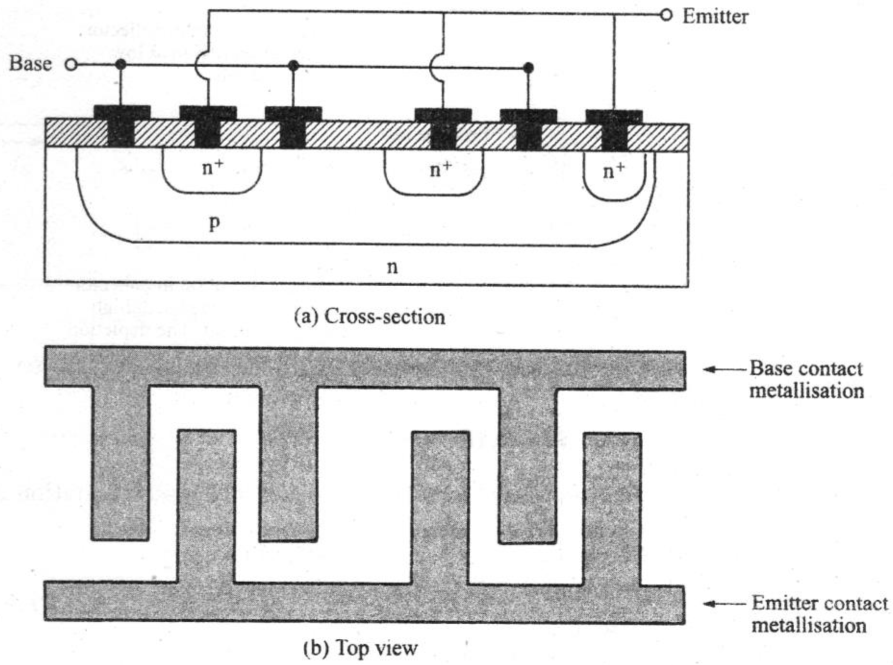
For the purpose of analysis we have assumed equal area for emitter and collector junctions which is actually not true. The area of emitter-base junction is less than that of collector base junction. Similarly, the doping of emitter is higher than that of collector. I_{CS} is higher than I_{ES} and α_F is higher than α_I.



(a) Base resistance

(b) Emitter current crowding

Fig. 4.16 Cross-section of p-n-p transistor showing



(a) Cross-section

(b) Top view

Figure 4.17 Interdigitated emitter structure

4.12.7 Kirk Effect

The current gain of BJT at higher current, drops due to a mechanism known as Kirk effect. Consider a p-n-p BJT. As the emitter-base forward bias increases, more charge carriers (holes) are injected into the base region and collected by collector region. Under this condition, depletion approximation is invalid in the collector-base junction. The positive charge on the base side of the depletion layer increases and negative charge on the collector side of the depletion layer decreases due to presence of holes as shown in Fig. 4.18.

Thus, for a given V_{CB}, fewer donors are required on the n-side to maintain the voltage across the junction. Therefore, the depletion layer on the base side shrinks and base width increases, reducing transport factor and current gain. The shrinking of depletion layer on the base side of collector-base junction due to the presence of mobile charge carriers is called Kirk effect.

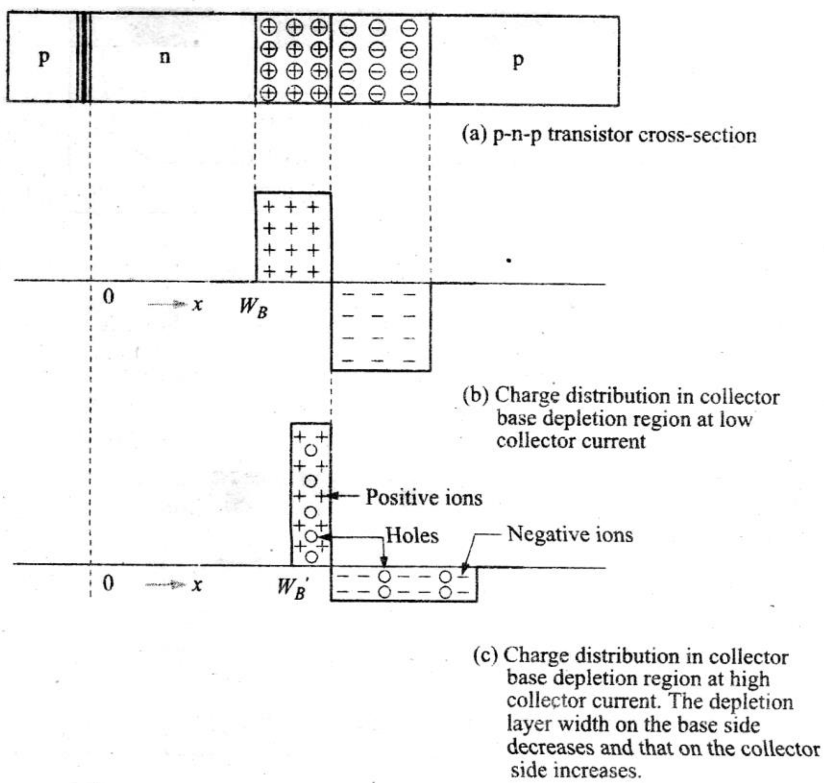


Fig. 4.18 Kirk effect

The electric field in the depletion layer may be expressed by Poisson equation as

$$\frac{dE}{dx} = \frac{q}{\epsilon}\left[\left(N_D^+ - N_A^-\right) + \frac{I_C}{qAv_d}\right] \quad \text{on the base side} \qquad (4.50)$$

where I_C - collector current

A - area of cross-section of collector-base junction

v_d - drift velocity of holes.

The last term represent mobile charge carrier density

$$\frac{I_C}{A} = pqv_d$$

$$p = \frac{I_C}{qAv_d} \tag{4.51}$$

Increase in I_C has the same effect as increasing doping level on the n-side (base) and (reducing doping level on the p-side. Kirk effect is observed in n-p-n transistors also.

4.13 TRANSISTOR CHARACTERISTICS

4.13.1 Common-base configuration

a. Input characteristics

The input characteristics of a p-n-p bipolar junction transistor in common-base configuration is shown in Fig. 4.19(b). It is a plot of I_E as a function of V_{EB} with V_{CB} held constant. By Ebers Moll equation, with reverse-bias on collector-base junction,

$$I_E = -I_{ES}\left(e^{V_{CB}/V_T} - 1\right) - \alpha_I I_{CS}$$

I_{ES} and I_{CS} are negative for p-n-p transistor and I_E is positive.

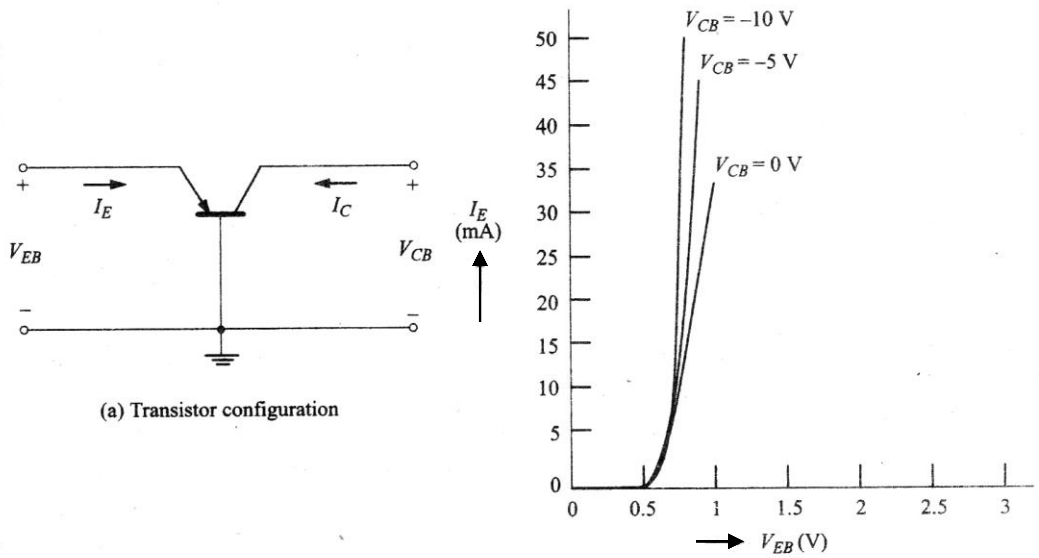


Fig. 4.19 Input characteristics of a p-n-p transistor in common base configuration

Even though the shape of this characteristics is similar to that of p-n junction, a change in V_{CB} shifts the characteristics. With increase in V_{CB}, the reverse-bias on collector-base junction increases, increasing depletion layer width and reducing base width. Reduced base width causes an increase in the slope of minority carrier distribution in the base region increasing I_C and I_E. Therefore, with increase in V_{CB} for a given V_{EB}, the emitter current increases. Thus the curve shifts towards the left with increase in V_{CB}. This is a consequence of early effect.

b. Output characteristics

The output characteristics of a p-n-p bipolar junction transistor is shown in Fig. 4.20. The output characteristics is a plot of I_C as a function of V_{CB} with I_E kept constant. From Ebers Moll model, by equation (4.40),

$$I_C = -\alpha_F I_E - I_{CBO}\left(e^{V_{CB}/V_T} - 1\right)$$

When $I_E = 0$, $I_C = I_{CBO}$ for sufficiently large collector-base reverse-bias V_{CB}.
With reverse-bias on collector-base junction,

$$I_C = -\alpha_F I_E - I_{CBO}$$

For a given I_E, I_C remains almost constant.
With forward bias on collector-base junction (V_{CB} positive) I_C decreases due to injection of holes from collector to base (slope of minority carrier distribution decreases). I_C becomes zero for a finite forward bias on collector base junction (V_{CB}) and is given by

$$V_{CB}\,|_{I_C=0} = V_T \ln\left[1 - \frac{\alpha_F I_E}{I_{CBO}}\right] \qquad (4.52)$$

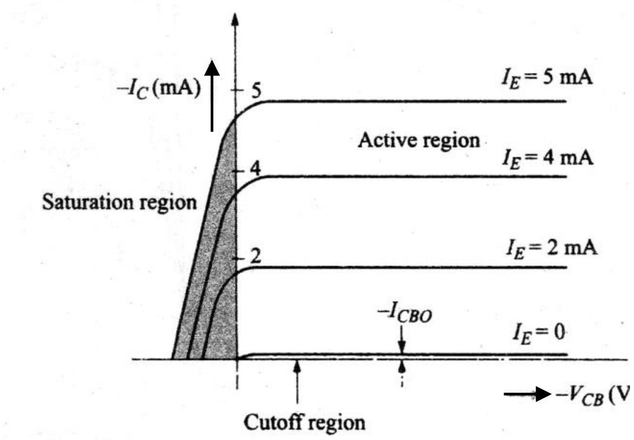


Fig. 4.20 Output characteristics of p-n-p transistor in common base configuration

The output characteristics for a single I_E and the minority carrier distribution in the base region for different points in the characteristics are shown in Fig. 4.21.

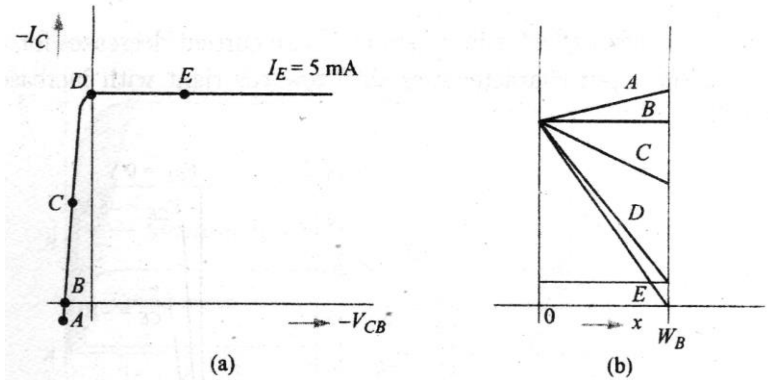


Fig. 4.21 (a) Output characteristics for a single I_E and (b) the minority carrier distribution in the base region for different points in the characteristics, neglecting change in base width

A - Inverse saturation

B - $I_C = 0$

C - Forward saturation, deep saturation

D - Forward saturation, weak saturation

E - Forward active

At point A, the forward bias on the collector-base junction is high compared to that on the emitter-base junction and it corresponds to inverse saturation. At point B, the injection from collector to base and emitter to base are equal and $I_C = 0$. Points C and D corresponds to forward saturation. C represents deep saturation and D weak saturation. Point E corresponds to active region of operation.

In the active region, collector current remains almost constant and the effect of base width modulation is negligible. In common-base configuration I_C and I_E increases with increase in V_{CB}. The increase in I_C is more compared to increase in I_E ($\Delta I_E = \Delta I_C - \Delta I_B$). But on keeping I_E constant, the change in I_C almost nullifies. Therefore, I_C increases only slightly with increase in V_{CB} in the output characteristics.

4.13.2 Common-Emitter Configuration

a. Input characteristics

Input characteristics in common-emitter configuration is the plot of I_B as a function of V_{BE}, keeping V_{CE} constant. From Ebers moll equations, by assuming large reverse-bias on collector-base junction

$$I_B = I_{ES}(1 - \alpha_F)\left(e^{V_{EB}/V_T} - 1\right) - I_{CS}(1 - \alpha_I) \qquad (4.53)$$

I_{ES} and I_{CS} are negative for p-n-p transistor and I_B is therefore negative.

Even though I_B appears to be independent of V_{CB} in equation 4.53, it decreases with increase in V_{CB} due to base width modulation. (With increase in V_{CE} ($V_{CE} = V_{BE} + V_{CB}$) base width reduces, transport factor increases and base current decreases for a given V_{BE} or I_E). Therefore, the input characteristics shift towards right with increase in V_{CE} as shown in Fig. 4.22b.

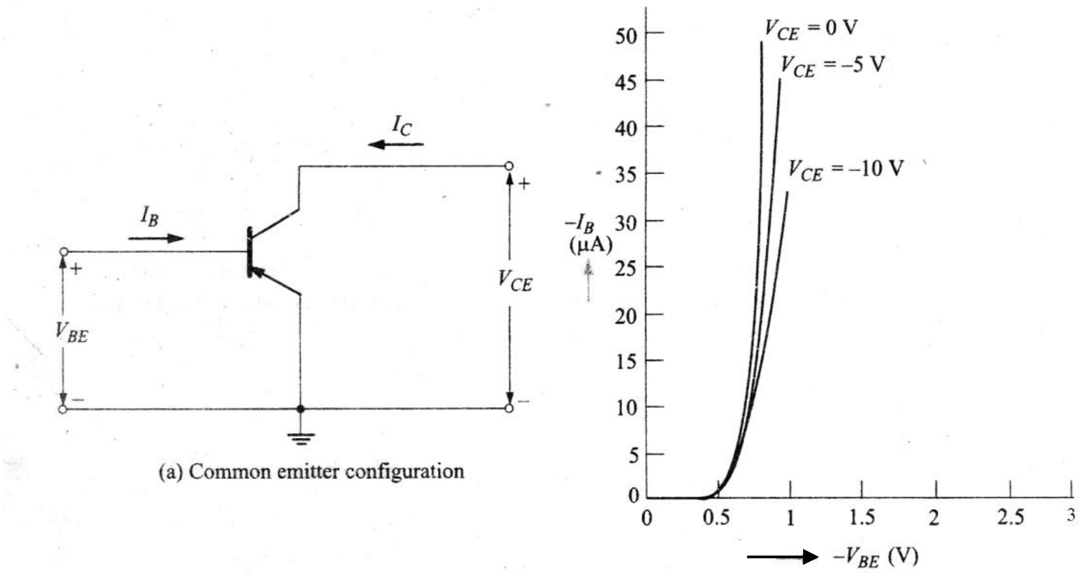


Fig. 4.22 Input characteristics of an p-n-p transistor in common emitter configuration

b. Output characteristics

The output characteristics in common-emitter configuration is a plot of I_C as a function of V_{CE} keeping I_E constant. The output characteristics, the minority carrier distribution in the base region for different points (marked in the characteristics) and expanded view of characteristics in saturation region are shown in Fig. 4.23.

$$V_{CE} = V_{CB} + V_{BE}$$
$$V_{BE} = V_{CE} - V_{CB}$$

When $V_{CE} = 0$; $V_{CB} = V_{BE}$, the collector-base junction and emitter-base junction are equally forward-biased. Under this condition injection from collector to base and emitter to base are equal and the collector current is zero.

With increase in V_{CE}, I_C increases due to reduced forward bias on collector-base junction upto point C. On further increasing V_{CE}, the collector-base junction become reverse-biased and the transistor enters the active region of operation. In this region, collector current increase slowly with increase in V_{CB} due to base width modulation.

When I_B is increased, the resistance between collector and emitter decrease with more injection of charge carriers into the base and slope of the characteristics in the active region increases.

When $I_B = 0$; $I_C = I_{CEO} = (1 + \beta)\, I_{CBO}$ (See Problem 4.1)

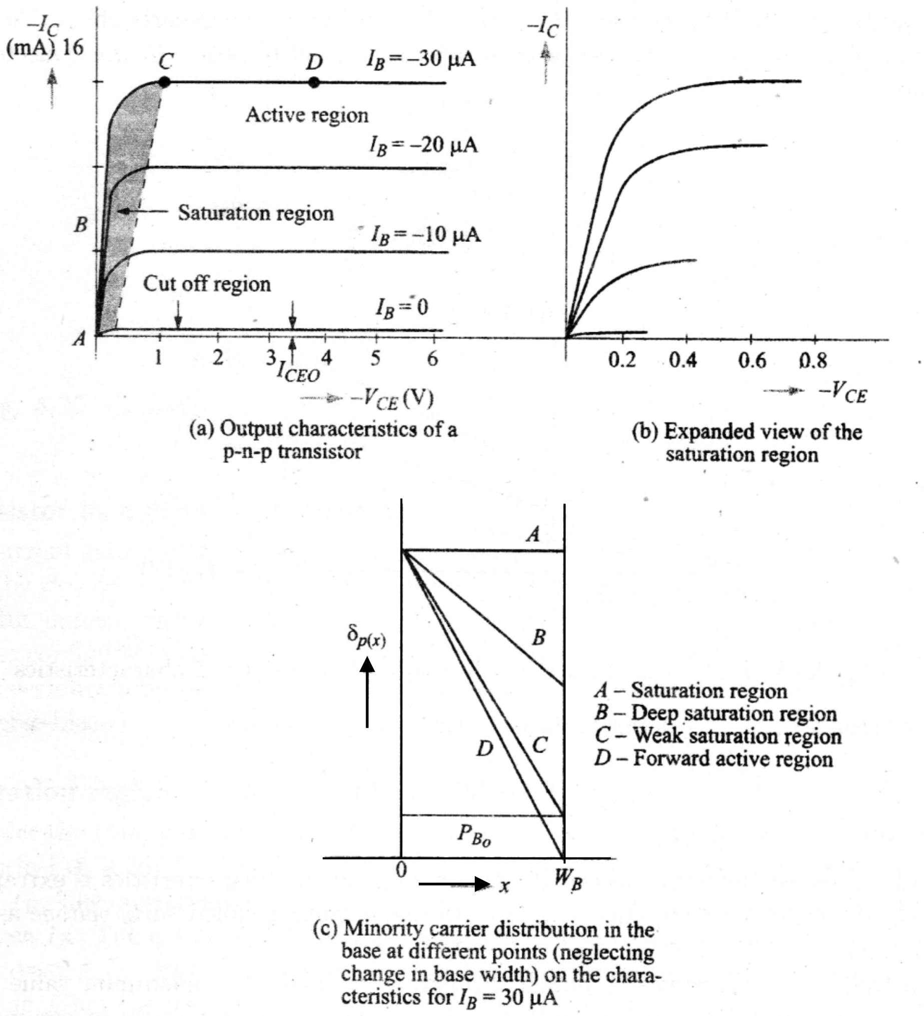


Fig. 4.23

I_{CEO} is the current between collector and emitter with base terminal open. This current is much higher than the reverse saturation current of collector-base junction with emitter open (I_{CBO}). It can be explained as follows.

With reverse-bias on collector-base junction, electrons drift from collector to base region. These electrons cannot move into the base terminal as it is open. Electrons accumulate in the base region and this produces a small forward bias on the emitter-base junction. Holes are injected from emitter to base and are collected by the collector terminal causing an increased current between collector and emitter.

Effect of Base Width Modulation

With increase in V_{CB}, I_C increases due to increased slope of minority carrier distribution in the base region.

In real transistors the increase of I_C in the active region is due to early effect. (At higher values of V_{CE}. the slope increases due to avalanche multiplication in the collector-base junction.)

$$I_C = -qAD_p \cdot \frac{dp}{dx}$$

$$= qAD_p \cdot \frac{\Delta_p E}{W_B}$$

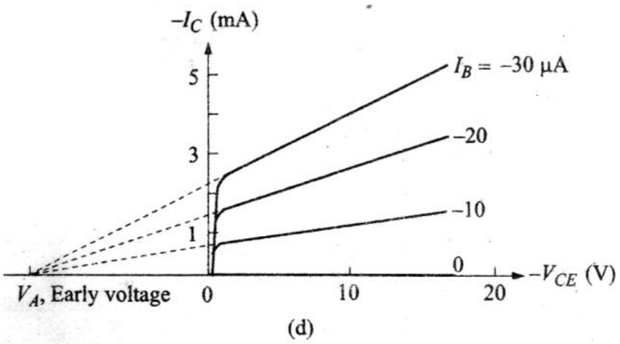


Fig. 4.24 The effect of base width modulation on output characteristics; I_C increases due to reduction in base width as V_{CB} increases.

$$I_C \;;\; I_S \left(e^{V_{EB}/V_T} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

where V_A is called the eariy voltage. If the common-emitter characteristics is extrapolated to meet voltage axis, all the curves meet at the same voltage called early voltage as shown in Fig. 4.24.

In a well designed transistor punch through is avoided. The maximum value of V_{CB} is decided by avalanche breakdown. The avalanche breakdown voltage in common-base configuration (V_{BrCBO}) and common-emitter configuration (V_{BrCEO}) are related as

$$V_{BrCEO} = \left(\frac{V_{BrCBO}}{(1+\beta)^{\frac{1}{n}}}\right) \tag{4.54}$$

n varies from 2 to 7.

V_{BrCEO} - Breakdown voltage in common-emitter configuration with base open
V_{BrCBO} - Breakdown voltage in common-base configuration with emitter open

Equation (4.54) shows that the breakdown voltage in common-base configuration is much higher than that in common-emitter configuration.

The effect of avalanche break down in the output characteristics of p-n-p transistor in common emitter configuration is shown in Fig. 4.25.

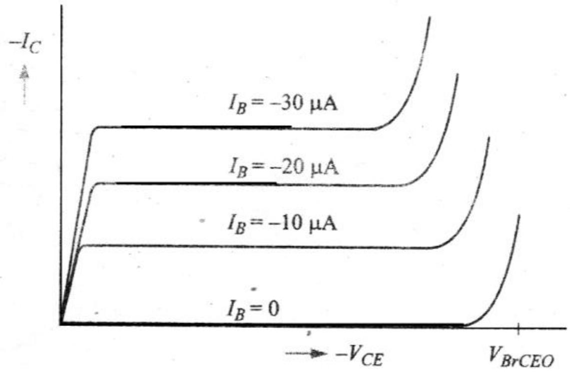


Fig. 4.25 Common emitter output characteristics showing the effect of avalanche multiplication in collector-base junction

Transistor as a Linear Current Amplifier

The current gain of a transistor in common-emitter configuration in forward active mode is given by $\beta = \frac{I_C}{I_B}$ which is independent of the bias voltage V_{EB} or V_{CB} .i.e., $I_C = \beta I_B$. The collector current varies linearly with base current I_B. Therefore, transistor in common emitter configuration acts as a linear current amplifier when it is operated in active mode or active region where the emitter-base junction is forward-biased and collector base junction is reverse-biased.

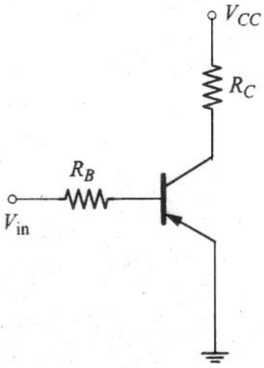
Saturation Region

Consider the common-emitter configuration of transistor and its output characteristics as shown in Fig. 4.26. The load line is fixed by V_{CC} and R_C. The operating point is fixed by R_B or I_B. The operating point is the intersection of load line and the output characteristics for given I_B. The maximum value of collector current is I_C (sat) which cannot be increased by increasing I_B. But in active region I_C can be increased by increasing I_B for a given load line. Once the transistor is in saturation region, I_C is saturated to its maximum value irrespective of the base current. Therefore "saturation" implies the saturation of collector current for a given circuit arrangement or load line. This current is fixed by external resistance (R_C) and voltage applied (V_{CC}). Therefore in saturation region $I_C \leq \beta I_B$.

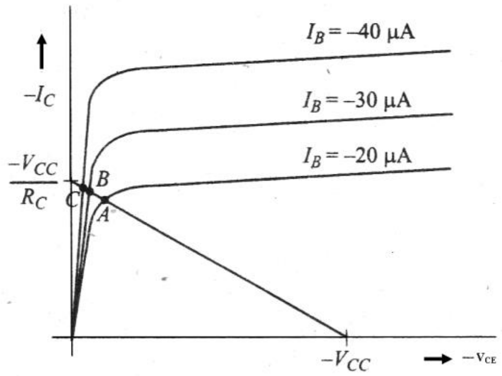
4.14 SWITCHING

An ideal switch has zero resistance (short-circuit) in ON condition and infinite resistance (open-circuit) in OFF condition. A BJT in common emitter configuration acts as a switch when it is operated in saturation and cutoff regions. In saturation region it acts as closed switch (low-resistance) and in cutoff region as an open switch (high-resistance).

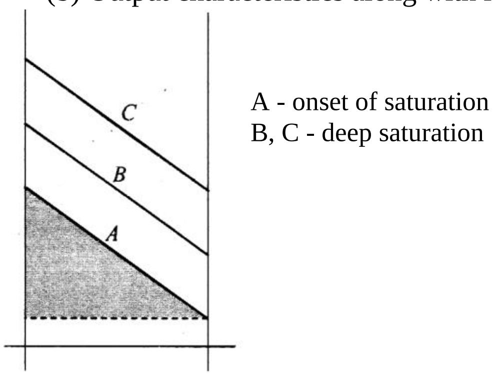
Fig. 4.27 shows the circuit arrangement of BJT as a switch and its characteristics. When I_B = 0, the BJT is in cutoff as shown by point A in the figure. The voltage drop V_{CE} is very large under this condition.



(a) Circuit arrangement



(b) Output characteristics along with load line



(c) Minority carrier distribution in the base region

Fig. 4.26 Operation of a p-n-p BJT in saturation region. As I_B is increased from 20 μA to 40 μA. the operating point is moved from A to C. The change in collector current is negligible

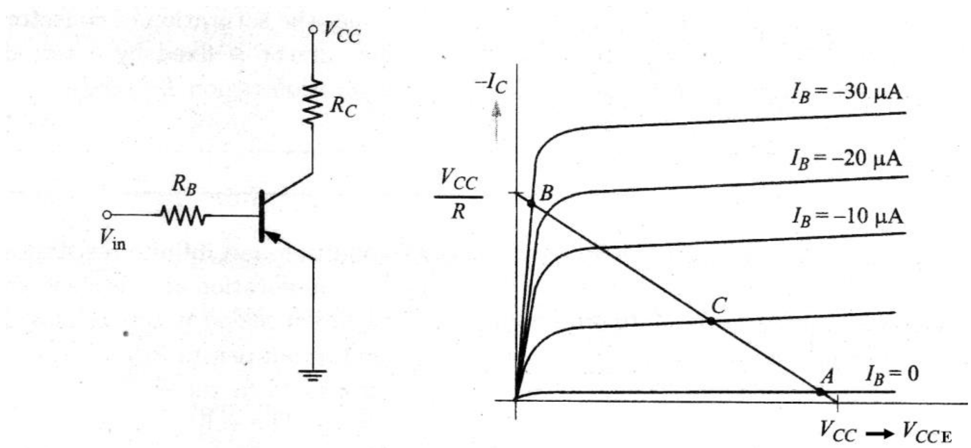


Fig. 4.27 Circuit arrangement and characteristics of BJT as a switch

If $|I_B|$ is made large $\left(|I_B| > \left|\frac{I_{C(sat)}}{\beta}\right|\right)$ the collector current reaches a maximum value for the given load line. At the onset of saturation (point B in the figure)

$$I_B = \frac{I_{C(sat)}}{\beta}$$

If I_B is increased further, the collector current remains constant (saturates). There is no further shift in operating point. Under this condition V_{CE} is very small and both junctions are forward-biased. This is the ON condition of the transistor.

A sudden change of I_B from zero to a high value will not switch the transistor from OFF state to ON state instantaneously. This requires the charging of storage (diffusion) capacitance which introduces a time delay in switching. The storage capacitance is due to the storage of minority carriers, mainly in the base region. The distribution of minority carriers in the base region under cut-off, saturation and active modes of operation are shown in Fig. 4.28.

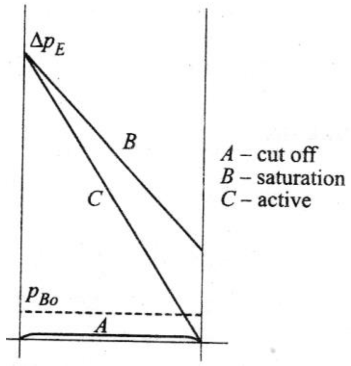


Fig. 4.28 Distribution of minority carriers in the base region of p-n-p transistor (A) cutoff (B) saturation and (C) active modes of operation

The time delay may also be represented in terms of the time of transit of the charge carriers (τ_{tB}) across the base. The collector current is related to the excess hole density by

$$I_C = -qAD_p \cdot \frac{d}{dx}\delta p(x)$$

$$= qAD_p \cdot \frac{\Delta_{pE}}{W_B} \tag{4.55}$$

The total excess holes stored in the base is

$$Q_B = -qA \cdot \frac{1}{2}\Delta_{pE} \cdot W_B$$

$$= \frac{qA\Delta_{pE}W_B}{2} \tag{4.56}$$

The collector current is equal to the rate of transfer of charge from base to collector and is given by

$$|I_C| = \frac{Q_B}{\tau_{tB}} \tag{4.57}$$

where τ_{tB} is the time of transit of charge carriers (holes) through the base.

$$\tau_{tB} = \left|\frac{Q_B}{I_C}\right| = \frac{\dfrac{qA\Delta_{pE}W_B}{2}}{\dfrac{qA\cdot D_p\cdot\Delta_{pE}}{W_B}} \quad \text{(from equations (5.55) and (4.56))}$$

$$= \frac{W_B^2}{2D_p} \tag{4.58}$$

Base current due to recombination in the base is given by

$$\left|I_{r\beta}\right| = \frac{Q_B}{\tau_p}$$

where τ_p is the minority carrier lifetime in the base region.

4.14.1 Charge Control Equations

The base current in the forward mode of operation $\left(I_B^F\right)$ is given by

$$-I_B^F = I_{rB} + I_{nE} + I_{CBO}$$

$$= I_{rB} + I_{nE} \quad \text{(neglecting } I_{CBO}\text{)}$$

$$= Q_B^F + I_{nE} = \frac{Q_B^F}{\tau_p}\left[1 + \frac{I_{nE}}{I_{rB}}\right]$$

$$\equiv \frac{Q_E^F}{\tau_{B(eff)}} \tag{4.59}$$

where $\tau_{B(eff)} = \dfrac{\tau_p}{1+\dfrac{I_{nE}}{I_{rB}}}$, effective lifetime in the base. (4.60)

$$\frac{I_C^F}{I_B^F} = \beta^F = \frac{\tau_{B(eff)}^F}{\tau_{tB}^F} \quad \text{(by equations (4.57) and (4.59))}$$

$$; \ \frac{\tau_p}{\tau_{tB}} \quad \text{(if } I_{nE} \text{ is neglected)}$$

If a time varying voltage υ_{EB} is applied between base and emitter, the base current and stored charge will also vary with time. Therefore, i_B^F will consist of a term due to the time variation of stored charge.

$$-i_B^F = \frac{Q_B^F}{\tau_{B(eff)}^F} + \frac{dQ_B^F}{dt} \tag{4.61}$$

The collector current in the forward mode is given by

$$-i_C^F = \frac{Q_B^F}{\tau_t^F} \tag{4.62}$$

and emitter current is given by

$$i_B^F = -\left(i_C^F + i_B^F\right)$$

$$= \frac{Q_B^F}{\tau_t^F} + \frac{Q_B^F}{\tau_{B(eff)}^F} + \frac{dQ_B^F}{dt} \tag{4.63}$$

Similarly the currents in inverse mode may be expressed in terms of charge stored in the base under inverse mode.

$$\text{Charge stored } Q_B^I = \frac{qAW_B \cdot \Delta_{pC}}{2} \tag{4.64}$$

$$-i_B^I = \frac{Q_B^I}{\tau_{B(eff)}^I} + \frac{dQ_B^I}{dt} \tag{4.65}$$

$$-i_E^I = \frac{Q_B^I}{\tau_t^I} \tag{4.66}$$

$$i_B^I = \left(i_B^I + i_E^I\right)$$

$$= \frac{Q_B^I}{\tau_{B(eff)}^I} + \frac{dQ_B^I}{dt} + \frac{Q_B^I}{\tau_t^I} \tag{4.67}$$

$$i_B = i_B^F + i_B^I \tag{4.68a}$$

$$i_C = i_C^F + i_C^I \tag{4.68b}$$

$$i_E = i_E^F + i_E^I \tag{4.68c}$$

4.14.2 Tum-ON-Time

Let a p-n-p bipolar junction transistor be switched from cut-off (I_B =0) to saturation ($I_B = I_{B1}$) at t = 0.

In equation (4.61), replacing $\tau_{B(eff)}^F$ by τ_p it may be written as

$$-i_B = \frac{dQ_B}{dt} + \frac{Q_B}{\tau_p} \tag{4.69}$$

Solution of this equation by applying boundary condition $Q_B = 0$ at t = 0 is

$$Q_B(t) = I_{B1}\tau_p(I - e^{-t/\tau_p}) \tag{4.69a}$$

The collector current is given by

$$i_C(t) = \frac{Q_B(t)}{\tau_{tB}}$$

$$= \frac{I_{B1}\tau_p(1 - e^{-t/\tau_p})}{\tau_{tB}} \text{ for } Q_B \le Q_{B(sat)} \tag{4.70}$$

$Q_{B(sat)}$ is the stored charge when $i_C = I_{C(sat)}$.

The charge Q_B continues to build-up with increase in base current but the collector current remain almost unchanged at $I_{C(sat)}$. Therefore equation (4.70) is valid only for $Q_B \le Q_{B(sat)}$. The variation of stored charge with time, the output characteristics along with load line and the variation of I_C with time are shown in Fig. 4.29.

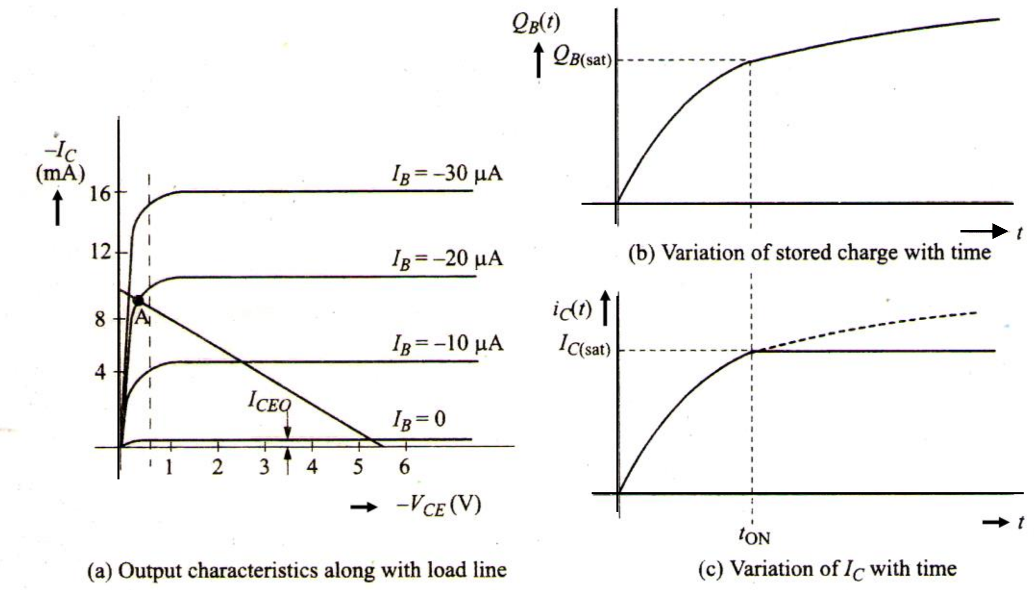


Fig. 4.29 Variation of stored charge and I_C with time.

The stored charge in the base region at point A on the load line is represented by $Q_{(sat)}$. Equation (4.70) can be written in steady-state ($t \Rightarrow 4\text{-}\ \infty$) as

$$i_C = \frac{I_{B1}\tau_p}{\tau_{tB}} = I_{B1} \times \beta_F \tag{4.71}$$

But on further increasing I_B the transistor is driven into deep saturation. The corresponding increase in I_C is negligible. Therefore in saturation region $I_C \leq \beta_F I_B$.

In saturation region, collector current is limited to

$$I_{C(sat)} = \frac{V_{CC} - V_{CE(sat)}}{R_C} \;;\; \frac{V_{CC}}{R_C} \tag{4.72}$$

($V_{CE(sat)}$ for silicon transistor is around 0.2 V. It is the difference in forward-bias between the emitter-base and collector-base junctions.)

In saturation region, the charge stored in the base region Q_B increases with increase in base current even though I_C remains constant. A transistor is assumed to be turned ON as I_C reaches $I_{C(sat)}$. Therefore equation (4.70) becomes (substituting for $Q_{B(sat)}$ from equation 4.69(a) by making $t = t_{ON}$)

$$I_{C(sat)} = \frac{Q_{B(sat)}}{\tau_{tB}} = \frac{I_{B_1}}{\tau_{tB}} \tau_p (1 - e^{-t_{ON}/\tau_p})$$

$$\therefore t_{ON} = \tau_p 1_n \left[\frac{1}{1 - \left(\frac{I_{C(sat)}}{I_{B_1}}\right)\left(\frac{\tau_{tB}}{\tau_p}\right)} \right] \tag{4.73}$$

Equation (4.73) shows that to minimise turn ON time, τ_p must be minimum, I_B must be maximum and I_C must be minimum, and transit time (τ_{tB}) must be minimum.

4.14.3 Turn-OFF Time

To turn OFF a transistor, the excess stored charge in the base has to be removed and collector current has to be reduced to zero. This is done by setting I_B = 0 or negative. When the base current is removed, the stored charge decay due to recombination and diffusion as there is no source to replenish it.

When I_B = 0, equation (4.69) becomes

$$\frac{dQ_B}{dt} = \frac{-Q_B}{\tau_p} \tag{4.74}$$

i.e., Q_B decreases with time. Transistor remains in ON condition until Q_B goes below $Q_{B(sat)}$. The turn-off transients are shown in Fig. 4.30. As Q_B reaches $Q_{B(sat)}$, I_C decays exponentially to zero.

The turn-off time is the time required to reduce the collector current to zero, from the instant the base current is removed. It is constituted by two components:

(i) time taken to reduce the stored charge to $Q_{B(sat)}$ known as storage delay (t_s) and

(ii) the time taken to reduce the stored charge from $Q_{(sat)}$ to 0.1 $Q_{(sat)}$, known as fall time (t_f).

Let the base current be made zero at t = 0. Therefore, the solution of equation (4.74) is

$$Q_B(t) = Q_B(0)e^{-t_s/\tau_p} \tag{4.75}$$

where $Q_B(0)$ is the stored charge in the base just before removal of base current pulse.

The storage delay time tg is obtained from equation (4.75) as

$$Q_B(t_s) = Q_{B(sat)} = Q_B(0)e^{-t/\tau_p}$$

$$t_s = \tau_p 1_n \left(\frac{Q_B(0)}{Q_{B(sat)}} \right) \tag{4.76}$$

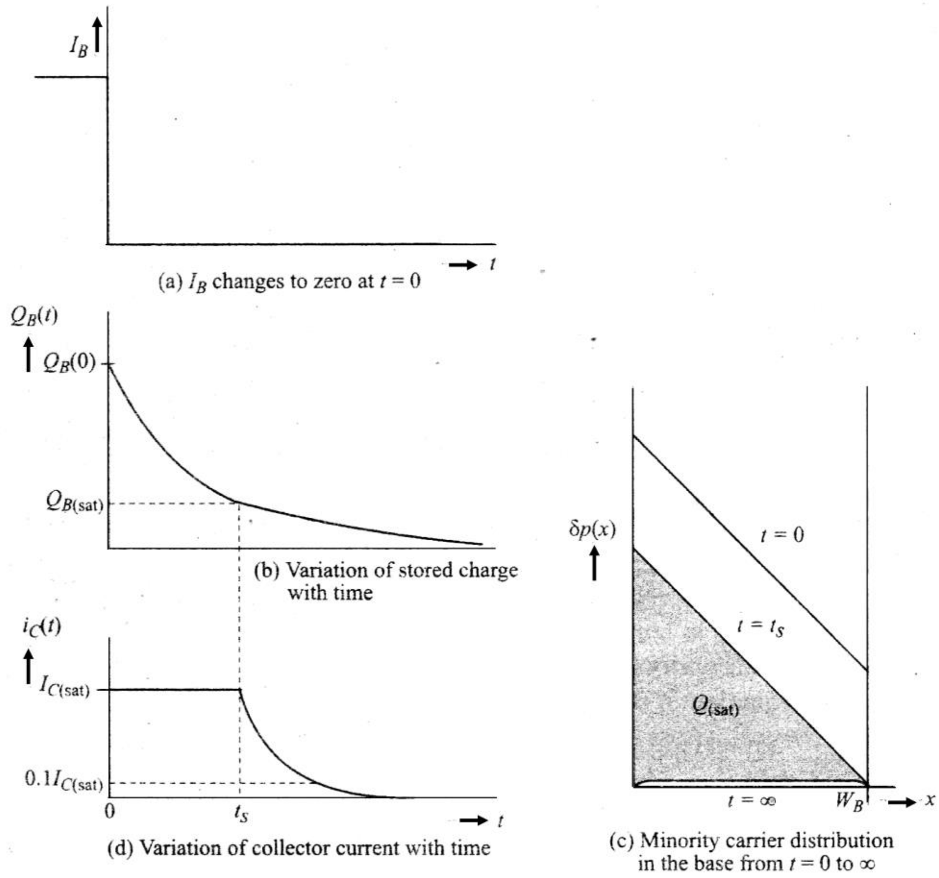


Fig. 4.30 Turn off transient

For $t > t_s$ transistor is in active region.

$$\therefore i_C(t) = \frac{Q_B(t)}{\tau_{tB}} = \frac{Q_{B(sat)}e^{-t/\tau_p}}{\tau_{tB}} \qquad \text{Where } t' = (t - t_s)$$

$$= I_{C(sat)}\ e^{-t'/\tau_p} \tag{4.77}$$

From equation (4.77) the time t_f for $i_C(t)$ to fall from $I_{C(sat)}$ to $0.1\ I_{C(sat)}$ is $2.3\ \tau_p$

$$\left[\begin{array}{l} \text{When} \quad t = t_s + t_f\,, \quad t' = t_f \text{ and } i_C(t) = 0.1I_{C(sat)} \\ \therefore \quad 0.1I_{C(sat)} = I_{C(sat)}e^{-t'/\tau_p}\ \therefore t_f = -\tau_p \ln(0.1) = 2.3\tau_p \end{array}\right]$$

Smaller lifetime (τ_p) and smaller I_B in ON condition reduces the turn OFF time. τ_p can be reduced in switching transistor by adding gold which increases the recombination rate.

Example 4.6 A base current pulse of 100 µA and width 300 ns is applied to the base of a switching transistor shown in Fig. Ex.4.6(a).

Given τ_{PB} = 0.25 µs, W_B = 2.5 µm and D_p = 6 cm^2/s. Sketch the waveforms of the collector current and the charge Q_B.

Solution

$$I_{B1} = 100\ \mu A$$

$$I_{C(sat)} \cong \frac{V_{CC}}{R_L} = \frac{-10}{10\times10^{-3}} = -1 mA$$

From equation (4.58)

$$\tau_{tB} = \frac{W_B^2}{2D_p} = \frac{(2.5\times10^{-4})^2}{2\times6} = 5.2 \times 10^{-9} s$$

$$Q_{B(sat)} = I_{C(sat)}\ \tau_{tB}$$
$$= 1 \times 10^{-3} \times 5.2 \times 10^{-9} = 5.2 \times 10^{-12}\ C$$

Turn ON time is the time at which $Q_B = Q_{B(sat)}$. By equation (4.73),

$$t_{ON} = \tau_p \ln\left[\frac{1}{1-\left(\frac{V_{CC}}{I_{B_1}R_L}\right)\left(\frac{\tau_t}{\tau_p}\right)}\right]$$

$$= 0.25 \times 10^{-6} \ \text{In} \left[\frac{1}{1-\frac{10}{100\times10^{-6}\times10\times10^{3}}\times\frac{5.2\times10^{-9}}{0.25\times10^{-6}}}\right]$$

$$= 5.83 \times 10^{-8}\ s$$

After $t = t_{ON}$, I_C remains constant at 1 mA and Q_B continues to increase until the base current is removed as shown in Fig. Ex.4.6.
By equation (4.69(a)),

$$QB(t) = I_{B_1}\tau_p\left[1-e^{-t/\tau_p}\right]$$

$$Q_B(t = 300\ ns) = 100 \times 10^{-6} \times 0.25 \times 10^{-6}\left[1-e^{-300\times10^{-9}/0.25\times10^{-6}}\right]$$

$$= 1.75 \times 10^{-11}\ C$$

The storage delay time by equation (4.75) is

$$t_s = \tau_p \ln\left[\frac{Q_B(0)}{Q_{B(sat)}}\right]$$

where $Q_B(0) = Q_B(t = 300\ ns)$

$$t_s = 0.25 \times 10^{-6} \ln\left[\frac{1.75\times10^{-11}}{5.2\times10^{-12}}\right] = 0.303\ \mu s$$

$$t_f = 2.3\ \tau_p = 575\ ns.$$

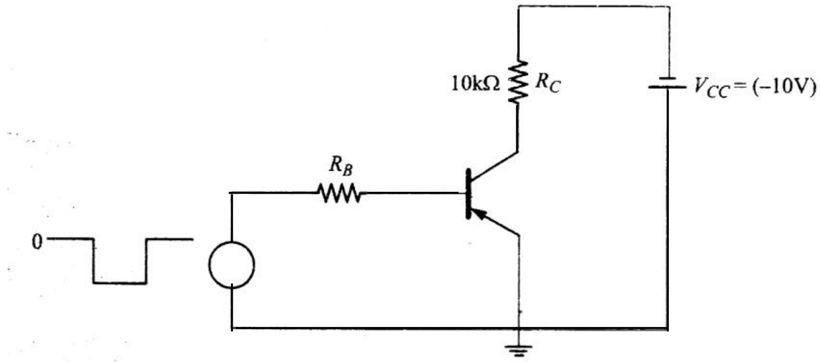


Fig. Ex.4.6(a)

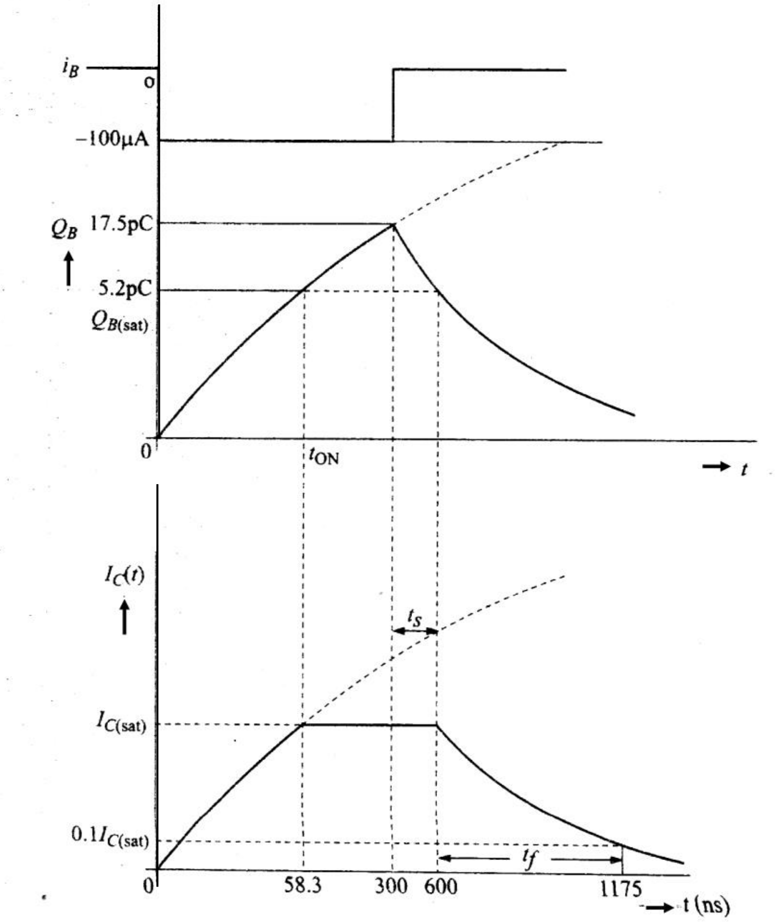


Fig. Ex.4.6(b)

4.15 SMALL SIGNAL EQUIVALENT CIRCUIT

The low-frequency small signal equivalent circuit of a BJT is shown in Fig. 4.31. In this equivalent circuit r_π represents the resistance offered by the base emitter junction to base current and is called small signal base resistance. g_mV_{BE} represents the change in collector current as a result of change in base emitter voltage.

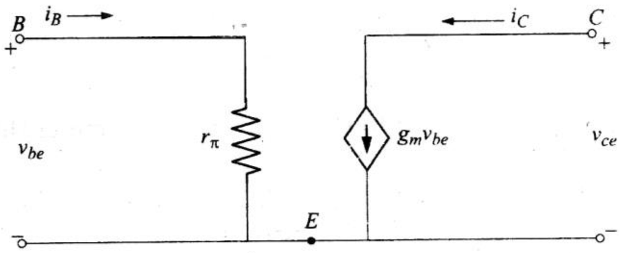


Fig. 4.31 Low-frequency small signal equivalent circuit of BJT

Transconductance $g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T}$

$$r_\pi = \frac{1}{\left(\frac{dI_C}{dV_{BE}}\right)} = \frac{\partial I_C}{\partial I_B} \cdot \frac{\partial I_{BE}}{\partial I_C} = \frac{\beta}{g_m}$$

High-frequency small signal equivalent circuit of BJT is shown in Fig. 4.32.

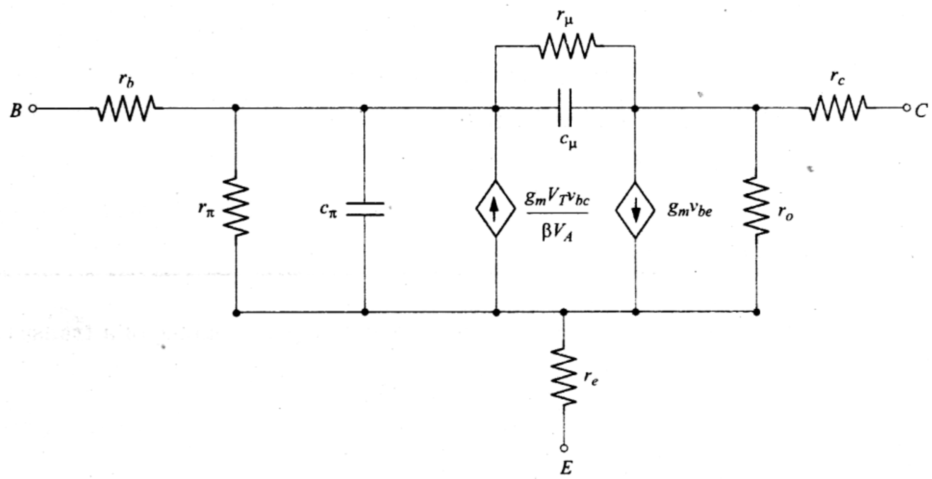


Fig. 4.32 High-frequency small signal equivalent circuit of BJT

The different elements of the equivalent circuit are as follows.

Transconductance $g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T}$

$$r_\pi = \frac{1}{\left(\frac{dI}{dV_{BE}}\right)} = \frac{\beta}{gm}$$

$$C_\pi = C_B + C_{jE}$$

$C_B = g_m\tau_{tB}$ is the base charging capacitance which is the capacitance due to change in stored charge in the base with change in V_{BE}.

C_{jE} is the capacitance of the emitter-base junction depletion layer.

$$C_\mu = \frac{-dQ_B}{dV_{CE}} = \tau_{tB}.\frac{I_C}{V_T} = \frac{V_T}{V_A} C_B$$

C_μ represents the capacitance due to change in stored charge in the base with change in V_{CB}.
r_μ accounts for the change in base current due to V_{CB}.

$$r_o = \frac{1}{\left(\frac{dI_C}{dV_{CE}}\right)} = \frac{V_A}{I_C}$$

r_o is the common-emitter output resistance and V_A. is the early voltage.
$\frac{g m V_T v_{CB}}{\beta_0 V_A}$ is the change in base recombination current resulting from a change in V_{CB}.
$g_m v_{BE}$ is the change in collector current caused by a change in V_{BE}.

r_b - Ohmic (bulk) resistance of the base region.
It is also known as base spreading resistance.

r_e - Ohmic resistance of emitter region

r_c - Ohmic resistance of collector region

4.16 FIGURE OF MERIT

Figure of merit (f_T) is a measure of the high-frequency performance of a transistor an is given by

$$f_T = \alpha f_\alpha = \beta f_\beta$$

where α - common-base current gain

f_α - cut-off frequency in common-base configuration (α-cutoff frequency)

β - common-emitter current gain

f_β - cut-off frequency in common-emitter configuration (β-cutoff frequency)

f_β - is also equal to the unity gain frequency i.e., $f_T = f_\beta$ if $|\beta(\omega)| = 1$.

The equivalent circuit of a high-frequency transistor may be modified as that in Fig. 4.33 by neglecting r_μ, r_c, r_e, and r_b.

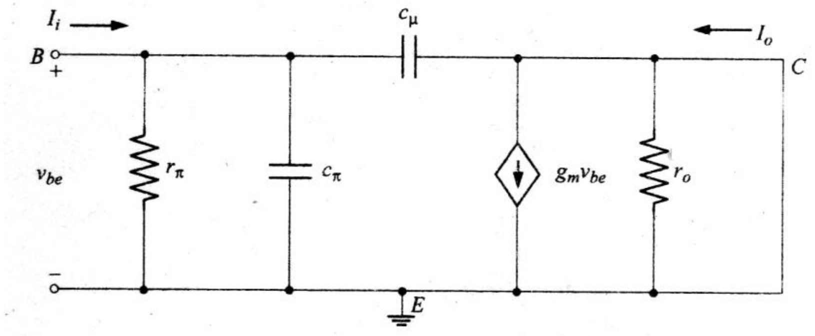


Fig. 4.33 Circuit arrangement to evaluate figure of merit of a BJT

The short-circuit current gain $\beta(\omega)$ is determined as $\frac{I_o}{I_i}$ by placing a short-circuit at the output.

$$I_o = g_m v_{be}$$

$$I_i = \frac{V_{be}}{\left[\dfrac{1}{\frac{1}{r_\pi} + jw(C_\pi + C_\mu)}\right]} = \frac{v_{be}[1 + j\omega r_\pi (C_\mu + C_\pi)}{r_\pi}$$

$$\therefore \quad \frac{I_o}{I_i} = \beta(\omega) = \frac{g_m r_\pi}{|1 + jwr_\pi (C_\mu + C_\pi)|}$$

At high-frequency the magnitude of the imaginary part is large

$$|\beta(\omega)| = \frac{g_m r_\pi}{\omega r_\pi (C_\mu + C_\pi)}$$

$$= \frac{g_m}{\omega [C_\mu + C_\pi]} \tag{4.78}$$

when $f = f_T$ $|\beta(\omega)| = 1$; $\omega = 2\pi f_T$

i.e.,
$$\frac{g_m}{2\pi f_T (C_\mu + C_\pi)} = 1$$

$$\therefore \quad f_T = \frac{g_m}{2\pi [C_\pi + C_\mu]} \tag{4.79}$$

Example 4.7 Consider a silicon p-n-p transistor operating at T = 300 K with I_C = 1 mA, V_{EB} = 0.7 V, V_{BC} = 5 V. The device has β = 200, $N_{DB} = 10^{17}$ cm^{-3}, $N_{AC} = 10^{16}$ cm^{-3}, W_B = 0.8 μm, V_A = 100 V, D_{PB} = 10 cm^2, $N_{AE} = 10^{18}$ cm^{-3}, $A = 10^{-5}$ cm^2.
Determine g_m, r_π, r_o, C_{jE}, C_B, C_π, C_μ and f_T.

Solution

$$g_m = \frac{I_C}{V_T}$$

$$= \frac{1 \times 10^{-3}}{0.026} = 38.46 \times 10^{-3}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$= \frac{200}{38.46 \times 10^{-3}} = 5.2 \times 10^3\ \Omega$$

$$r_o = \frac{V_A}{I_C}$$

$$= \frac{100}{1 \times 10^{-3}} = 10^5\ \Omega$$

$$V_{EB_o} = \frac{kT}{q} \ln\left(\frac{N_{AE} N_D}{n_i^2}\right)$$

$$= 0.026 \ln\left(\frac{10^{18} \times 10^{17}}{(1.5\times 10^{10})^2}\right) = 0.877V$$

$$W_{EB} = \sqrt{\frac{2\in(V_{EBo} - V_{EB})}{q}\left(\frac{1}{N_{AE}} + \frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2\times 8.854\times 10^{-14}\times 11.8(0.877\text{-}0.7)}{1.6\times 10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{17}}\right)}$$

$$= 5.04 \times 10^{-6} \text{ cm}$$

$$C_{jE} = \frac{11.8\times 8.854\times 10^{-14}\times 10^{-5}}{5.04\times 10^{-6}}$$

$$= 20.72 \times 10^{-13} \text{ F}$$

$$\tau_{tB} = \frac{W_B^2}{2D_p}$$

$$= \frac{(0.8\times 10^{-14})^2}{2\times 10} = 3.2 \times 10^{-10} \text{ s}$$

$$C_B = g_m \tau_{tB}$$

$$= 38.46 \times 10^{-3} \times 3.2 \times 10^{-10}$$

$$= 1.23 \times 10^{-11} \text{ F}$$

$$C_\mu = \tau_{tB}.\ \frac{I_C}{V_A}$$

$$= 3.2 \times 10^{-10} \times \frac{1\times 10^{-3}}{100}$$

$$= 3.2 \times 10^{-15} \text{F}$$

$$C_\mu = C_B + C_{jE}$$

$$= 1.23 \times 10^{-11} + 20.72 \times 10^{-13}$$

$$= 1.437 \times l6^{-11} \text{ F}$$

$$f_T = \frac{g_m}{2\pi[C_\pi + C_\mu]}$$

$$= \frac{38.46\times 10^{-3}}{2\pi\left[1.437\times 10^{-11} + 3.2\times 10^{-15}\right]}$$

$$= 4.258 \times 10^8 \text{ Hz.}$$

Equations Modified for npn BJT

Emitter injection efficiency $\gamma = \dfrac{I_{nE}}{I_{nE} + I_{pE}}$

Base transport factor $\alpha_T = \dfrac{I_{pC}}{I_{pC} + I_{rB}}$

$I_{nE} = qA \dfrac{D_n}{L_n}\left[\Delta_{nE}\coth\left(\dfrac{W_B}{L_n}\right) - \Delta_{pC}\cos ech\left(\dfrac{W_B}{L_n}\right)\right]$

$I_{pE} = qA \dfrac{D_{pE}}{L_{pE}}\left[\Delta_{pE}\coth\left(\dfrac{W_E}{L_{pE}}\right)\right]$

$I_{nC} = qA \dfrac{D_n}{L_n}\left[\Delta_{nE}\cos ech\left(\dfrac{W_B}{L_n}\right) - \Delta_{pC}\coth\left(\dfrac{W_B}{L_n}\right)\right]$

$I_{pC} = -\,qA \dfrac{D_{pC}}{L_{pC}}\left[\Delta_{pC}\coth\left(\dfrac{W_C}{L_{pC}}\right)\right]$

$\Delta_{nE} = n_{B0}\left(e^{V_{EB}/V_T} - 1\right)$

$\Delta_p E = p_{E0}\left(e^{V_{EB}/V_T} - 1\right)$

$\Delta_p C = p_{C0}\left(e^{V_{CB}/V_T} - 1\right)$

$\Delta_n C = p_{C0}\left(e^{V_{CB}/V_T} - 1\right)$

$\gamma = \dfrac{1}{1 + \frac{D_{pE}}{D_n} \times \dfrac{N_A}{N_{DC}} \times \frac{W_B}{W_E}}$

$\alpha_T = \dfrac{1}{\cosh\left(\frac{W_B}{L_n}\right)} = \dfrac{1}{1 + \dfrac{1}{2}\left(\frac{W_B}{L_n}\right)^2}$, if $W_B << L_n$

$\alpha = \dfrac{1}{1 + \frac{D_{pE}}{D_n} \times \dfrac{N_A}{N_D e} \times \frac{W_B}{W_E}\ \dfrac{1}{2}\left(\frac{W_B}{L_n}\right)^2}$

$\beta = \dfrac{1}{1 + \frac{D_{pE}}{D_n} \times \dfrac{N_A}{N_D e} \times \frac{W_B}{W_E}\ \dfrac{1}{2}\left(\frac{W_B}{L_n}\right)^2}$

$= \dfrac{2L_n^2}{W_B^2}$

$V_{PT} = \dfrac{qN_A(N_{DC} + N_A)W_{B_o}^2}{2\grave{o}N_{DC}}$

$\tau_{tB} = \dfrac{W_B^2}{2D_n}$

$I_{rB} = \dfrac{Q_B}{\tau_n}$

$Q_B = qA\Delta_{nE}\dfrac{W_B}{2}$

$t_s = \tau_n \ln\left(\dfrac{Q_B(0)}{Q_B(sat)}\right)$

$t_f = 2.3\ \tau_n$

Solved Problems

Problem 4.1

Prove the following relations

a. $\beta = \dfrac{\alpha}{1-\alpha}$, b. $\dfrac{\beta}{1+\beta}$,

c. $I_C = -\alpha I_E + I_{CBO}$, d. $I_C = \beta I_B + (1+\beta)\, I_{CBO}$,

e. $I_C = \beta I_B + I_{CEO}$

Solution

a. Alpha (α) of a transistor is defined as the forward short-circuit current gain in common-base configuration.

$$\alpha = \frac{-I_C}{I_E}, V_{CB} = 0$$

(α is positive for n-p-n and p-n-p transistors as I_C and I_E have opposite signs)

Beta (β) of a transistor is defined as the base to collector current amplification factor or the common-emitter current gain.

$$\beta = \frac{I_C}{I_B}$$

$$I_E + I_C + I_B = 0$$

or $$I_B = -(I_E + I_C)$$

$$\beta = \frac{I_C}{-I_E - I_C}$$

Dividing numerator and denominator by $-I_E$

$$\beta = \frac{\frac{I_C}{-I_E}}{1+\frac{I_C}{I_E}}$$

$$= \frac{\alpha}{1-\alpha}$$

b. $$\beta = \frac{\alpha}{1-\alpha}$$

$$\beta\,(1-\alpha) = \alpha$$

$$\beta - \alpha\beta = \alpha$$

$$\beta = \alpha\,(1+\beta)$$

or $$\alpha = \frac{\beta}{1+\beta}$$

c. $I_C = -(I_{pC} - I_{CBO})$

when $V_{CB} = 0$, $I_{CBO} = 0$, $I_C = -I_{pC}$

$$\therefore \quad \alpha = \left.\frac{-I_C}{I_E}\right|_{V_{CB}=0} = \frac{I_{pC}}{I_E}$$

$$= \frac{-(I_C - I_{CBO})}{I_E}$$

$\alpha I_E \quad = - I_C + I_{CBO}$

or $\quad I_C \quad = -\alpha I_E + I_{CBO}$

d. $\quad I_E \quad = (I_B + I_C)$

$I_C \quad = -\alpha I_E + I_{CBO}$

$I_C \quad = \alpha(I_B + I_C) + I_{CBO}$

$I_C(1-\alpha) = \alpha I_B + I_{CBO}$

$$\therefore \quad I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO}$$

$$= \beta I_B + (1+\beta) I_{CBO} \left[\frac{1}{1-\alpha} = \frac{1}{1-\frac{\beta}{1+\beta}} = 1+\beta \right]$$

e. But $(1 + \beta)\, I_{CBO} = I_{CEO}$ which is the reverse saturation current or leakage current between collector and emitter with base terminal open.

$$\therefore \quad I_C = \beta I_B + I_{CEO}$$

Problem 4.2

The current components in a transistor are $I_{nE} = 2.712 \times 10^{-6}$ A, $I_{pE} = 0.678$ mA, $I_{nC} = 9.4 \times 10^{-15}$ A and $I_{pC} = 0.6779$ mA. Determine a. γ, b. α_T, c. β and d. I_{CBO}.

Solution

Injection efficiency $\quad \gamma = \dfrac{I_{pE}}{I_{nE} + I_{pE}}$

$$= \frac{0.678 \times 10^{-3}}{2.712 \times 10^{-6} + 0.678 \times 10^{-3}}$$

$$= 0.996$$

$$\alpha_T = \frac{I_{pC}}{I_{pE}}$$

$$= \frac{0.6779}{0.678} = 0.9998$$

$$\alpha = \alpha_T \cdot \gamma$$

$$= 0.9998 \times 0.996$$

$$= 0.9958$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9958}{1-0.9958} = 237$$

$$I_C = -[I_{pC} + I_{nC}]$$

$$I_{CBO} = I_C + I_{pC}$$

$$= -[I_{pC} + I_{nC}] + I_{pC}$$

$$= -I_{nC}$$

$$= -9.4 \times 10^{-15} \text{ A.}$$

Problem 4.3

A symmetrical Si p^+np^+ Si BJT has the following parameters.

	Emitter	Base
$A = 10^{-4}$ cm^2	$N_A = 10^{17}$ cm^{-3}	$N_D = 10^{15}$ cm^{-3}
$W_B = 1\ \mu$ m	$\tau_n = 0.1\ \mu$ s	$\tau_p = 10\ \mu$ s
	$\mu_p = 200$ cm^2/Vs	$\mu_n = 1300$ cm^2/Vs
	$\mu_n = 700$ cm^2/Vs	$\mu_p = 450$ cm^2/Vs

a. Calculate I_{ES}, I_{CS}
b. Calculate I_B when $V_{EB} = 0.4$ V and $V_{CB} = -25$ V.
Assume perfect injection efficiency.

Solution

$$p_{no} = \frac{n_i^2}{N_D}$$

$$= \frac{(1.5\times10^{10})}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

$$D_{PB} = \frac{kT}{q}.\mu_{pB}$$

$$= 0.026 \times 450 = 11.7 \text{ cm}^2\text{/s}$$

$$L_p = \sqrt{D_p \tau_p}$$

$$= \sqrt{11.7\times10\times10^{-6}}$$

$$= 0.0108 \text{ cm}$$

$$= 1.08 \times 10^{-2} \text{ cm}$$

$$I_{ES} = I_{CS} = qA\frac{D_{pB}}{L_{pB}}p_{no}\coth\left(\frac{W_B}{L_{pB}}\right)$$

$$= 1.6\times 10^{-19}\times 10^{-4}\times \frac{11.7}{1.08\times10^{-2}}\times 2.25\times 10^5 \coth\left(\frac{10^{-4}}{1.08\times10^{-2}}\right)$$

$$= 4.21 \times 10^{-13} \text{ A}$$

$$\Delta_{pE} = p_{no}\left(e^{V_{EB}/V_T} - 1\right); \quad \Delta_{pC} \cong 0$$

$$= 2.25 \times 10^5 \left(e^{0.4/0.026} - 1\right)$$

$$= 1.08 \times 10^{12}$$

$$I_B = \frac{\text{Stored charge}}{\text{Lifetime}} = \frac{qAW_B\Delta_{pE}}{2\tau_p}$$

$$= \frac{1.6\times10^{-19}\times10^{-4}\times1\times10^{-4}\times1.08\times10^{12}}{2\times10\times10^{-6}}$$

$$= 8.64 \times 10^{-11} \text{ A.}$$

Problem 4.4

A silicon n-p-n bipolar junction transistor has $N_{DC} = 10^{18}$ cm^{-3}, $N_{AB} = 10^{15}$ cm^{-3} and $W_B = 1\mu$m. Determine the following at 300 K.
a. The punch through voltage.

b. The average value of electric field at punch through.
c. How can punch through voltage be increased?

Solution

a. $V_{CBo} = \frac{kT}{q}\ln\left(\frac{N_{AB}N_{DC}}{n_i^2}\right)$

$$= 0.026 \ln\left(\frac{10^{15}\times 10^{18}}{(1.5\times 10^{10})^2}\right) = 0.757 \text{ V}$$

At punch through, width of depletion layer at collector-base junction equals W_B.

$$W_B = \sqrt{\frac{2\epsilon(V_{CBo}+V_{PT})}{q}\left(\frac{1}{N_{AB}}+\frac{1}{N_{DC}}\right)}$$

$$V_{CBo} + V_{PT} = \frac{W_B^2\times q}{2\epsilon\left(\frac{1}{N_{AB}}+\frac{1}{N_{DC}}\right)}$$

$$= \frac{(1\times 10^{-4})2\times 1.6\times 10^{-19}}{2\times 8.854\times 10^{-14}\times 11.8\left(\frac{1}{10^{15}}+\frac{1}{10^{18}}\right)} = 6.884 \text{ V}$$

$$V_{PT} = 6.884 - 0.757$$
$$= 6.127\text{V}$$

b. $E_{av} = \frac{V_{PT}+V_{CBo}}{W_B}$

$$= \frac{6.127+0.757}{1\times 10^{-4}} = 6.884\times 10^4 \text{ V/cm}$$

c. The punch through voltage can be increased by (1) increasing base doping, (2) increasing base width, (3) reducing collector doping.

But increase in base doping will reduce injection efficiency and transport factor. Increase in base width will reduce transport factor. Punch through voltage can also be increased by reducing collector doping. This increases collector resistance.

Problem 4.5

A p-n-p transistor shown in Fig. Sp.4.5 has uniform doping in the emitter, base and collector regions and are 10^{19}, 10^{17} and 10^{15}/cm3 respectively. The minority carrier diffusion lengths are L_E = 5 µm, L_p = 100 µm. Assuming low-level injection conditions and using the law of junctions, calculate the collector current density and base current density due to base recombination (suitable approximations may be made). In all regions D_p = 8 cm^2/s, D_n = 16 cm^2/s, $n_i = 1.5\times 10^{10}$ cm^{-3}, kT/q = 26 mV, q = 1.6 ×10^{-19} C.

Solution

$$p_{Bo} = \frac{n_i^2}{N_D}$$

$$= \frac{(1.5\times 10^{10})}{10^{17}} = 2.25\times 10^3$$

$$\Delta_{pE} = p_{Bo}\left(e^{V_{EB}/V_T}-1\right)$$

$$= 2.25 \times 10^{3}\left(e^{0.78/0.026} - 1\right)$$

$$= 2.4 \times 10^{16}\ \text{cm}^{-3}$$

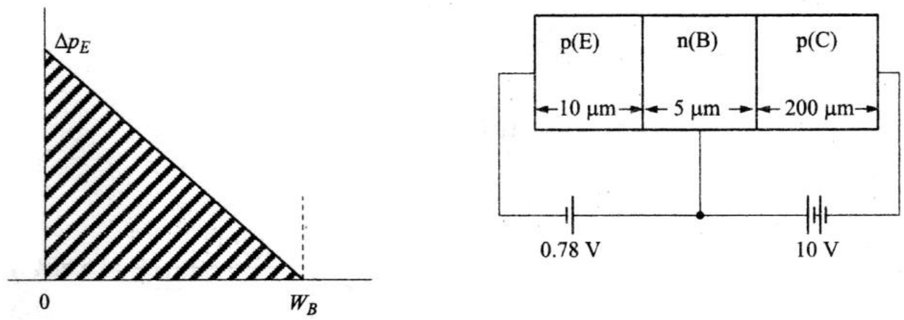


Fig. Sp.4.5

Collector current density,

$$J_C = -qD_p \cdot \frac{dp}{dx} = q.D_p.\frac{\Delta_{pE}}{W_B}$$

$$= 1.6\times10^{-19}\times8\times\frac{2.4\times10^{16}}{5\times10^{-4}} = 61.44\ \text{A/cm}^2$$

Base current density due to recombination,

$$J_B = \frac{\text{stored charge in the base}}{\text{minority carrierlifetime in the base}}$$

$$= \frac{\text{Area of shaded triangle } \times \text{ q}}{\tau_p}$$

$$\tau_p = \frac{L_p^2}{D_p} = \frac{\left(100\times10^{-4}\right)^2}{8} = 1.25 \times 10\ \ \text{s}$$

$$\therefore\ J_B = \frac{\frac{1}{2}\Delta_{pE}.W_B\times q}{\tau_p}$$

$$= \frac{\frac{1}{2}\times2.4\times10^{16}\times5\times10^{-4}\times1.6\times10^{-19}}{1.25\times10^{-5}}$$

$$= 76.8\ \text{mA/cm}^2$$

Problem 4.6

For a typical n-p-n transistor as shown in Fig. Sp.4.6, we have the following data available at 300 K.

a. W_C = 20 µm, collector doping = 5 × 10^{18} cm^{-3}

b. W_E = 1µm, emitter doping = 10^{19} cm^{-3}

c. base doping = 5 × 10^{15} cm^{-3}

d. minority carrier lifetime in the base region τ_{nB} = 0.5 µs.

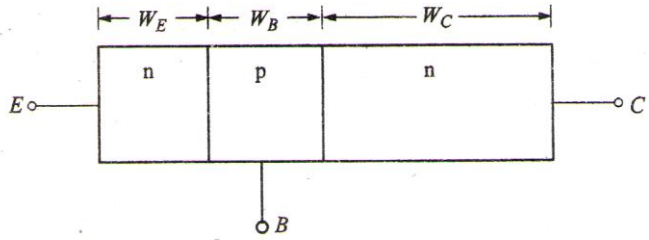


Fig. Sp.4.6

Under punch through condition, V_{BC} = 10 V + V_o where V_0 is the built-in potenital of the base collector junction. Emitter junction efficiency can be assumed as 1 for this transistor. Evaluate base width and current gain α.

Solution

Given q = 1.6×10^{-19} C, D_{nB} = 30 cm^2 /s

$\int_r\int_o$ = 10^{-12} F/cm, n_i = 1.5×10^{10} cm^{-3}

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026 \times \ln\left[\frac{5\times10^{15}\times5\times10^{18}}{\left(1.5\times10^{10}\right)^2}\right] = 0.8408 \text{ V}$$

$$L_{nB} = \sqrt{D_{nB}\tau_{nB}}$$

$$= \sqrt{30\times0.5\times10^{-6}}$$

$$= \sqrt{30\times0.5\times10^{-6}} = 3.872 \times 10^{-3} \text{ cm}$$

$$= 38.72 \text{ µm}$$

Punch through occurs when depletion layer width equals base width.

$$W_B = \sqrt{\frac{2\grave{o}(V_o + V_{Br(PT)})}{qN_A}}$$

$$= \sqrt{\frac{2\times10^{-12}(0.8408+10)}{1.6\times10^{-19}\times5\times10^{15}}} = 1.646 \times 10^{-4}\text{cm}$$

$$\beta = 2\left(\frac{L_{nB}}{W_B}\right) = 2\left(\frac{3.872\times10^{-3}}{1.646\times10^{-4}}\right) = 1106$$

$$\alpha = \frac{\beta}{1-\beta}$$

$$= \frac{1106}{1106+1} = 0.99909$$

Problem 4.7

The reverse saturation current of the collector-base junction (I_{CBO}) of a BJT is found to be 10 nA at low collector voltages. The low voltage current amplification factor (α) is 0.98. Find out the change in collector current with its base open (I_{CEO}) when the collector voltage is increased such that α increases by 1%.

Solution

$$(I_{CBO}) = 10\text{n A}, \ \alpha = 0.98$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

I_{CEO} = $(1+\beta)\, I_{CBO} = (1+49)10 = 500$ nA

Let $\Delta\alpha$ be the change in α.

$$\frac{\Delta\alpha}{\alpha}\times 100 = 1\% \quad \therefore \frac{\Delta\alpha}{\alpha} = 0.01$$

or $\Delta\alpha = 0.001\alpha = 0.0098$

$\alpha' = \alpha + \Delta\alpha = 0.98 + 0.0098 = 0.9898$

$$\beta' = \frac{\alpha'}{1-\alpha'} = \frac{0.9898}{1-0.9898} = 97$$

$I_{CEO} = (1+\beta')\, I_{CBO} = 98 \times 10 = 980$ nA

% change in I_{CEO} = $\dfrac{\Delta I_{CEO}}{I_{CEO}}\times 100$

$$= \frac{980-500}{500}\times 100 = 96\%$$

Problem 4.8

The emitter current of a p-n-p transistor with $\alpha_F = \alpha_I$ is 1 mA, when the emitter-base junction is forward-biased and collector is left-open. When the collector is shorted to the base, the current rises to 100 mA. Calculate β and base width of the transistor assuming a minority carrier diffusion length of 25 μm in the base and the emitter injection efficiency to be unity.

Solution

Substituting $I_C = 0$ (collector open) in Ebers Moll equation (equation (4.37)),

$$I_{CS}\left(e^{V_{CB}/V_T} - 1\right) = \alpha_F I_{ES}\left(e^{V_{EB}/V_T} - 1\right)$$

Substituting this in equation (4.36)

$$I_E = -I_{ES}(1 - \alpha_F\alpha_1)\left(e^{V_{EB}/V_T} - 1\right) = 1\text{ mA} \tag{4.88}$$

When collector is shorted to base, $V_{CB} = 0$. Therefore from equation (4.36)

$$I_E = -I_{ES}\left(e^{V_{EB}/V_T} - 1\right) = 100\text{ mA} \tag{4.89}$$

Equation (4.88) ÷ by (4.89) gives

$$1 - \alpha F\alpha_1 = \frac{1}{10}$$

$\alpha_F = \alpha_1$ Let $\alpha_F = \alpha_1 = \alpha$

$$\therefore 1-\alpha = \frac{1}{100}$$

$$\alpha = 0.995$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.995}{1-0.995} = 199$$

$$\beta = 2\left(\frac{L_N}{W_B}\right)^2$$

$$\therefore \quad W_B = \sqrt{\frac{2L^2B}{\beta}}$$

$$= \sqrt{\frac{2\times(25\times10^{-4})^2}{199}} = 2.50\mu m$$

Problem 4.9

Explain a method to measure I_{ES} and I_{CBO} of sa n-p-n transistor.

Solution

a. Measurement of I_{ES}

Plot the characteristics of the emitter-base junction using the setup shown in Fig. Sp.4.9(a). In this configuration the current I = I_{ES} $\left(e^{V_{EB/V_T}} - 1\right)$. Determine the reverse saturation current as explained in Section 3.5.3, which gives I_{ES}.

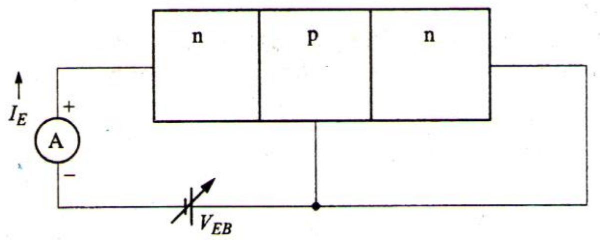


Fig. Sp. 4.9(a)

b. Measurement of I_{CBO}

In the configuration shown in Fig.Sp.4.9(b) I - I_{CBO} = $\left(e^{V_{CB/V_T}} - 1\right)$. Plot the forward characteristics and determine the reverse saturation current.

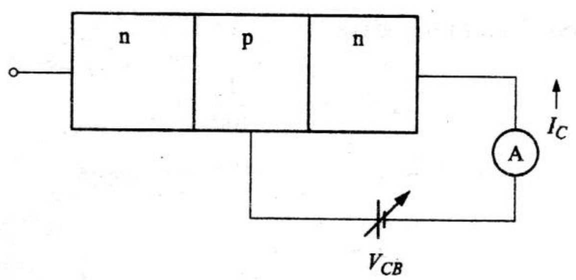


Fig. Sp.4.9(b)

Problem 4.10

Show that the transport factor of a BJT is given approximately by

$$\alpha_T = \frac{1}{1+\dfrac{\tau_{tB}}{\tau_p}}$$

Solution

For a p-n-p transistor if I_{nC} is neglected; $I_{nC} \simeq I_C$

$$I_C = \frac{\text{Excess stored charge}}{\text{transit time}}$$

$$= \frac{qA\Delta_{pE}W_B}{2\tau_{tB}}$$

$$I_{rB} = \frac{\text{stored charge}}{\text{life time}}$$

$$= \frac{qA\Delta_{pE}W_B}{2\tau_{tB}}$$

$$\alpha_T = \frac{I_{pC}}{I_{pE}} = \frac{I_{pC}}{I_{pC}+I_{rB}} = \frac{I_C}{I_C+I_{rB}}$$

$$= \frac{1}{1+\dfrac{I_{rB}}{I_C}}$$

$$= \frac{1}{1+\dfrac{\tau_{tB}}{\tau_p}}$$

Problem 4.11

A base current pulse of 250 μA with a duration of 500 ns is used to turn ON a silicon p-n-p BJT in the circuit in Fig. 4.26(a). Given V_{CC} = -5.2 V, R_C = 1 kΩ, τ_{pB} = 1 μs, W_B = 5 μm, D_{pB} = 10 cm²/s. Determine the following at 300 K. a. turn ON time b. storage time.

$$I_{B_1} = -250\mu A$$

$$\tau_{tB} = \frac{W_B^2}{2D_{pB}}$$

By equation (4.73),

$$t_{ON} = \tau_{pB} In\left[\frac{1}{1-\left(\dfrac{V_{CC}}{I_{B_1}R_C}\right)\left(\dfrac{\tau_{tB}}{\tau_{pB}}\right)}\right]$$

$$= 10^{-6}\, In\left[\frac{1}{1-\left(\frac{5.2}{250\times10^{-6}\times10^{3}}\right)\left(\frac{1.25\times10^{-8}}{1\times10^{-6}}\right)}\right]$$

$= -\,0.30\ \mu s$

$Q_{B(sat)} = I_{C(sat)}\ \tau_{tB}$

$I_{C(sat)}\ ;\ \frac{V_{CC}}{R_C} = \frac{-5.2}{10^3} = -5.2mA$

$Q_{B(sat)} = 5.2 \times 10^{-3} \times 1.25 \times 10^{-8}$

$= 6.5 \times 10^{-11}$ C

From equation (4.69(a)),

$$Q_B(t = 500ns) = I_{B_1}\tau_{pB}\left(1-e^{-t/\tau_{PB}}\right)$$

$= 250 \times 10^{-6} \times 10^{-6}\ (1 - e^{-500\times10^{-9}/10^{-6}})$

$= 9.84 \times 10^{-11}$ C

$t_s = \tau_{pB} I_n\left(\frac{Q_B(0)}{Q_{B(sat)}}\right)$; where $Q_B(0) = Q_B$ (t = 500 ns)

$= 10^{-6} ln\left(\frac{9.84\times10^{-11}}{6.5\times10^{-11}}\right)$

$= 0.41\ \mu S.$

Problem 4.12

Plot the energy band diagram and potential distribution of n-p-n transistor in active region.

Solution

Energy band diagram and potential distribution of n-p-n transistor in active region is as shown in Fig. p.4.12 Note that the potential decreases as energy increases.

$$V = \frac{E}{(-q)}$$

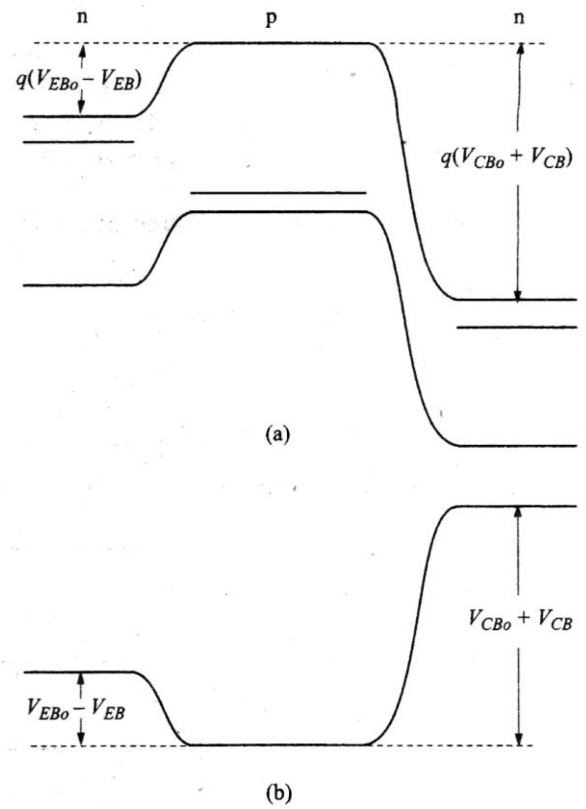


Fig. Sp.4.12 (a) Energy band diagram and (b) Potential distribution in n-p-n BJT in forward active mode of operation

Problem 4.13

Plot the minority carrier distribution in the base region of p-n-p transistor with a forward bias applied to the emitter-base junction under the following conditions:

a. Collector shorted to the base.

b. Collector shorted to the emitter.

c. Collector terminal is kept open.

All these configurations act as diodes. Which of these configurations as a diode gives best performance?

Solution

Since, the emitter-base junction is forward-biased, holes are injected from emitter into the base region.

a. When collector is shorted to the base, $V_{BE} = 0$ and $\Delta_{pC} = 0$. The concentration plot is shown in Fig. Sp.4.13(a).

b. When the collector is shorted to the emitter, both the junctions are forward-biased by same voltage. Therefore equal amount of excess holes are injected from both collector and emitter to the base region ($\Delta_{pE} = \Delta_{pC}$). The holes recombine in the base resulting in a dip in the concentration at the centre of the base as shown in Fig. Sp.4.13(b).

c. When collector terminal is kept open, Ic = 0. Therefore from Ebers Moll equation, it can be shown that V_{CB} becomes positive. So the collector-base junction gets a forward-bias, but V_{CB} is lower than V_{EB}. The collector-base junction does not collect or inject holes so that the slope of hole profile is zero at the junction. The hole distribution is shown in Fig. Sp.4.13(c).
Configuration (a) gives the best diode as the stored charge is minimum.

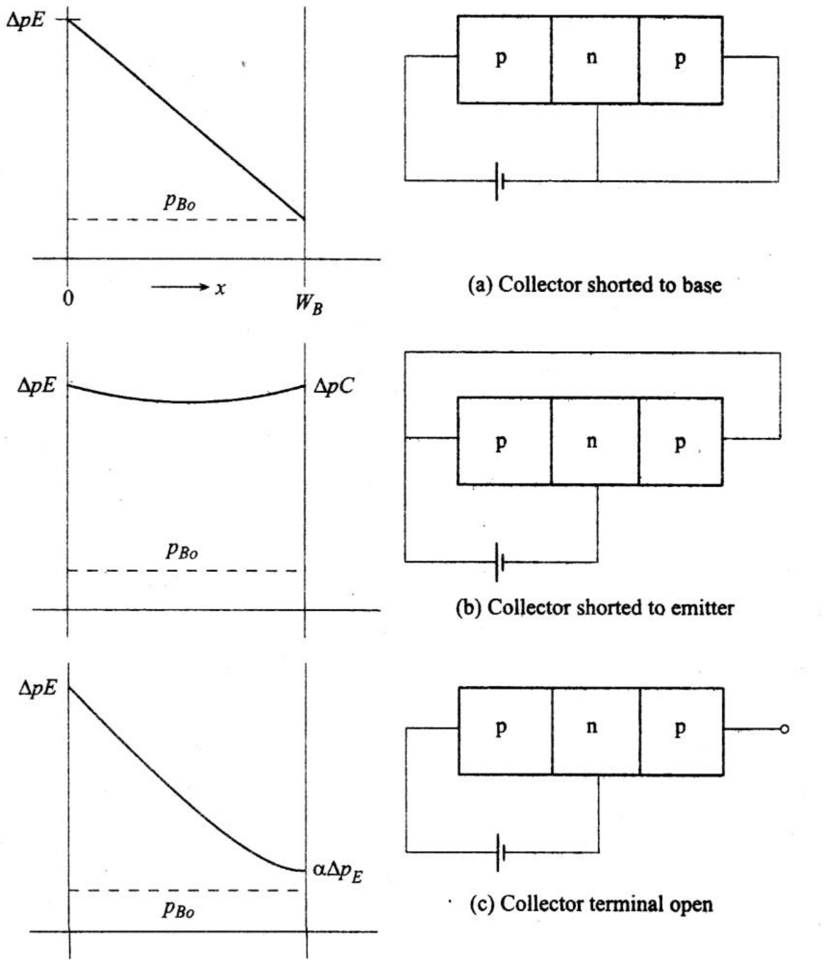


Fig. Sp. 4.13 Minority carrier distribution in the base of p-n-p transistor

Problem 4.14

Prove that $V_{BrCEO} = \frac{V_{BrCBO}}{\sqrt{\beta_F}}$

Solution

$$I_c = -\alpha_F I_E + I_{CBO}$$

If carrier multiplication in the collector region is taken into account

$I_C = M(-\alpha_F I_E + I_{CBO})$ (4.90)

where avalanche multiplication factor M is given by

$$M = \frac{1}{1-\left(\frac{V}{V_{Br}}\right)^n}$$

Breakdown voltage in common-base configuration is V_{BrCBO}

$$\therefore M = \frac{1}{1-\left(\frac{V}{V_{BrCBO}}\right)^n} \qquad (4.91)$$

V_{BrCBO} is the breakdown voltage of collector-base junction with emitter open ($I_E = 0$).

$\therefore$ $I_C = MI_{CBO}$ by equation (4.90)

For common emitter configuration breakdown voltage is V_{BrCEO}
Replacing I_E with $-(I_C + I_B)$ m equation (4.90)

$$I_C = M\,[\alpha_F\,(I_C + I_B) + I_{CBO}]$$

$$I_C = \frac{M\left(\alpha_F I_B + I_{CBO}\right)}{1-M\alpha_F} \qquad (4.92)$$

since V_{BrCEO} is breakdown voltage with base open ($I_B = 0$) equation (4.92) becomes

$$I_C = \frac{MI_{CBO}}{1-M\alpha_F}$$

Breakdown takes place when $M\alpha_F = 1$

or $$M = \frac{1}{\alpha_F}$$

Substituting this in equation (4.90) and assuming $V_{CB} \simeq V_{CE}$($\because$ $V_{CB} = V_{CE} - V_{BE} \simeq V_{CE}$) and designating this voltage at breakdown as V_{BrCEO},

$$\frac{1}{\alpha_F} = \frac{1}{1-\left(\frac{V_{BrCEO}}{V_{BrCBO}}\right)^n}$$

$$\alpha_F = \frac{1}{1 - \left(\dfrac{V_{BrCEO}}{V_{BrCBO}}\right)^n}$$

$$\frac{V_{BrCEO}}{V_{BrCBO}} = (1 - \alpha_F)^{1/n}$$

$$= \frac{1}{(\beta_F)^{1/n}}$$

$$\text{or } V_{BrCEO} = \frac{V_{BrCBO}}{(\beta_F)^{1/n}}$$

Problem 4.15

A PNP silicon BJT has $N_{AE} = 10^{18}$ cm^{-3}, $N_{AC} = 10^{15}$ cm^{-3}, $N_DB = 10^{16}$ cm^{-3} the metallurgical base with =1.0 μm and A = 3mm^2. For V_{EB} = 00.5V and V_{CB} = -5V, determine at T = 300 K, the effective width of the neutral base.

$$W_{Beff} = W_{\text{metallurgical}} - \text{deplition layer width}$$

$$= W_m - (X_{BEn} + X_{BCn})$$

$$X_{BEn} = X_{BE}\frac{N_{AE}}{N_{AE} + N_{DB}}$$

$$W_{BE} = \sqrt{\frac{2ò\left(V_{EB_0} - V_{EB}\right)}{q}\left(\frac{1}{N_{AE}} + \frac{1}{N_{DB}}\right)}$$

$$V_{EB0} = \frac{kT}{q} ln \frac{N_{AE}N_{DB}}{n_i^2}$$

$$= 0.026 \text{ In } \frac{10^{18} \times 10^{16}}{(1.5 \times 10^{10})^2} = 0.817V$$

$$W_{BE} = 2.044 \times 10^{-5} \text{ cm} = 0.204 \text{ μm}$$

$$X_{BE_n} = 2.044 \times 10^{-5} \times \frac{10^{18}}{10^{18} + 10^{16}} = 0.202\mu m$$

$$W_{BC} = \sqrt{\frac{2ò\left(V_{CB_0} - V_{CB}\right)}{q}\left(\frac{1}{N_{AC}} + \frac{1}{N_{DB}}\right)}$$

$$V_{CB0} = 0.026\, ln \frac{10^{15} \times 10^{16}}{(1.5 \times 10^{10})^2} = 0.637V$$

$$W_{BC} = \sqrt{\frac{2 \times 8.854 \times 10^{-14} \times 11.8(0.637 + 5)}{1.6 \times 10^{-19}}}\left(\frac{1}{10^{15}} + \frac{1}{10^{16}}\right)$$

$= 2.845 \times 10^{-4}$ cm

$$X_{BCn} = W_{BC} \frac{N_{AC}}{N_{AC} + N_{DB}}$$

$$= 2.845 \times 10^{-4} \times \frac{10^{15}}{10^{15} + 10^{16}} = 2.586 \times 10^{-5} \, cm$$

$= 0.259$ μm

$W_{eff} = 1 - (0.202 + 0.259) = 0.539$μm

Problem 4.16

A silicon npn BJT has τ_{pB} = 1μs and μ_{pB} = 440 cm^2/Vs. Determine W_B so that the transport factor is 0.995 at T = 300 K. Assume $W_B/L_p << 1$.

Solution

By equation (4.32),

$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{L_p}\right)^2}$$

$$\frac{D_p}{\mu_{pB}} = \frac{kT}{q}$$

$D_p = 0.026 \times 440 = 11.44$cm^2/s

$$L_p = \sqrt{D_p \tau_p}$$

$$= \sqrt{11.44 \times 1 \times 10^{-6}} = 3.38 \times 10^{-3} \, cm$$

$$0.995 = \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{3.38 \times 10^{-3}}\right)^2}$$

$W_B = 3.38 \times 10^{-4}$ cm = 3.38μm

Problem 4.17

An npn BJT has $N_{AB} = 5 \times 10^{16}$ cm^{-3}, W_B = 1μm, A = 10^{-3} mm^2 and life time of minority carriers in all three regions is 0.1 μs. For operation in the active region and at T = 300 K, determine I_C for V_{BE} = 0.5 V. μ_n = 1000 cm^2/Vs.

Solution

By equation (4.26), neglecting e^{V_{CB}/V_T} term in the active region for npn transistor,

$$I_C = qA \frac{D_n}{L_n} \frac{n_i^2}{N_A} \cos ech\left(\frac{W_B}{L_n}\right)\left(e^{V_{EB/V_T}} - 1\right)$$

Assume $n_i = 1.5 \times 10^{10}$cm^{-3}

$$\frac{D_n}{\mu_n} = \frac{kT}{q}$$

$D_n \quad = 0.026 \times 1000 = 26\text{cm}^2\ /\text{s}$

$L_n \quad = \sqrt{D_n \tau_n} = \sqrt{26 \times 0.1 \times 10^{-6}} = 1.61 \times 10^{-3}\, cm$

$$I_C = 1.6 \times 10^{-19} \times 10^{-3} \times 10^{-2} \times \frac{26}{1.61 \times 10^{-3}}$$

$$\times \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} \cos ech\left(\frac{1 \times 10^{-4}}{1.61 \times 10^{-3}}\right)\left(e^{0.5/00.026} - 1\right)$$

$$= 2.41 \times 10^{-5} = 0.241\mu\text{A}$$

Problem 4.18

An npn BJT has $N_{DE} = 5 \times 10^{17}$ cm^{-3}, $N_{AB} = 10^{16}$ cm^{-3}, $W_B = 5$ μm, $W_E = 2.25 \times 10^{-3}$cm. If $\mu_{pE} = 200$cm^2/Vs and $\mu_{nB} = 1250$ cm^2/Vs, determine β_F at T = 300K for (a) $\tau_{nB} = 1$ μs (b) $\tau_{nB} = 10\mu_s$.

Solution

By equation (4.35), for npn transistor

$$\beta_F = \frac{1}{\dfrac{N_{AB} D_{PE}}{N_{DE} D_{nB}} \dfrac{W_B}{W_E} + \dfrac{1}{2}\left(\dfrac{W_n}{L_n}\right)^2}$$

$$\frac{D_{nB}}{\mu_{nB}} = \frac{kT}{q} = 0.026$$

$$D_{nB} = \mu_{nB} \frac{kT}{q}$$

$$= 1250 \times 0.026 = 32.5\text{cm}^2/\text{s}$$

$$D_{PE} = \mu_{nE} \frac{kT}{q}$$

$$= 200 \times 0.026 = 5.2\ \text{cm}^2/\text{s}$$

(a)

$\tau_{nB} = 1\mu$s

$L_{nB} = \sqrt{D_{nB} \tau_{nB}}$

$= \sqrt{32.5 \times 1 \times 10^{-6}} = 5.7 \text{ x } 10^{-3}$ cm

$$\beta_F = \frac{1}{\dfrac{10^{16}}{5 \times 10^{17}} \times \dfrac{5.2}{32.5} \times \dfrac{5 \times 10^{-4}}{2.25 \times 10^{-3}} + \dfrac{1}{2}\left(\dfrac{5 \times 10^{-4}}{5.7 \times 10^{-3}}\right)^2}$$

$= 219.7$

(b) $\tau_{nB} = 10\mu$s

$L_{nB} = \sqrt{32.5 \times 10 \times 10^{-6}} = 18.02 \text{ x } 10^{-3}$

$$\beta_F = \frac{1}{\frac{10^{16}}{5\times10^{17}}\times\frac{5.2}{32.5}\times\frac{5\times10^{-4}}{2.25\times10^{-3}}+\frac{1}{2}\left(\frac{5\times10^{-4}}{18.02\times10^{-3}}\right)^2}$$

$= 912.45$

Problem 4.19

A silicon npn BJT has $N_{AB} = 10^{17}$ cm^{-3}, $N_{DC} = 10^{16}$ cm^{-3} and $W_{B0} = 0.25$ μm.
Determine at T = 300 K,
(a) the punch through voltage
(b) the average value of electric field intensity at punch through.

Solution

By equation (4.48), for npn transistor

$$V_{PT} = \frac{qN_A(N_{DC}+N_A)W_{B_0}^2}{2\epsilon N_{DC}}$$

$$= \frac{1.6\times10^{-19}\times10^{17}\left(10^{16}+10^{17}\right)\left(0.25\times10^{-4}\right)^2}{2\times8.854\times10^{-14}\times11.8\times10^{16}}$$

$= 52.64$ V

(b)Electric field $E = \frac{V_{PT}}{W_{B_0}}$

$$= \frac{52.64}{0.25\times10^{-4}}$$

$= 2.10\text{x}10^6$ V/cm

Problem 4.20

An npn silicon BJT at T = 300 K has heavy collector doping and $N_A = 10^{16}$ cm^{-3}.
Given $W_B = 1.0$ μm. Determine
(a) the breakdown voltage for active operation in the common-base mode. The breakdown field in silicon is 3 x 10^5 V/cm.
(b) The punch through voltage.

Solution

By equation (3.89)

$$V_{BrCBO} = \frac{\epsilon E_{crit}^2}{2qN_A}$$

$$= \frac{8.854\times10^{14}\times11.8\times\left(3\times10^5\right)^2}{2\times1.6\times10^{19}\times10^{16}}$$

$$= 29.38 \text{ V}$$

By equation (4.8)

$$V_{PT} = \frac{qN_A W_{B_0}^2}{2\epsilon}$$

For npn BJT with $N_{DC} >> N_{AB}$

$$V_{PT} = \frac{1.6 \times 10^{-19} \times 10^{16} \times \left(1 \text{ x } 10^{-4}\right)^2}{2 \times 8.854 \times 10^{-14} \times 11.8}$$

$$= 7.65 \text{ V}$$

Problem 4.21

For a silicon pnp transistor operating at T = 300 K and at I_C = 1 mA, V_{EB} = 0.7 V and V_{BC} = 5 V. The device has β_0 = 200, $N_{DB} = 10^{17}$ cm^{-3}, $N_{AC} = 10^{16}$ cm^{-3}, W_B = 0.8 μm, V_A = 100 V and D_{PB} = 10 cm^2/s. Determine (a) g_m (b) τ_π, (c) τ_0.

Solution

(a)

$$g_m = \frac{I_C}{V_T} = \frac{10^{-3}}{0.026} = 38.46 \times 10^{-3} \text{ A/V}$$

(b)

$$\tau_\pi = \frac{\beta_0}{g_m} = \frac{200}{38.46 \times 10^{-3}} = 5.2 \times 10^3 \Omega = 5.2 \text{k}\Omega$$

(c)

$$\tau_0 = \frac{V_A}{I_C} = \frac{100}{10^{-3}} = 10^5 \Omega$$

Problem 4.22

A GaAs pnp BJT has $N_{AE} = 10^{20}$ cm^{-3}, $N_{DB} = 10^{16}$ cm^{-3}, D_{pB} = 30 cm^2/s, τ_p = 10μs, A = 10^{-3} cm^2 and W_B = 12 μm. At T = 300 K and for I_C = 2 mA in the active region. Determine

(a) the base storage capacitance, C_B

(b) the emitter junction capacitance

(c) if βo = 100 and C_μ = 0.25 pF determine r_π, and f_T(n_i, = 2 × 10^6 cm^{-3})

Solution

$$g_m = \frac{I_C}{V_T} = \frac{2 \times 10^{-3}}{0.026} = 76.92 \times 10^{-3} A/V$$

(a)

$$C_B = g_m \tau_{tB}$$

$$g_m = \frac{I_C}{V_T} = \frac{2 \times 10^{-3}}{0.026} = 76.92 \times 10^{-3} A/V$$

$$\tau_{tB} = \frac{W_B^2}{2D_p} = \frac{(12 \times 10^{-4})}{2 \times 30} = 2.4 \times 10^{-8} s$$

$$= 24 \text{ ns}$$

$C_B = 76.92 \times 10^{-3} \times 2.4 \times 10^{-8}$

$= 184.6 \times 10^{-11}$ F = 1.853nF

(b) $C_jE = \dfrac{\grave{o}A}{W_{BE}}$

$$W_{BE} = \sqrt{\frac{2\grave{o}\left(V_{BE_0} - V_{BE}\right)}{qN_{DB}}}$$

$$V_{BE_0} = \frac{kT}{q} ln \frac{N_A N_D}{n_i^2}$$

$$= 0.026 \ln \frac{10^{20} \times 10^{16}}{(2\times10^6)^2} = 1.4V$$

From equation (4.25) neglecting e^{V_{CB}/V_T} term

$$I_C = \frac{qAD_p p_o}{W_B} e^{qV_{EB}/kT}$$

$$p_o = \frac{n_i^2}{N_{DB}} = \frac{(2\times10^6)^2}{10^{16}} = 4\times10^{-4} cm^{-3}$$

$$2\times10^{-3} = \frac{1.6 \times 10^{-19} \times 10^{-3} \times 30 \times 4 \times 10^{-4}}{12\times10^{-4}} e^{V_{EB}/V_T}$$

V_{EB} = 1.08V

$$W_{BE} = \sqrt{\frac{2 \times 8.854 \times 10^{-14} \times 13.1 \times (1.4 - 1.08)16}{1.6 \times 10^{-19} \times 10^{16}}}$$

$= 2.15 \times 10^{-5}$ cm

$$C_{jE} = \frac{8.854 \times 10^{-14} \times 13.1 \times 10^{-3}}{2.15\times10^{-5}}$$

$=5.38 \times 10^{-11}$

= 53.8 pF

(c) $r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{76.92\times10^{-3}} = 1.3\times10^3 = 1.3k\Omega$

$$f_T = \frac{1}{2\pi(C\pi + C\mu)}$$

$C\pi = 5.38 \times 10^{-11} + 184.6 \times 10^{-11}$

$=1.899 \times 10^{-9} = 1.89$ nF

$$f_T = \frac{1}{2\pi\left(1.89 \text{ x } 10^{-9} + 0.25 \times 10^{-12}\right)}$$

= 83.7MHz

Problem 4.23

A base current pulse of 250 μA with a duration of 300 ns is used to turn ON a silicon pnp BJT in the circuit of Figure 4.26(a). Given V_{CC} = 5.2 V, R_C = 1 KΩ, τ_p = 1 μs, W_B = 5 μm and D_p = 10 cm²/s, determine at T = 300 K; (a) turn ON time (b) storage time.

Solution

(a) In equation (4.73), substituting $I_{Csat} = \dfrac{V_{CC} - V_{CEsat}}{R_C}$

$$t_{ON} = \tau_p ln \frac{1}{1 - \left(\dfrac{V_{CC} - V_{CEsat}}{I_B R_C}\right)\tau_{tB} / \tau_p}$$

$$\tau_{tB} = \frac{W_B^2}{2D_p}$$

$$= \frac{\left(5 \text{ x } 10^{-4}\right)^2}{2 \times 10} = 1.25\ 10^{-8}\text{s}$$

$$t_{ON} = 10^{-6} \text{ In } \frac{1}{1 - \left(\dfrac{50}{2} 50 \times 10^{-6} \times 1 \times 10^3\right)\dfrac{1.25 \times 10^{-8}}{10^{-5}}}$$

$$= 0.3 \times 10^{-6} s = 0.3 \mu s$$

(b) By equation (4.76),

$$t_{sd} = \tau_p \text{In} \frac{Q_B(0)}{Q_B sat}$$

$$Q_{Bsat} = I_{Csat}\tau_{tB}$$

$$= \frac{5.0}{1 \times 10^3} \times 1.25 \times 10^{-8} = 6.25 \times 10^{-11} C$$

$$Q_B(t) = I_B \tau_p [1 - e^{-t/\tau_p}]$$

$$Q_B(0) = Q_B (t = 300 \text{ ns})$$

$$= 250 \times 10^{-6} \times 10^{-6} \left[1 - e^{\frac{-300 \times 10^{-9}}{10^{-6}}}\right]$$

$$= 6.47 \times 10^{-11} \text{ C}$$

$$t_{sd} = 10^{-6} \times \text{In } \frac{6.47 \times 10^{-11}}{6.25 \times 10^{-11}}$$

$$= 34.59 \times 10^{-9} \text{ s}$$

$$= 34.6 \text{ns}$$

▪ <u>Points to Remember</u>

All the defenitions and expressions given below are for p-n-p transistors.

- Emitter injection efficiency (γ) is the ratio of emitter current due to injection from emitter to base to the total emitter current

$$\gamma = \frac{I_{pE}}{I_E}$$

- Base transport factor (α_T) is the effectiveness of base in transporting charge carriers through the base to collector, which are injected from the emitter

$$\alpha_T = \frac{I_pC}{I_{pE}}$$

- In BJT, emitter region is heavily doped, base is moderately doped and collector region is lightly doped.
- The currents in BJT mainly depend on the minority carrier distribution in the base region. The expressions for terminal currents and other important parameters of a pnp, BJT are as follows.

$$I_{pE} = qA\frac{D_p}{L_p}\left[\Delta_{pE}Coth\left(\frac{W_B}{L_p}\right) - \Delta_{pC}cosech\left(\frac{W_B}{L_p}\right)\right]$$

$$I_{pC} = qA\frac{D_p}{L_p}\left[\Delta_{pE}Co\sec h\left(\frac{W_B}{L_p}\right) - \Delta_{pC}\cot h\left(\frac{W_B}{L_p}\right)\right]$$

$$I_{nE} = qA\frac{D_{nE}}{L_{nE}}\Delta_{nE}coth\left(\frac{W_E}{L_{nE}}\right)$$

$$I_{nC} = -qA\frac{D_{nC}}{L_{nC}}\Delta_{nC}coth\left(\frac{W_C}{L_{nC}}\right)$$

$I_E = I_{pE} + I_{nE}$

$I_C = I_{pC} + I_{nC}$; $\quad I_B = -(I_E + I_C)$

$$\gamma = \frac{1}{1+\frac{D_{nE}}{D_p}\times\frac{N_D}{N_{AE}}\times\frac{W_B}{W_E}}$$

$$\alpha_T = \frac{1}{\cosh\left(\frac{W_B}{L_p}\right)}$$

for $W_B \ll L_p$, $\alpha_T = \dfrac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_p}\right)^2}$

Common-base current gain, α_T ; $\dfrac{1}{1+\frac{D_nE}{D_p}\times\frac{N_D}{N_{AE}}\times\frac{W_B}{W_E}+\frac{1}{2}\left(\frac{W_B}{L_p}\right)^2}$

Common-emitter current gain, β ; $\dfrac{2L_p^2}{W_B^2}$

- **Ebers Moll equations**

$$I_E = -\left[I_{ES}\left(e^{V_{EB}/V_T}-1\right) - \alpha_1 I_{ES}\left(e^{V_{EB}/V_T}-1\right)\right]$$

$$I_C = -\left[I_{ES}\left(e^{V_{CB}/V_T}-1\right) - \alpha_F I_{ES}\left(e^{V_{EB}/V_T}-1\right)\right]$$

$$\alpha_F I_{ES} = \alpha_I I_{CS}$$

$$I_{ES} = \frac{I_{EBO}}{1-\alpha_F \alpha_1}$$

$$I_{CS} = \frac{I_{CBO}}{1-\alpha_F \alpha_1}$$

- Punch through occurs when depletion layer width towards the base equals the base width.

$$V_{pT} = \frac{qN_D(N_{AC}+N_D)W_{BO}^2}{2\varepsilon N_{AC}}$$

$$V_{BrCEO} = \frac{V_{BrCBO}}{(1+\beta)^{1/n}}$$

$$\tau_{tB} = \frac{W_B^2}{2D_p}$$

$$I_{rB} = \frac{Q_B}{\tau_p}$$

$$I_C = \frac{Q_B}{\tau_{tB}}$$

$$Q_B = \frac{qA\Delta_{pE} W_B}{2}$$

By charge control analysis, $i_B = \frac{dQ_B}{dt} + \frac{Q_B}{\tau_p}$

Turn-ON time of a BJT switch

$$t_{ON} = \tau_p ln\left[\frac{1}{1-\left(\frac{I_{C(sat)}}{I_{B_1}}\right)\left(\frac{\tau_{t\beta}}{\tau_p}\right)}\right]$$

Turn-OFF time $t_{OFF} = t_s + t_f$

$$t_s = \tau_p ln\left(\frac{Q_B(0)}{Q_{B(sat)}}\right)$$

$$t_f = 2.3\,\tau_p$$

Transconductance of BJT

$$g_m = \frac{I_c}{V_T}$$

$$\tau_\pi = \frac{\beta}{g_m}$$

$$C_B = g_m \tau_{tB}$$

$$C_\pi = C_B + C_{jE}$$

$$C_\mu = \frac{-dQ_B}{dV_{CB}} = \frac{V_T}{V_A} C_B$$

Figure of merit (f_T) = $\alpha f_\alpha = \beta f_\beta$

$$= \frac{g_m}{2\pi[C_\pi + C_\mu]}$$

Exercise Problems

(1) Plot the current components in an n-p-n bipolar junction transistor and explain.

(2) The following parameters are given for an n-p-n transistor. I_{nE} = -5 mA, I_{pE} = -0.01 mA, I_{nC}= -4.99 mA, I_{pC}= -0.001 mA. Determine α_T, γ, β, I_B, I_C and I_E.

Ans: 0.998, 0.998, 0.996, 249, 0.019 mA, 4.991 mA, -5.01 mA.

(3) A symmetrical germanium p-n-p transistor with a diameter of 1mm for emitter-base and collector-base junctions has the following specifications.

$N_{DB} = 5 \times 10^{15}$ cm^{-3}, $N_{AC} = N_{AE} = 10^{19}$ cm^{-3}

$W_B = 10 \mu$m, $\tau_{pB} = 4\ \mu$s, $\tau_{nE} = 10^{-8}$ s,

D_{pB} == 47 cm^2/s, D_{nE} = 52 cm^2/s.

Determine α and β of the transistor, assuming that $W_E >> L_{nE}$

Ans: 0.9965, 285.

(4) Draw the energy band diagram of an n-p-n transistor at equilibrium and inverse active mode of operation.

(5) A symmetrical silicon p-n-p BJT has $N_{AE} = 10^{18}$ cm^{-3}, $N_{DB} = 5 \times 10^{16}$ cm^{-3}, $W_B = 2\ \mu$m, τ_{pB} = 1 μs, D_{pB} = 12 cm^2/s, A = 0.01 cm^2 and is operating in the active region. For V_{EB} = 0.65 V and V_{CB} = -3 V, determine at 300 K (a) I_C (b) β_p.

Ans: 600, 31.1 mA.

(6) Explain an experimental setup to plot β as a function of I_C. What is the expected shape? Why?

(7) Given a silicon BJT in forward active mode of operation at 300 K with $N_{DE} = 10^{19}$ cm^{-3}, $N_{AB} = 10^{14}$ cm^{-3}, D_{nB} = 10 cm^2/s, $W_B = 4\ \mu$m, $\tau_{nB} = 2\ \mu$s. Find J_E if V_{EB} = -0.5 V.

Ans: 250. 2.02 A/cm^2.

(8) A p-n-p silicon BJT has $N_D = 10^{15}$ cm^{-3} on the base and the collector is heavily doped. Determine the base width if the avalanche breakdown voltage V_{BrCBO} equals the punch through voltage. Assume $\varepsilon_{crit} = 3 \times 10^5$ V/cm. What will be V_{BrCEO} it β = 150 and n = 4.

Ans: 19.59 μm, 83.82 V.

(9) A Ge alloyed p-n-p transistor has I_{ES} = -2μA, I_{CS} = -3 μA and α = 0.95. The transistor is connected to a 5 V battery in series with a 1 KΩ resistor such that positive of the battery is connected to the emitter, the negative to the collector, and the base is open circuited. Calculate fhe current through the circuit and voltage drop across each of the two junctions.

Ans: 23.9 μA, 64.38 mV, 4.91 V.

(10) A base current pulse of 100 μA. and width 500 ns is applied to the base of a switching transistor shown in Fig. EP.4.10.

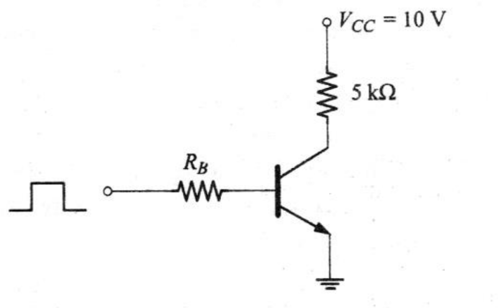


Fig. EP.4.10

Given τ_{pB} = 1 μs, W_B =5 μm, D_p = 6 cm^2/s. Determine t_{ON},t_s and $Q_{B(sat)}$. Plot $Q_B(t)$ and $I_C(t)$.
Ans: 0.53 μs, 0.868 μs, 4.166 × 10^{-11} C.

▪ <u>Review Questions</u>

(1) Why is it not possible to use two p-n junctions connected as in Fig. 4.1(b) as a transistor?
(2) Explain the fabrication sequence of a monolithic BJT.
(3) What is the role of buried layer in BJT?
(4) What are the different modes of operation of a BJT?
(5) Draw the current components in BJT.
(6) List the current components in a BJT. How are they related to the terminal currents?
(7) Define injection efficiency and transport factor of a BJT. How are they related to α and β?
(8) What are the doping and dimensional requirements of emitter, base and collector regions of a BJT?
(9) Draw a typical doping profile in BJT and explain its significance.
(10) Draw the energy band diagram of an n-p-n transistor with uniform doping in all regions under (a) equilibrium (b) forward active (c) saturation and (d) cutoff.
(11) Plot the minority carrier distribution in a pnp BJT in different regions and label it properly.
(12) What are the approximations made in the derivation of terminal currents of BJT?
(13)Derive expression for I_C, I_E and I_B of a n-p-n BJT.
(14) Derive expressions for α_T, γ, α and β of an n-p-n BJT.
(15) Draw Ebers Moll model of pnp BJT and write the Ebers Moll equations. Explain the terms involved.
(16) Describe an experimental procedure to measure I_{CS} and I_{ES} of a BJT.
(17) How will you experimentally determine I_{CBO} and I_{CEO} of a BJT.
(18) Show that $\alpha_F I_{ES} = \alpha_I I_{CS}$
(19) Derive relationship between I_{CS} and I_{CBO}.
(20) Derive relationship between I_{ES} and I_{EBO}
(21) Plot the minority carrier distribution in n-p-n BJT in (a) forward active mode (b) saturation mode (c) cutoff mode and (d) inverse active mode.
(22) List the non-idealities in a BJT.
(23) Draw Gummel plot and explain.
(24) Explain effect of non-uniform doping in the base region of BJT.
(25) What is early effect? What are its effects on I_C, I_B, I_E, α and β of a transistor?

(26) What is meant by punch through in a BJT? How is it avoided?

(27) What is meant by emitter crowding?

(28) What is Kirk effect?

(29) Plot the input and output characteristics of a p-n-p transistor in common-base configuration and explain. Mark different regions of operation.

(30) Plot the input and output characteristics of a pnp transistor in common-emitter configuration and explain. What are the effects of base width modulation as seen in the characteristics?

(31) Explain early effect. What is early voltage?

(32) Explain how a transistor works as a current amplifier.

(33) What is the difference between transit time and lifetime with respect to base region of a BJT? Derive expression for transit time.

(34) Derive charge control equations for I_B^F, I_C^F and I_E^F.

(35) Derive expression for turn-ON time of transistor switch in Fig. 4.27.

(36) What are the factors which determine the tum-OFF time of a transistor switch?

(37) Derive expression for storage delay of a pnp transistor switch.

(38) Draw the low-frequency small signal equivalent circuit of BJT and explain.

(39) Draw the high-frequency small signal equivalent circuit of a BJT and explain different parameters.

(40) Define figure of merit of a BJT. Derive an expression for the same.

(41) What are the short comings of Ebers Moll model? How are these overcome in Gummel poon model of BJT?