PART - III

Basic Semiconductor Devices

3 Junction Diodes

INTRODUCTION

Diode is a two terminal electronic device which conducts well in one direction of current flow and does not conduct in the opposite direction. This property of diode is used for rectification of signals. As diodes have low resistance in one direction (closed switch) and very high resistance in the opposite direction (open switch), they are also used as electronic switches. The predecessor of semiconductor diodes used for the same purposes are the vacuum tubes (vacuum diodes) which are costy and bulky.

A semiconductor diode is a junction between p-type and n-type semiconductors. There fore, it is also designated as a p-n junction. There are diodes which arc formed by contact between a metal and a semiconductor called metal-semiconductor contact or Schottky diode. This type of diode is discussed in Section 3.10.

Based on the method of fabrication, p-n junction diodes may be of the following types:

Grown junction: During the process of crystal growth dopant is added to the melt. An abrupt change of dopant (say from n-type to p-type or vice versa) results in the formation of a p-n junction. This type of p-n junction is called grown junction.

Alloyed junction: When a semiconductor (substrate) containing one type of dopant is alloyed with a metal containing opposite type of impurity, p-n junction formation takes place. This type of p-n junction is called alloyed junction.

Diffused junction: Impurity atoms will diffuse into a semiconductor if a p-type/n- type semiconductor is kept at high temperature (1000ºC) in a gaseous atmosphere of donor/acceptor impurities. This will result in the formation of a p-n junction. This technique is widely used in the fabrication of devices in integrated circuits. Thousands of devices can be fabricated simultaneously by using this technique. A diffused junction fabrication has been explained in detail in Section 2.3.1.

Ion implanted junction: In this method a beam of impurity ions is accelerated to very high kinetic energies (several kcVs) and is directed to the surface of the semiconductor. The ions will come to rest by giving up their energy by collision and replace semiconductor atoms. Thus, by implanting p-type impurity ions to n-type semiconductor or vice versa. a p-n junction may be formed.

The doping profile (doping concentration as a function of distance along the direction of current flow) of the junction differ from diode to diode. Typical doping profiles are shown in Fig. 3.1. The acceptor doping on the p-side is shown as a constant. The donor doping on the n-side is a function of distance from the junction.

$$N_D(x) = Gx^m \tag{3.1}$$

where, G is a constant called grading constant and m is another constant.

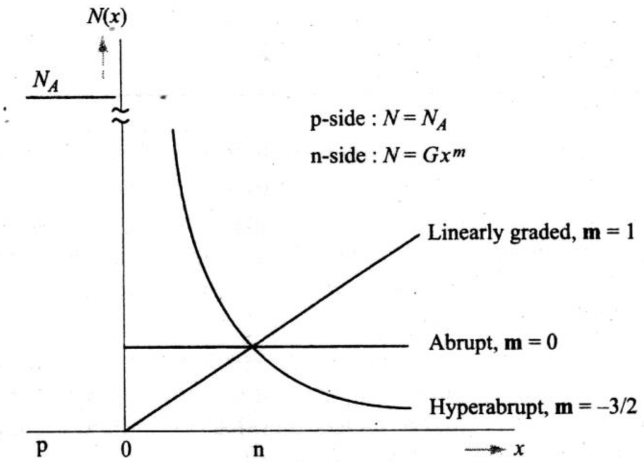


Fig. 3.1 Doping profiles of p-n junction

Abrupt p-n Junction

If m = 0, $N_D(x) = G$, a constant, i.e., in an abrupt junction the doping on the p-side and n-side remain constant and uniform. The type of impurity abruptly changes from acceptor to donor. But, the amount of doping may or maynot be equal. This type of junction also called step graded junction, does not exist in practice.

Graded Junction

If $m \neq 0$, the junction is said to be a graded p-n junction. When m = 1, $N_D(x) = Gx$ i.e., the doping on the n-side increases linearly with increase in distance x. A p-n junction with this type of doping profile is called linearly graded diode. If m = $\frac{-3}{2}$, then the diode is said to be hyper abrupt. For simplicity of analysis our discussion hereafter is with respect to abrupt p-n junction.

3.2 P-N JUNCTION UNDER THERMAL EQUILIBRIUM

Let a p-material with doping concentration 10^{16} cm^{-3} and n-material with doping 10^{15} cm^{-3} are brought together to form a contact at room temperature. The carrier concentrations in the p and n materials just before formation of junction are shown in Fig. 3.2(a).

It is evident from the figure that large gradient in hole concentration exists between p-material and n-material. Therefore, holes diffuse from p-side to n-side of the junction. The diffused holes recombine with electrons in the n-material. As a result, uncompensated acceptor ions are left behind in the p-side and donor ions on the n-side as shown in Fig. 3.2(c). Similarly, electrons diffuse from n-side to p-side which recombines with holes on the p-side. This also forms uncompensated ions on both n and p sides. A electron hole pair recombines equal number of negative ions and positive ions are formed.

The diffusion of charge carriers across the junction forms a region of uncompensated negative ions on the p-side and positive ions on the n-side as shown in Fig. 3.2(c). This region of immobile charges is called depletion region or space charge region.

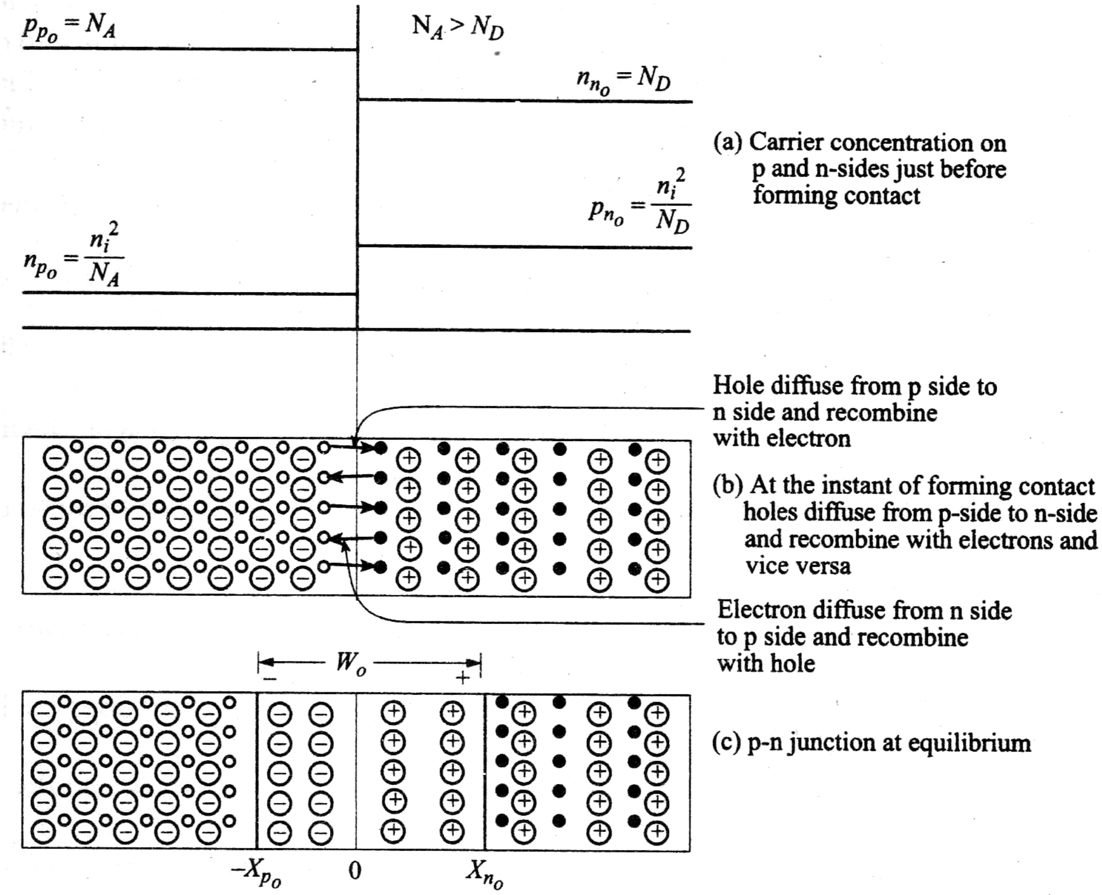


Fig. 3.2 Formation of depletion layer in p-n junction

The dipole effect of depletion layer charges result in a potential difference between the two ends of the depletion layer. This develops an electric field across the depletion layer which is directed from n-side to p-side (negative). This electric field aids the flow of minority carriers (drift current) across the junction. At the same time, the presence of depletion layer reduces the gradient of carrier concentration, resulting in decrease of diffusion current. As the diffusion process continues, the depletion layer widens and the electric field across the depletion layer increases i.e., the diffusion current decreases and drift current increases. Thermal equilibrium is attained when drift current balances with diffusion current. Since, the direction of drift and diffusion currents are opposite to each other, the net current at equilibrium is zero.

The potential appearing across the two ends of the depletion layer at equilibrium is called built-in potential or barrier potential or contact potential (V_o) or diffusion potential. This potential is called built-in potential because it is "built-in" the semiconductor and not an externally applied potential. The difference in potential causes a difference in energy ($\Delta E = qV_o$) between the edges of depletion layers. This energy difference between the p and n sides of the conduction band acts as a barrier for electron flow from n-side to p-side. Similarly, the same energy difference exist between the valence band edges which acts as barrier for hole movement from p-side to n-side. This potential thus acts as a barrier for the flow of majority carriers, hence, the name barrier potential. This potential is also called contact potential as it develops due to the physical contact between two semiconductors. This potential is developed as a result of diffusion of majority carriers across the junction. Hence, the name diffusion potential.

The region which is depleted of mobile charge carriers at the junction is called the depletion layer or the space charge layer. It has the following properties:

(1) The depletion region is depleted of mobile charges (electrons and holes).

(2) The p-side of depletion layer is negatively charged and n-side of depletion layer is positively charged.

(3) Number of negative ions on the p-side equals the number of positive ions on the n-side. Or the depletion region as a whole is neutral.

(4) A potential difference exists between the two ends of depletion layer because of the dipole effect of oppositely charged immobile ions.

(5) The width of the depletion layer decrease with increase in dopings.

(6) If the dopings are equal in p and n sides, the widths of depletion layers are equal on p and n sides ($X_p = X_n$).

(7) If one side is lightly doped and the other is heavily doped, the lightly doped side will have more width and heavily doped side less width.

(8) The width of the depletion layer increases with increase in reverse-bias and decreases with increase in forward-bias.

(9) If a voltage is applied across the p-n junction, it drops across the depletion layer. This is because the depletion layer has very high resistance compared to the resistance of the region outside the depletion layer (neutral region).

The direction of currents across a p-n junction under equilibrium are shown in Fig. 3.3. Under thermal equilibrium the net current across the junction must be zero. Thus, the electron and hole components of currents must be separately equal to zero.

$$J_n = J_{n\,(diff)} + J_{n\,(drift)} = 0$$
$$J_p = J_{p\,(diff)} + J_{p\,(drift)} = 0$$

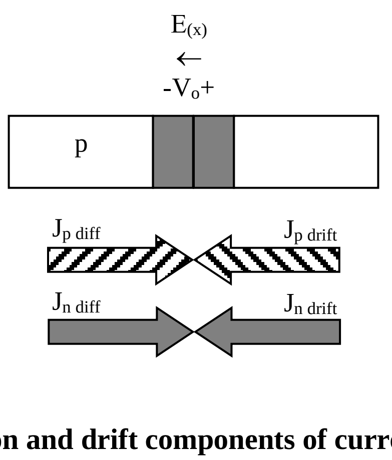


Fig. 3.3 Directions of diffusion and drift components of current in a p-n junction under thermal equilibrium

Constancy Of Fermi Level at Equilibrium

At thermal equilibrium net current through the diode is zero.

$$J_p = J_n = 0$$

$$J_p = q p_o \mu_p E_x - qD_p \frac{dp_o}{dx} = 0$$

$$p_o E_{(x)} = \frac{D_p}{\mu_p}\frac{dp_o}{dx} \quad \text{(A)}$$

$$p_o = n_i\, e^{(Ei-E_F)/kT}$$

$$\frac{dp_o}{dx} = \frac{n_i}{kT} e^{(Ei-E_F)/kT}\left[\frac{dE_i}{d_x} - \frac{dE_F}{dx}\right]$$

$$E_{(x)} = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{E_i}{-q}\right) = \frac{1}{q}\frac{dE_i}{d_x}$$

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

Substituting this in eqn (A)

$$\therefore\ p_o \frac{1}{q}\frac{dE_i}{d_x} = \frac{kT}{q}.\frac{n_i}{kT} e^{(Ei-E_F)/kT}\left[\frac{dE_i}{d_x} - \frac{dE_F}{dx}\right]$$

$$= \frac{1}{q} p_o \left[\frac{dE_i}{d_x} - \frac{dE_F}{dx}\right]$$

i.e., $$\frac{dE_F}{dx} = 0$$

3.2.1 Equilibrium Energy Band Diagram

The equilibrium energy band diagram of a p-n junction can be drawn with the help of the following principles:

(1) The Fermi level is a single horizontal line $\left(\frac{dE_F}{dx} = 0\right)$ under thermal equilibrium.

(2) Depletion layer is depleted of mobile charge carriers and the regions outside the depletion layer is neutral. (This is called depletion approximation).

(3) The electric field in the neutral region is zero. Therefore, the bands are flat in the neutral n and p regions. The Fermi level position in the neutral regions depend only on the dopings. (This is true for diodes with bias also).

(4) The energy bands bend upward in the direction of the electric filed in the depletion layer as explained in Section 1.13.9, or the direction of energy barrier is opposite to the direction of potential barrier.

The steps involved are as shown in Fig. 3.4 and listed below.

(1) Draw the equilibrium Fermi level.
(2) Mark the depletion region (where the electric field exists and band bends).
(3) Draw the valence band edge E_V on the p-side (E_{Vp}) and conduction band edge E_C on the n-side (E_{Cn}) relative to E_F.
(4) Draw the other edges of the band, E_{Cp} and E_{Vn}, keeping constant band gap on both sides.
(5) Connect E_{Cp} to E_{Cn} and E_{Vp} to E_{Vn} which completes the energy band diagram. We may derive the following information from energy band diagram in Fig. 3.4(b).
(1) The energy barrier for electron movement from conduction band on n-side to conduction band on p-side is E_{Cp} - E_{Cn}. The energy barrier for the hole movement from p-side valence band to n-side valence band is E_{Vp} - E_{Vn}.

The gradient in all energies (E_C, E_V and E_i) are identical in the energy band diagram.

i.e., $$E_{Cp} - E_{Cn} = E_{Vp} - E_{Vn} = E_{ip} - E_{in}$$

Thus, the potential across the junction is

$$V_o = \frac{E_{ip} - E_{in}}{q}$$

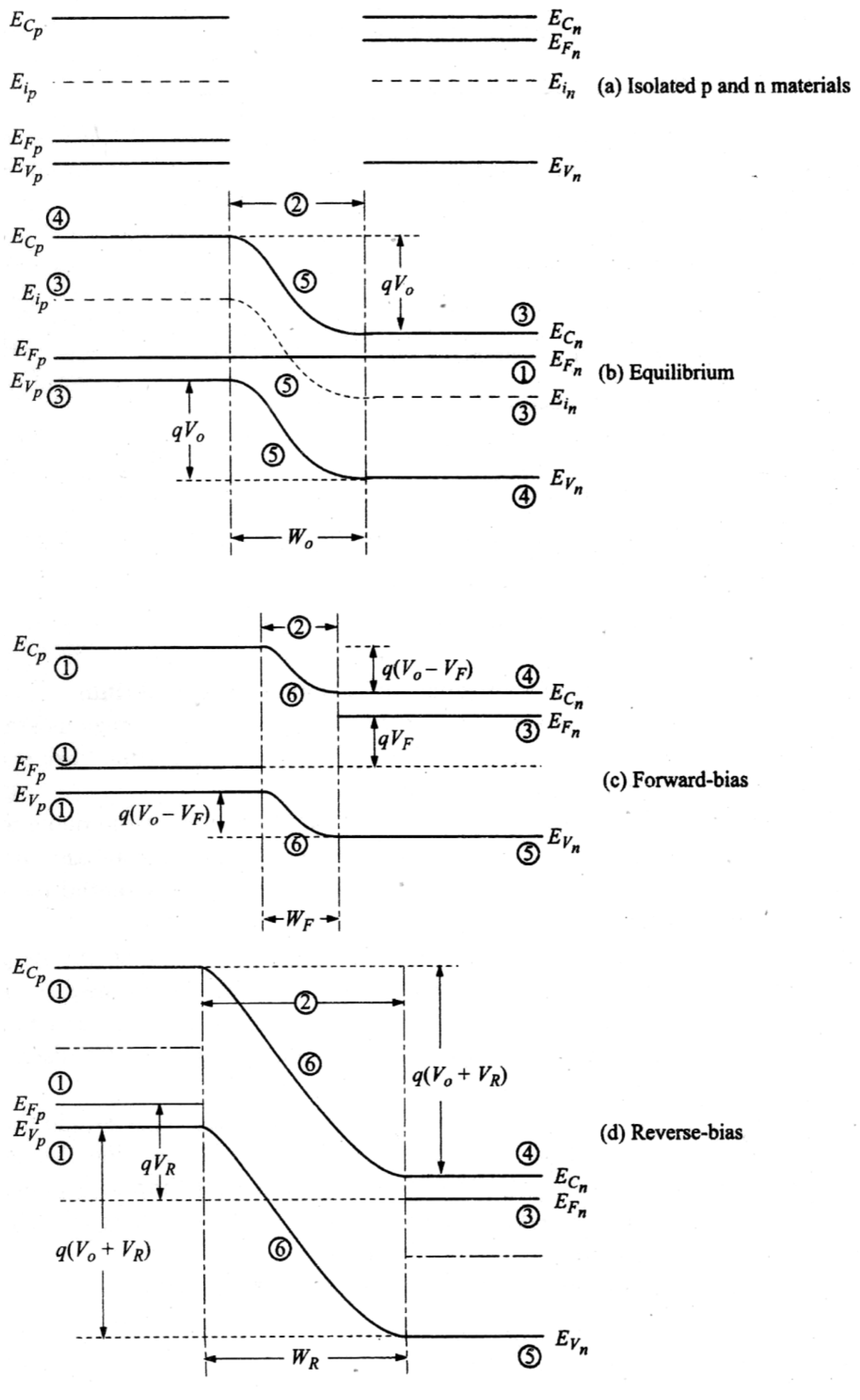
$$= \frac{(E_{ip} - E_F)}{q} - \frac{(E_{in} - E_F)}{q}$$

$$= \phi F_p - \phi F_n \tag{3.3}$$

where ϕF_p and ϕF_n are the Fermi potentials on the p and n sides of the junction. Notice that Fermi potential on the n-side (ϕF_n) is negative.

(2) The Fermi potentials increase with increase in doping. Therefore, the built-in potential also increases with increase in doping on the p and n sides.

(3) The equilibrium energy band diagram confirms that an electric field exists in the depletion layer which is directed from n-side to p-side (energies bend upward in the direction of electric field).



3.4 Energy band diagram of p-n junction. The circled numbers represent the sequence of drawing the diagram

3.2.2 Distribution of Carrier Concentration, Potential, Electric Field and Charge Density

Fig. 3.5 depicts the distribution of carrier concentrations, potential, electric field and charge, density. In an abrupt p-n junction under thermal equilibrium, meanings of the symbols used are:

p_{po}	-	equilibrium hole concentration on the p-side.
n_{po}	-	equilibrium electron concentration on the p-side.
n_{no}	-	equilibrium electron concentration on the n-side.
p_{no}	-	equilibrium hole concentration on the n-side.
W_o	-	equilibrium depletion layer width.
X_{no}	-	equilibrium depletion layer width towards n-side.
X_{po}	-	equilibrium depletion layer width towards p-side.
V_{no}	-	equilibrium potential at neutral n-side.
V_{po}	-	equilibrium potential at neutral p-side.

Suffixes n and p stands for n and p regions and o stands for equilibrium.

Under thermal equilibrium, the carrier concentrations in the neutral regions are same as the carrier concentrations in the isolated p and n materials. The depletion layer is depleted of mobile charge carriers. (By depletion approximation).

The potential rises from V_{po} on the p-side to V_{no} on the n-side of the depletion layer. The region outside depletion layer is neutral and potential remains constant throughout the neutral regions. The built-in potential equals the difference in potential between the two sides of depletion layer, $V_o = V_{no} - V_{po}$.

The electric field in the neutral region is zero, as there is no gradient in potential (negligible potential drop due to very high conductivity). The electric field is maximum (E_m) at the interface due to the abrupt transition from negative charges on n-side to positive charges on p-side of depletion layer. For abrupt p-n junction its distribution is linear as shown in Fig. 3.5(d).

The charge density distribution is shown in Fig. 3.5(e). The density is zero in the neutral region. Charge is negative on the p-side and positive on the n-side. The charge density is charge per unit volume. This is equal to the charge of ionized impurity multiplied by the doping concentration (by depletion pproximation). Therefore, charge density on the p-side is $-qN_A$ and charge density on the n-side is $+qN_D$.

The magnitude of charge on both sides are equal so that net charge is zero

$$\left|Q_{D_p}\right| = \left|Q_{D_n}\right|$$

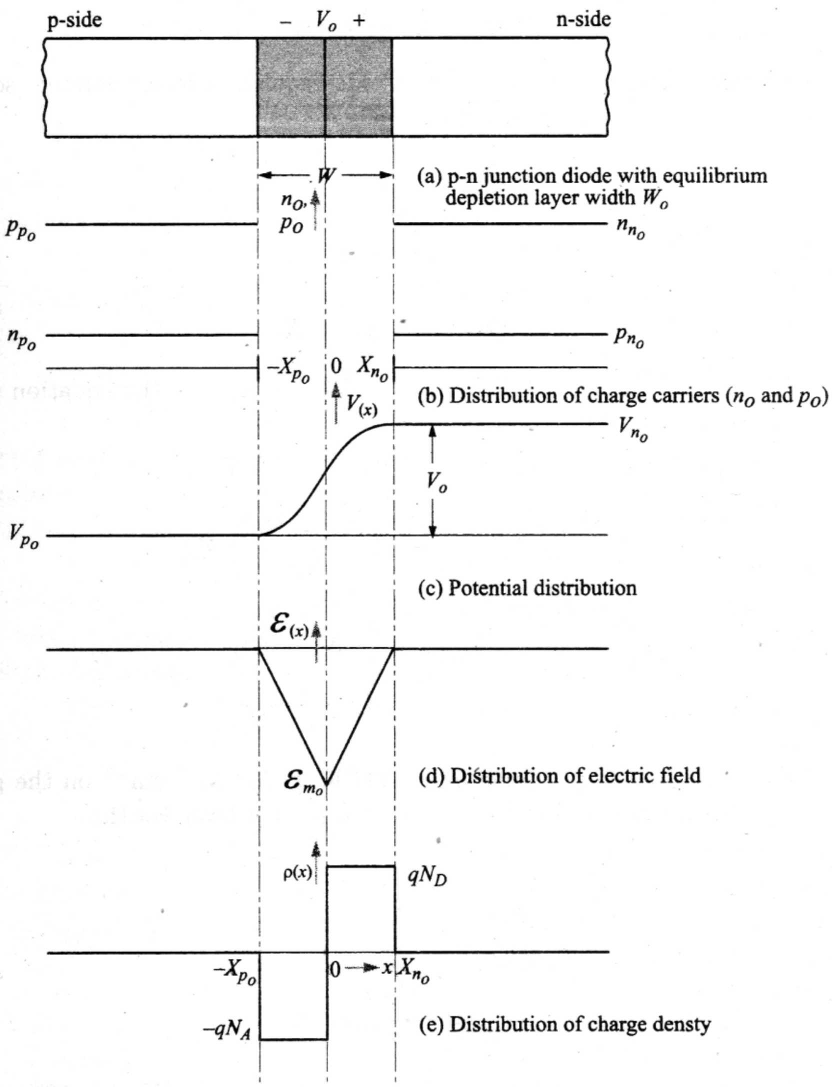


Fig. 3.5

$|Q_{D_p}|$ = Total negative charge on the p-side of depletion layer

$|Q_{D_n}|$ = Total positive charge on the n-side Of depletion layer

$|Q_{D_p}|$ = |charge density| × volume of depletion layer on p-side

$$= qN_AX_{po}A \tag{3.4}$$

where A is the area of cross-section of diode which is equal for both sides of depletion layer. i.e.,

$$|Q_{D_n}| = qN_DX_{no}A \tag{3.5}$$

$$qN_AX_{po}A = qN_DX_{no}A$$

$$N_A X_{po} = N_D X_{no} \tag{3.6}$$

To occupy equal charge, the heavily doped side requires a lesser volume, so that the depletion layer width is less on heavily doped side.

$$\frac{N_A}{N_D} = \frac{X_{n_o}}{X_{p_o}} \tag{3.6a}$$

We also know that

$$W_o = X_{n_o} + X_{p_o}$$

$$= \frac{N_A}{N_D} X_{p_o} + X_{p_o} \qquad \text{(by equation (3.(6a))}$$

$$= \left(\frac{N_A}{N_D} + 1 \right) X_{p_o}$$

$$= \left(\frac{N_A + N_D}{N_D} \right) X_{p_o}$$

$$X_{p_o} = W_o \frac{N_A}{N_A + N_D} \tag{3.7}$$

Similarly,

$$X_{n_o} = W_o \frac{N_A}{N_A + N_D} \tag{3.8}$$

Example 3.1 For a silicon p-n junction at 300 K, $N_A = 10^{18}$ cm^{-3} on the p-side and $N_D = 10^{14}$ cm^{-3} on the n-side. Find the ratio of depletion layer widths.

Solution

$$\frac{X_{n_o}}{X_{p_o}} = \frac{N_A}{N_D}$$

$$= \frac{10^{18}}{10^{14}} = -10^4$$

This shows that depletion width on the lightly doped side (X_{no}) is 10^4 times larger than the width of depletion layer on the heavily doped side (X_{po}) or $X_{no} = 10^4 X_{po}$.

3.2.3 The Built-in Potential

Built in potential is the potential across the depletion layer of a p-n junction under thermal equilibrium.

Under thermal equilibrium,

$$J_p = 0 \quad \text{and} \quad J_n = 0$$

Let us start with one of these equations to arrive at an expression for built-in potential (V_o), as it is a thermal equilibrium quantity.

$$J_p = -qD_p . \frac{dp_o(x)}{dx} + qp_o(x)\, \mu_p\, E(x) = 0 \tag{3.9}$$

$$\therefore \quad E_{(x)} = \frac{D_p}{\mu_p} \times \frac{1}{p_o(x)} \cdot \frac{dp_o(x)}{dx}$$

(3.10)

But, $\frac{D_p}{\mu_p} = \frac{kT}{q}$ and $E_{(x)} = -\frac{dV}{dx}$

$\therefore$ Equation (3.10) reduces to

$$-\frac{dV}{dx} = \frac{kT}{q} \frac{1}{p_o(x)} \cdot \frac{dp_o(x)}{dx} \qquad (3.11)$$

Integrating equation (3.11) from edge of depletion layer on the p-side to edge of depletion layer on the n-side.

$$-\int dV = -\int \frac{kT}{q} \cdot \frac{1}{p_o(x)} . dp_o(x)$$

On L.H.S integration is with respect to potential and on R.H.S integration is with respect to hole concentration. By substituting values of potential and carrier concentrations at edges of depletion layer from Fig. 3.5(b) and (c).

$$-\int_{V_{po}}^{V_{no}} dV = \frac{kT}{q} \int_{p_{po}}^{p_{no}} \frac{dp_o(x)}{p_o(x)}$$

$$-\left[V_{no} - V_{po}\right] = \frac{kT}{q} \ln[p_o(x)]_{p_{po}}^{p_{no}}$$

$$= \frac{kT}{q}\left[\ln p_{no} - \ln p_{po}\right]$$

$$V_{no} - V_{po} = \frac{kT}{q}\left[\ln p_{po} - \ln p_{no}\right]$$

$$= \frac{kT}{q} \ln \frac{p_{po}}{p_{no}}$$

From Fig. 3.5, $V_{no} - V_{po}$ = V_o

$$\therefore \quad V_o = \frac{kT}{q} \ln \frac{p_{po}}{p_{no}} \qquad (3.12)$$

p_{po} = N_A (equilibrium majority carrier concentration on p-side)

p_{no} = $\frac{n_i^2}{N_D}$ (equilibrium minority carrier concentration on n-side)

$\therefore$ Equation (3.12) becomes

$$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \qquad (3.13)$$

Equation (3.12) may also be written as

$$\frac{p_{po}}{p_{no}} = e^{qV_o/kT} \qquad (3.14a)$$

Similarly if we start with $J_n = 0$; it can be shown that

$$\frac{n_{no}}{n_{po}} = e^{qV_o/kT} \tag{3.14b}$$

Equation (3.13) shows that

(1) The built-in potential increases with increase in dopings (N_A and N_D) for a given material at given temperature (given n_i, and T).
(2) For given dopings (N_A and N_D) and temperature, built-in potential increases with increase in band gap because n_i, decreases with increase in band gap.
(3) For a given material and doping built-in potential decreases with increase in temperature. This is because n_i is a highly temperature dependant quantity which increases exponentially with increase in temperature, reducing V_0.

Built-in Voltage in Terms of Fermi Potentials

Fermi potential on the p side

$$\phi_{F_p} = \frac{E_{i_p} - E_{F_p}}{q}$$

Fermi potential on the n side

$$\phi_{F_n} = \frac{E_{i_n} - E_{F_n}}{q} \ (-ve)$$

Built-in potential

$$V_0 = \phi_{F_p} - \phi_{F_n}$$

$$= \frac{E_{i_p} - E_{i_n}}{q} \ (\because E_{Fp} - E_{F_n} \text{ under equilibrium})$$

Example 3.3 also illustrates this relation but in a different approach.

Example 3.2 An abrupt p-n junction made of silicon has $N_A = 10^{18}$ cm^{-3} on the p-side and $N_D = 10^{15}$ cm^{-3} on the n-side. At 300 K,

a. Find the position of Fermi levels on p and n sides,
b. Draw the equilibrium energy band diagram and determine the built-in potential from the diagram,
c. Compare the built-in potential obtained from the EB diagram, with the calculated value using the expression for V_o.

Solution

$$p_{po} = N_A = 10^{18} \text{ cm}^{-3}$$
$$n_{no} = N_D = 10^{15} \text{ cm}^{-3}$$
Let $$n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

a. On p-side

$$p_{po} = n_i\, e^{(E_{ip} - E_{Fp})/kT}$$

$$E_{ip} - E_{Fp} = kT \ln \frac{p_{po}}{n_i}$$

$$= 0.026 \ln \frac{10^{18}}{1.5\times10^{10}}$$

$$= 0.468 \text{ eV}$$

On n-side

$$n_{no} = n_i\, e^{(E_{Fn}-E_{in})/kT}$$

$$E_{Fn} - E_{in} = kT \ln \frac{n_{no}}{n_i}$$

$$= 0.026 \ln \frac{10^{15}}{1.5\times10^{10}}$$

$$= 0.289 \text{ eV}$$

b. From Fig. Ex.3.2

$$qV_o = E_{ip} - E_{Fp} + E_{Fn} - E_{in}$$

$$= 0.468 + 0.289$$

$$= 0.757 \text{ eV}$$

$$\therefore \quad V_0 = 0.757\text{V}$$

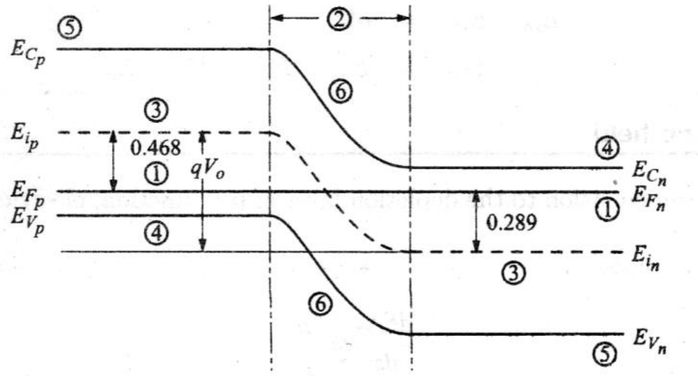


Fig. Ex.3.2

c. $$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$= \frac{10^{18}\times10^{15}}{(1.5\times10^{10})^2}$$

$$= 0.757 \text{ V}$$

This is same as the value obtained from the energy band diagram.

Example 3.3 Show that $qV_o = E_{V_p} - E_{V_n} = E_{i_p} - E_{i_n}$

Solution

From equation (3.14)

$$\frac{p_{p_o}}{p_{n_o}} = e^{qV_o/kT} \tag{A}$$

Under equilibrium Epp = Epn. By equation (1.48)

$$\frac{p_{p_o}}{p_{n_o}} = \frac{N_V e^{-(E_{Fp}-E_{Vp})/kT}}{N_V e^{-(E_{Fn}-E_{Vn})/kT}}$$

$$= e^{(E_{Vp}-E_{Vn})/kT} \tag{B}$$

From (A) and (B)

$$E_{V_p} - E_{V_n} = qV_o$$

Also, by equation (1.54)

$$\frac{p_{p_o}}{p_{n_o}} = \frac{n_i e^{(E_{ip}-E_{Fp})/kT}}{n_i e^{(E_{in}-E_{Fn})/kT}}$$

$$= e^{(E_{ip}-E_{in})/kT} \tag{C}$$

From (A) and (C)

$$E_{ip} - E_{in} = qV_o$$

$$\therefore qV_o = E_{V_p} - E_{V_n} = E_{ip} - E_{in}$$

3.2.4 Electric Field

Applying Poisson equation to the depletion layer of p-n junction, electric field distribution can be obtained

Poisson equation is

$$\frac{d\text{E}_{(x)}}{dx} = \frac{\rho}{\grave{o}} \tag{3.15}$$

which relates gradient in electric field to charge density.

Applying this to a semiconductor $\frac{d\text{E}_{(x)}}{dx} = \frac{q}{\grave{o}}\left(N_A^+ - N_A^- + p - n\right)$

For $-X_{po} < x < 0$ (Refer Fig. 3.5(e)) (p-side of depletion layer)

$$\rho = -qN_A \text{ (by depletion approximation p = n = 0)}$$

$$\frac{d\text{E}_{(x)}}{dx} = \frac{-qN_A}{\grave{o}}$$

$$\text{E}_{(x)} = \int \frac{-q}{\grave{o}} N_A dx + C$$

$$= \frac{-q}{\grave{o}} N_A x + C$$

But $\text{E}_{(x)} = 0$ at $x = -X_{po}$

$$\therefore C = \frac{-q}{\grave{o}} N_A X_{po}$$

$$E_{(x)} = \frac{-q}{\grave{o}} N_A (x + X_{po}); -X_{po} < x < 0 \qquad (3.16)$$

Similarly, applying Poisson equation to the n-side of the depletion layer, we get

$$E_{(x)} = \frac{-q}{\grave{o}} N_D (x - X_{no}); -0 < x < X_{no} \qquad (3.17)$$

Electric field is maximum at x = 0. Therefore, by equations (3.16) and (3.17).
Maximum electric field

$$E_{m_o} = \frac{-q}{\grave{o}} N_A X_{po} = \frac{-q}{\grave{o}} N_D X_{no} \qquad (3.18)$$

$$\therefore \quad E_{(x)} = \varepsilon_{m_o} \left(1 + \frac{x}{X_{p_o}}\right); \ -X_{p_o} < x < 0 \qquad (3.18a)$$

$$E_{(x)} = \varepsilon_{m_o} \left(1 + \frac{x}{X_{n_o}}\right); \ 0 < x < X_{n_o} \qquad (3.18b)$$

Equations (3.18) shows that electric field varies linearly with distance in the depletion region.

3.2.5 Potential Distribution

The potential distribution on the p-side may be evaluated by assuming the potential at x = -X_{p_o} as zero.

$$V_p(x) = -\int E_{(x)} dx \left(-X_{p_o} \le x \le\right) \qquad (3.19)$$

$$= -\int E_{m_o} \left(1 + \frac{x}{X_{p_o}}\right) dx$$

$$= -E_{m_o} \left(x + \frac{x^2}{2X_{p_o}}\right) + C$$

At $x = -X_{po}$, $\quad V_p(x) = 0$

$$\therefore \quad C = -E_{m_o} \frac{X_{p_o}}{2}$$

$$\therefore \quad V_p(x) = -E_{m_o} \left(x + \frac{x^2}{2X_{p_o}} + \frac{X_{p_o}}{2}\right); -X_{p_o} < x < 0 \qquad (3.20)$$

$$V_n(x) = -\int E_{(x)} dx; \quad (0 \le x \le -X_{n_o}$$

$$= -E_{m_o} \left(x - \frac{x^2}{2X_{n_o}}\right) + \ C \ \text{ by equation (3.18b)}$$

Since, the potential is continuous at x = 0,

$$V_n = V_p \quad \text{at} \quad x = 0$$

i.e., $$C = \frac{-E_{m_o} X_{p_o}}{2}$$

$$\therefore\ V_n(x) = -E_{m_o}\left(x - \frac{x^2}{2X_{n_o}} + \frac{X_{p_o}}{2}\right); 0 < x < X_{n_o} \tag{3.21}$$

The zero level for potential is set at x = X_{po}.
Therefore, the potential drop in the depletion layer is obtained by setting x = X_{no} in $V_n(x)$.

i.e., $$V_o = V_n(X_{n_o}) = -E_{m_o}\left[X_{n_o} - \frac{x_{n_o}^2}{2X_{n_o}} + \frac{X_{p_o}}{2}\right]$$
$$= \frac{E_{m_o}}{2}\left(X_{n_o} + X_{p_o}\right)$$
$$= \frac{E_{m_o} W_o}{2} \tag{3.22}$$

3.2.6 Width of Depletion Layer

$V_o = \int_{-X_{po}}^{X_{no}} E(x)dx$ = area of triangle formed by $E_{(x)}$ with x axis as in Fig. 3.6.

$$V_o = \frac{-1}{2} E_{m_o} W_o \textit{ where } E_{m_o} = \frac{-q}{\text{ò}} N_A X_{po}$$
$$= \frac{-1}{2} \times \frac{-q}{\text{ò}} N_A X_{po} . W_o$$

But, $$X_{p_o} = W_o . \frac{N_D}{N_A + N_D} . W_o^2$$

$$\therefore\ V_o = \frac{q}{2\text{ò}} \frac{N_A N_D}{N_A + N_D} . W_o^2$$

and $$W_o = \sqrt{\frac{2\text{ò}V_o}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)} \tag{3.23}$$

where ò = ò$_0$ ò$_r$
ò - permittivity of semiconductor material
ò$_0$ - permittivity of free space (8.854×10^{-14} F/cm)
ò$_r$ - relative permittivity of material.

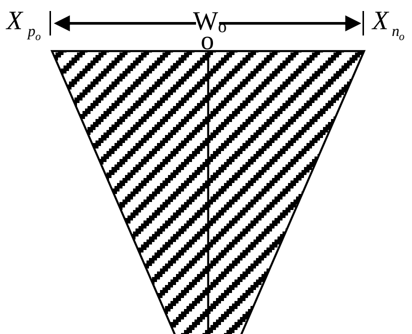


Fig. 3.6

Equation (3.23) shows that the width of depletion layer decrease with increase in doping concentrations.

Example 3.4 An abrupt silicon (Si) p-n junction has $N_A = 10^{17}$ cm^{-3} on the p-side and $N_D = 10^{15}$ cm^{-3} on the n-side. The area of cross-section of the diode is 10^{-4} cm^2.
The relative permittivity of Si is 11.8. Determine the following at 300 K:

a. the built-in voltage,

b. the depletion layer width W_o, X_{no}, and X_{po},

c. the maximum electric field,

d. the charge on one side of depletion layer.

Plot electric field and charge density to scale.

Let n_i for Si at 300 K be 1.5×10^{10} cm^{-3}

Solution

a. Built-in voltage

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026 \text{ In } \frac{10^{17} \times 10^{15}}{\left(1.5 \times 10^{10}\right)^2} = 0.697\text{V}$$

b. Depletion layer width

$$V_o = \sqrt{\frac{2 \in_0 \in_r Vo}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2 \times 8.854 \times 10^{-14} \times 11.8 \times 0.697}{1.6 \times 10^{-19}}\left(\frac{1}{10^{17}} + \frac{1}{10^{15}}\right)}$$

$= 9.588 \times 10^{-5}$ cm

$X_{no} \quad = W_o \dfrac{N_A}{N_A + N_D}$

$= 9.588 \times 10^{-5} \times \dfrac{10^{17}}{10^{17}+10^{15}}$

$= 9.493 \times 10^{-5}$ cm

$X_{po} \quad = W_o \dfrac{N_D}{N_A + N_D}$

$= 9.588 \times 10^{-5} \times \dfrac{10^{15}}{10^{17} - 10^{15}}$

$= 9.493 \times 10^{-7}$ cm

c. Maximum electric field

$$E_{m_o} = \frac{-q}{\varepsilon_0 \varepsilon_r} N_A X_{po}$$

$$= \frac{-1.6\times10^{-19}\times10^{17}\times9.493\times10^{-7}}{8.854\times10^{-14}\times11.8}$$

$= -14.54 \times 10^{3}$ V/cm

d. Charge on one side of depletion layer

$|Q_D|$ = Charge density $\times$ volume of depletion layer
$= qN_A \times X_{po} \times A$
$= 1.6 \times 10^{-19} \times 10^{17} \times 9.493 \times 10^{-7} \times 10^{-4}$
$= 15.189 \times 10^{-13}$ C

Charge density on p-side $= -qN_A$
$= -1.6 \times 10^{-19} \times 10^{17}$
$= -1.6 \times 10^{-2}$ C/cm^3

Charge density on n-side $= qN_D$
$= 1.6 \times 10^{-19} \times 10^{15}$
$= 1.6 \times 10^{-4}$ C/cm^3

Figure Ex. 3.4 shows the electric field and charge density distribution.

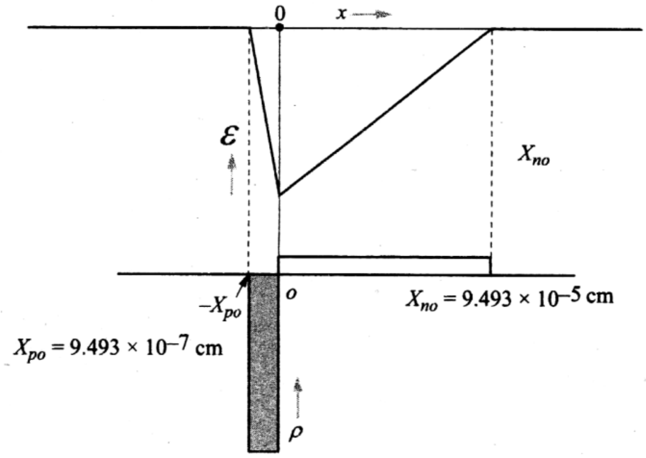


Fig. Ex.3.4

3.3 BIASING OF P-N JUNCTION

Under thermal equilibrim, p-side of a p-n junction is at a lower potential with respect to n-side as shown in Fig. 3.7(a). Application of external potential (V_a) to this junction is called biasing.

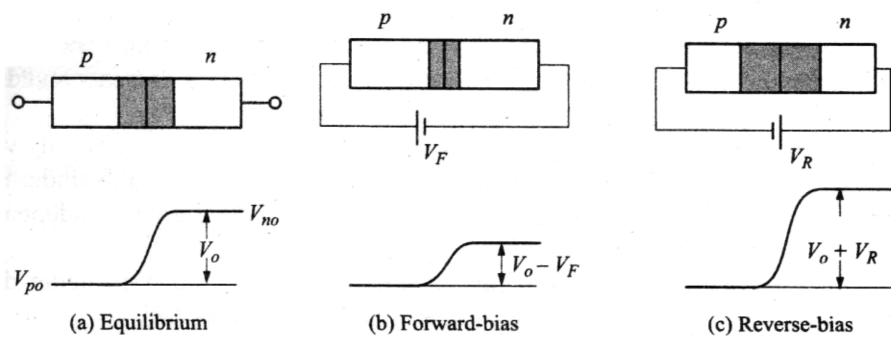


Fig. 3.7 Biasing of p-n junction and potential distribution

Forward-bias: Applying positive potential (V_F) to p-side with respect to n-side is called forward-biasing. Now the potential on the n-side lowers with respect to that on p-side as shown in Fig. 3.7(b). The potential barrier at the junction reduces to V_o - V_F.

Reverse-bias: Applying negative potential ($-V_R$) to p-side with respect to that on the n-side is called reverse-biasing. Now the potential on the n-side increases with respect to the equilibrium potential. Or the potential barrier at the junction increases to ($V_o + V_R$).

Application of bias voltage (V_a) across the junction changes the parameters of p-n junction as follows.

$V_o \rightarrow V_o - V_a$

V_a - applied voltage (positive for forward-bias and negative for reverse-bias)

$$W_o \rightarrow W = \sqrt{\frac{2\epsilon(V_0 - V_a)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$X_{po} \rightarrow X_n = W. \frac{N_D}{N_A + N_D}$$

$$X_{no} \rightarrow X_n = W. \frac{N_A}{N_A + N_D}$$

$$E_{m_o} \rightarrow E_m = \frac{-q}{\epsilon} N_A X_p = \frac{-q}{\epsilon} N_D X_n$$

With increase in forward-bias ($V_a = +V_F$), barrier potential reduces, width of depletion layer decreases, maximum electric field decreases and charge in depletion layer decreases. With increase in reverse-bias ($V_a = V_R$) all the above quantities increases.

The energy band diagram of p-n junction under thermal equilibrium, forward-bias and reverse-bias are shown in Fig. 3.8. It also shows the movement of charge carriers by drift and diffusion across the junction. Under equilibrium the energy barrier is in such a way that drift and diffusion currents are equal. With forward-bias the energy barrier reduces, so that diffusion currents exceeds drift current and net current flows from p-side to n-side. With increase in forward-bias, barrier for majority carrier movement decrease and diffusion current increases, increasing the net current from p to n.

With reverse-bias, barrier for majority carrier movement increase, so that only very few electrons can crossover the barrier. Therefore, diffusion current is negligible under reverse-bias. Therefore, there will be a net current from n to p due to the drift of minority carriers across the junction, which remains almost independent of the applied reverse-bias.

Fig. 3.8 illustrates qualitatively the current flow across a p-n junction under different bias conditions.

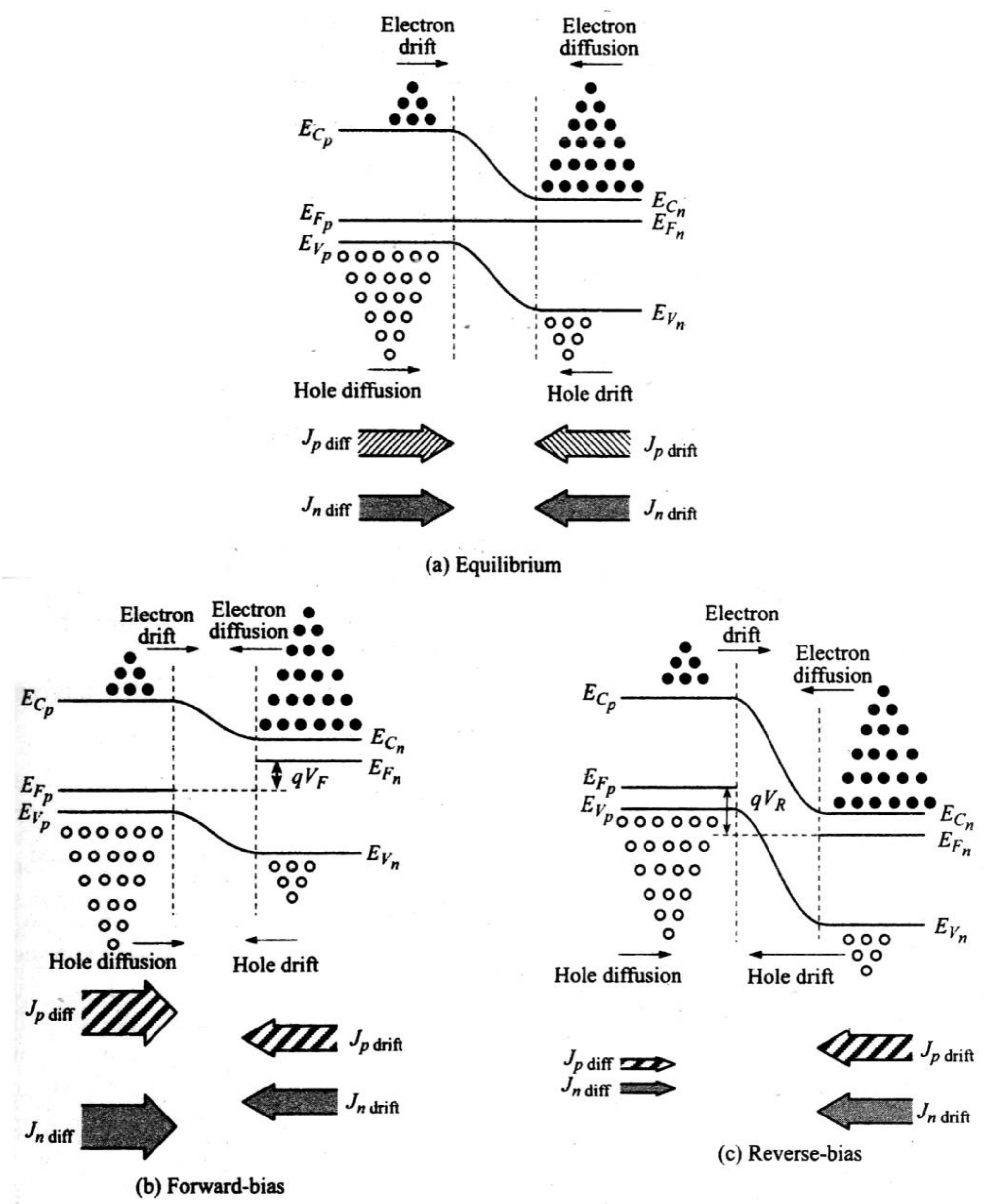


Fig. 3.8 Qualitative description of current flow across p-n junction. The drift components of current remain almost unchanged with bias voltage and diffusion component vary with bias due to change in energy barrier

3.3.1 Energy band diagram of biased p-n junction

The energy band diagrams of forward-biased and reverse-biased p-n junctions are shown in Figs. 3.4 and 3.8. The following guidelines may be used to draw the energy band diagrams.

Forward-bias

Step 1: draw energy levels on one side (p-side) same as that in equilibrium condition.
Step 2: Leave space for depletion layer (less than that in thermal equilibrium).
Step 3: With forward-bias, energy barrier reduces by qV_F. Therefore E_{Cn}, E_{Fn} and E_{Vn} move up by qV_F relative to equilibrium.
Step 4: Complete energy band diagram on n-side. Connect E_{Cp} to E_{Cn}, E_{Vp} to E_{Vn}.

Reverse-bias

Steps (1) and (2) are same as above (depletion layer width is more in this case). Step (3) energy barrier increases by qV_R. Therefore, E_F and E_V on the n-side move down by qV_R relative to equilibrium. Step (4) is same as that in the case of forward-bias.

It may be noted from the energy band diagram that the potential energy of electron changes in opposite direction to the applied potential. If positive potential (V_F) is applied to p-side, the potential on the p-side (V_p) increase, reducing the energy barrier to $q(V_o - V_F)$. If negative potential (V_R) is applied to p-side, potential on p-side (V_p) decrease, increasing energy barrier to $q\,(V_o + V_R)$.

It can also be noticed that the shift in Fermi level equals q times the applied voltage ($\Delta E_F = qV_a$).

Fermi level shifts downward on the higher potential side (with increase in electrostatic potential, electron potential energy decrease $E = (-q)V$) and vice versa.

3.3.2 Qualitative Description of Current Flow Through a Diode

Under thermal equilibrium, the electron and hole components currents are separately zero and the net current is also zero. This is because the drift and diffusion components of electron and hole currents are equal and opposite.

When a forward-bias is applied across the junction, the energy barrier reduces, so that more electrons diffuse over the barrier from n-side conduction band to p-side conduction band and more holes diffuse from p-side valence band to n-side valence band (the carrier gradient across the depletion layer increases with increase in forward-bias due to reduced width of depletion layer). This increases the electron and hole diffusion components of currents. The drift current depends on the number of charge carriers involved in the current flow mechanism. This current is due to the thermally generated minority carriers, the concentration of which remain constant at a given temperature. Therefore, the drift current remain almost constant irrespective of bias. Hence, the net current which is the difference between diffusion and drift current increase with increase in forward-bias.

When a reverse-bias is applied across the junction, the diffusion components of currents are negligible due to increased barrier. The net current, therefore is the relatively small drift current from n to p (negative) which is almost independent of the reverse voltage. This current due to the thermally generated minority carriers is called reverse saturation current or leakage current. This current can be increased by increasing the minority carrier concentration (by increasing temperature, by optical generation etc).

3.4 THE IDEAL DIODE EQUATION

The ideal diode equation represents the analytical relationship between applied voltage and current in a diode in terms of the diode parameters. We derive the equation based on the following approximations.

(1) Diode is long. Length of the neutral regions on the n and-p sides are large compared to minority carrier diffusion lengths.
(2) The depletion region is completely depleted of mobile charge carriers and the region outside the depletion layer is perfectly neutral (potential drop in this region is negligible, so that the minority carrier current in this region is by diffusion only).
(3) The p-n junction is abrupt (step-graded), so that the carrier concentrations are constant on neutral regions.
(4) The contacts at the two ends are perfect ohmic contacts and minority carrier density at the contact equals the equilibrium value.
(5) Low-level injection condition exist when the diode is forward-biased (injected excess carrier concentrations are low compared to the equilibrium majority carrier concentrations).
(6) The temperature and doping are in such a way that the impurities are completely ionized.
(7) There is no generation or recombination within the depletion region, so that electron and hole currents are constant throughout this region.
(8) The diode is in steady-state condition (the applied voltage is d.c.).

For deriving the diode equation we assume the origin of distances on the n-side of junction as the edge of the depletion layer on the n-side and the distances on the n-side are denoted by x_n. On the p-side also the origin is taken as the edge of the depletion layer and distances are denoted by x_p towards negative x-direction.

The current in a p-n junction is the sum of electron current and hole current. By approximation 7, the electron and hole current remain constant in the depletion region. The total diode current is $I_n + I_p$ at any point in the depletion region including the edges of depletion layers ($x_n = 0$ and $x_p = 0$). Therefore the current in a p-n junction can be evaluated as the sum of minority carrier diffusion currents at the edges of the depletion layer. (See also Fig. 3.11)

$$
\begin{aligned}
I &= I_p + I_n \\
&= I_p\,(x_n = 0) + I_n\,(x_p = 0) \text{ by approximation (7)} \\
&= I_{p\ diff}\,(x_n = 0) + I_{n\ diff}(x_p = 0) \text{ by approximation (2)}
\end{aligned}
$$

Fig. 3.9 shows the minority carrier distribution in a forward biased pn junction. Δ_{pn} represents excess hole concentration at $x_n = 0$ and Δ_{np} represents excess electron concentration at $x_p = 0$.

We have already shown that for a p-n junction under thermal equilibrium,

$$\frac{p_{p_o}}{p_{n_o}} = e^{qV_o/kT} \qquad \text{(by equation (3.14))} \qquad (3.24)$$

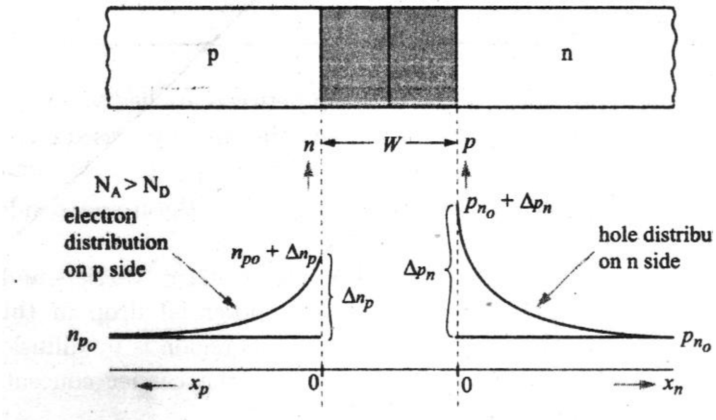


Fig. 3.9 Minority carrier distribution across a forward-biased p-n junctiol

With bias V_a applied, in the above equation V_o changes to $V_o - V_a$, p_{po} changes to $p_p(x_p = 0)$ (the total hole concentration at the edge of the depletion layer on p-side with bias V_a applied) and pn, changes to $p_n(x_n = 0)$.

$\therefore$ Equation (3.24) changes to

$$\frac{p_p(x_p = 0)}{p_n(x_n = 0)} = e^{q(V_o - V_a)/kT} \tag{3.25}$$

The majority carrier concentration remain almost unchanged for low-level injection.

$\therefore$ $p_p(x_p = 0) \cong p_{p_o}$ by approximation (5)

Equation (3.24) divided by equation (3.25) is

$$\frac{p_n(x_n = 0)}{p_{n_o}} = e^{qV_a/kT}$$

$$p_n(x_n = 0) = p_{n_o} e^{qV_a/kT}$$

Total hole concentration p_n at $x_n = 0$ is the sum of the equilibrium minority carrier (hole) concentration (pn.) and the injected excess minority carrier concentration (Δp_n)

$$p_n(x_n = 0) = p_{n_o} + \Delta p_n$$

or

$$\begin{aligned} \Delta p_n &= p_n(x_n = 0) - p_{n_o} \\ &= p_{n_o} e^{qV_a/kT} - p_{n_o} \\ &= p_{n_o}\left(e^{qV_a/kT} - 1\right) \end{aligned} \tag{3.26}$$

Similarly, excess electron concentration at $x_p = 0$ is given by

$$\Delta n_p = n_{p_o}\left(e^{qV_a/kT} - 1\right) \tag{3.27}$$

Equations (3.26) and (3.27) show that injected excess minority carrier concentrations increase exponentially with increase in forward-bias. Its magnitude also depends on the equilibrium minority carrier concentration i.e., the injected minority carrier concentration is more on the lightly doped side. Or there is more injection from heavily doped side to lightly doped side.

By approximations (2) and (8), the continuity equation for holes on the n-side under steady-state, assuming that the minority carrier current is by diffusion only, reduces to

$$\frac{d^2 \delta p(x_n)}{dx_n^2} = \frac{\delta p(x_n)}{Lp^2} \tag{3.28}$$

Solution to this equation is

$$\delta p(x_n) = C_1 \, e^{-x_n/L_p} + C_2 e^{x_n/L_p} \tag{3.29}$$

Applying the boundary conditions

(1) at $x_n = 0$; $\delta p(x_n) = \Delta p_n$

(2) at $x_n = \infty$ (by approximation 1); $\delta p(x_n) = 0$.

$$\Delta p_n = C_1 + C_2$$

$$0 = C_1 \times 0 + C_2 e^{\infty}$$

i.e., $C_2 = 0$

$$C_1 = \Delta p_n$$

Solution to equation (3.28) becomes

$$\delta p(x_n) = \Delta p_n \, e^{x_n/L_p} \tag{3.30}$$

This equation represents the hole distribution in the n-region and shows that the hole concentration decays exponentially from its initial value Δ_{pn} given by equation (3.26).

Hole diffusion current on the n-side is

$$I_{p\,diff}(x_n) = -q\,AD_p \frac{d}{dx_n}\delta p(x_n)$$

$$= -q\,AD_p \frac{d}{dx_n}\left(\Delta p_n e^{-x_n/L_p}\right)$$

$$= +q\,A \frac{D_p}{L_p}\Delta p_n e^{-x_n/L_p} \tag{3.31}$$

Similarly, by solving continuity equation for electrons on p-side, the electron diffusion current on the n-side can be evaluated as

$$I_{n\,diff}(x_p) = -q\,A \frac{D_n}{L_n}\Delta n_p e^{-x_p/L_n} \tag{3.32}$$

This current flows towards the negative x-direction.

The total diode current is the sum of the hole diffusion current at $x_n = 0$ and the electron diffusion current at $x_p = 0$ along the x-direction.

i.e., $I = I_{p\,diff}(x_n = 0) + I_{n\,diff}(x_p = 0)$

$$= \text{qA}\frac{D_p}{L_p}\Delta p_n + qA\frac{D_n}{L_n}\Delta n_p \tag{3.33}$$

(The second-term is also positive because the current given by equation (3.32) is in the negative x-direction).

By substituting the values of Δ_{pn} and Δ_{np} from equations (3.26) and (3.27), we get

$$\text{I} = \text{qA}\left[\frac{D_p}{L_p} p_{n_o}\left(e^{qV_a/kT} - 1\right) + \frac{D_n}{L_n} n_{p_o}\left(e^{qV_a/kT} - 1\right)\right]$$

$$= \text{qA}\left[\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right]\left(e^{qV_a/kT} - 1\right)$$

$$= I_s\left(e^{qV_a/kT} - 1\right) \tag{3.34}$$

where $$I_s = qA\left[\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right] \tag{3.35a}$$

is called the reverse saturation current.

$$p_{n_o} = \frac{n_i^2}{n_{n_o}} = \frac{n_i^2}{N_D}$$

$$n_{p_o} = \frac{n_i^2}{p_{p_o}} = \frac{n_i^2}{N_A}$$

$$\therefore \text{ I}_s = qAn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right) \tag{3.35b}$$

Equation (3.35b) shows that reverse saturation current increases with increase in temperature as n_i increase exponentially with increase in temperature.

At a given temperature, reverse saturation current is less for a diode made of wider band gap material due to lower value of intrinsic carrier concentration.

The I-V characteristics of an ideal diode represented by equation (3.34) is shown in Fig. 3.10. With $V_a > 0$, the diode is forward-biased and current increases exponentially with increase in forward voltage. The change in forward current is large for a small change in forward voltage. Therefore the resistance of the diode under forward-bias condition is very small.

With reverse-bias, the current remains constant at I_s. The reverse saturation current I_s is a very small current and is of the order of several microamperes for a germanium diode and nano amperes for a silicon diode.

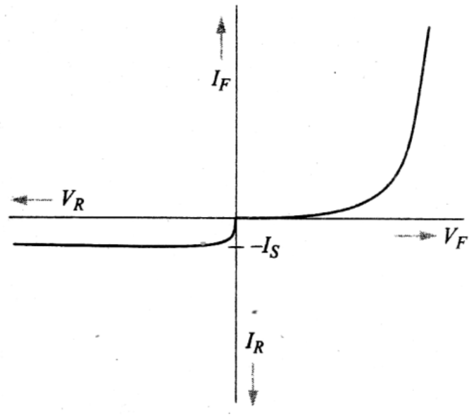


Fig. 3.10 Characteristics of ideal diode

3.4.1 Minority and Majority Carrier Currents

Equations (3.32) and (3.33) represent the distribution of minority carrier diffusion currents in the neutral region of p-n junction. The currents remain constant in the depletion region. The equations show that the minority carrier currents are maximum at the edge of depletion layer and they decay exponentially to zero at the ohmic contacts.

Total current in a diode,

$$I = qA\left(\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right)\left(e^{qV_a/kT} - 1\right)$$

On n-side, minority carrier current is,

$$I_{p\,diff} = qA\frac{D_p}{L_p}\Delta p_n e^{-x_n/L_p}$$

Therefore, majority carrier current $I_n = I - I_{p\,diff}$

On p-side, minority carrier current is,

$$I_{n\,diff} = qA\frac{D_n}{L_n}\Delta n_p e^{-x_p/L_n}$$

Therefore, majority carrier current $I_p = I - I_n$diff.

Fig. 3.11 shows the distribution of majority and minority components of current through the diode.

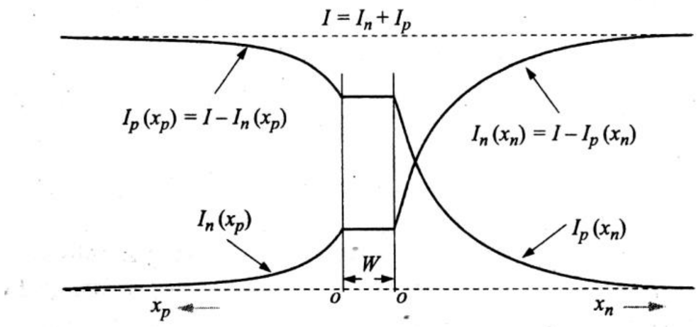


Fig. 3.11 Minority and majority carrier currents in a p-n junction

Example 3.5 The following data are given for a silicon abrupt p-n junctior at 300 K. A = 1 cm^2 V_a = 0.6 V.

p-side	n-side
$N_A = 10^{18}$ cm^{-3}	$N_D = 10^{16}$ cm^{-3}
τ_n = 50 µs	τ_p = 10 µs
D_n = 34 cm^2/s	D_p = 13 cm^2/s

Calculate $I_p(x_n = 0)$; $I_n(x_p = 0)$ and the total diode current $\frac{kT}{q}$ = 0.026 V.

Solution

Let $n_i = 1.5 \times 10^{10}$ cm^{-3}

$$L_n = \sqrt{D_n \tau_n} = \sqrt{34 \times 50 \times 10^{-6}} = 0.041 \text{ cm}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{13 \times 10 \times 10^{-6}} = 0.0114 \text{ cm}$$

$$n_{p_o} = \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 cm^{-3}$$

$$p_{n_o} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 cm^{-3}$$

$$I_p(x_n = 0) = qA \frac{D_p}{L_p} p_{n_o} \left(e^{qV_a/kT} - 1\right)$$

$$= 1.6 \times 10^{-19} \times 1 \times \frac{13}{0.0114} \times 2.25 \times 10^4 (e^{0.6/0.026} - 1)$$

$$= 4.32 \times 10^{-2} \text{ A}$$

$$I_n(x_p = 0) = qA \frac{D_n}{L_n} n_{p_o} \left(e^{qV_a/kT} - 1\right)$$

$$= 1.6 \times 10^{-19} \times 1 \times \frac{34}{0.041} \times 2.25 \times 10^2 (e^{0.6/0.026} - 1)$$

$$= 3.14 \times 10^{-4} \text{ A}$$

Total diode current

$$
\begin{aligned}
I &= I_p(x_n = 0) + I_n\,(x_p = 0)\\
&= 4.32 \times 10^{-2} + 3.14 \times 10^{-4}\\
&= 4.351 \times 10^{-2}\\
&= 43.5\text{mA.}
\end{aligned}
$$

Minority Carrier Distribution in a Reverse-Biased p-n Junction

The minority carrier (hole) concentration on n-side $p_n = p_{no} + \delta p_n$ and minority carrier concentration on p-side $n_p = n_{po} + \delta n_p$

$$
\begin{aligned}
\delta p_n &= \Delta p_n\, e^{-x_n/L_p}\\
\delta n_p &= \Delta n_p\, e^{-x_p/L_n}\\
\Delta p_n &= p_{no}\left(e^{qV_a/kT} - 1\right)\\
\Delta n_p &= n_{po}\left(e^{qV_a/kT} - 1\right) \text{ from equations (3.24) and (3.25)}
\end{aligned}
$$

Under reverse bias

$$
\begin{aligned}
\Delta p_n &= -p_{no} \quad \left(\text{Q } e^{qV_a/kT} \cong 1\right)\\
\Delta n_p &= -n_{po}\\
\therefore \quad \delta p_n &= -p_n\, e^{-x_n/L_p}\\
\delta n_p &= -n_p\, e^{-x_p/L_n}\\
\therefore \quad p_n\,(x_n) &= p_{no}\left(1 - e^{-x_n/L_p}\right) \qquad (3.36a)\\
n_p(x_p) &= n_{po}\left(1 - e^{-x_p/L_n}\right) \qquad (3.36b)
\end{aligned}
$$

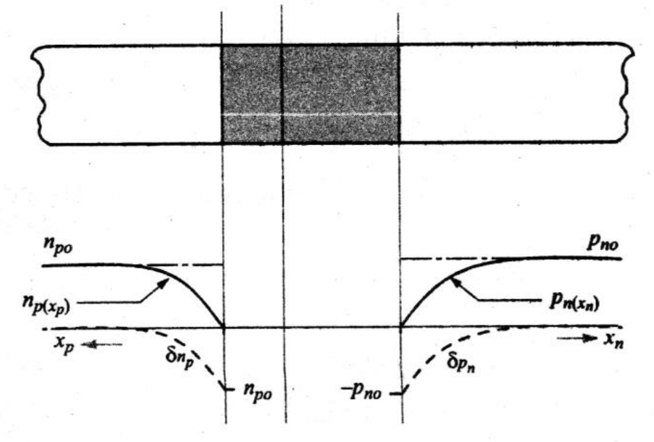


Fig. 3.12 Minority carrier distribution in reverse-biased p-n junction

Fig. 3.12 shows that the minority carrier concentration at the edge of the depletion layer under reverse-bias is less than the equilibrium minority carrier concentration. Minority carriers (holes) from n-side drift towards p-side and minority carriers (electrons) from p-side drift towards n-side. This makes the minority carrier concentrations less than the equilibrium value near the edges of depletion layer. This is called minority carrier extraction.

3.4.2 More General Form of Diode Equation

Equation (3.32) is valid for a long diode only. If we assume the length of neutral regions on the p and n sides of the diode as W_P and W_N respectively, as shown in Fig. 3.13, the boundary conditions for continuity equations for holes on n-side may be modified as

(1) at $x_n = W_N$; $\delta p(x_n) = 0$, assuming perfect ohmic contact
(2) at $x_n = 0$; $\delta p(x_n) = \Delta p_n$.

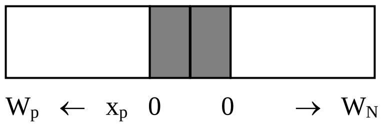


Fig. 3.13 A p-n junction with width of neutral regions W_P and W_N

On applying this to equation (3.29),

$$0 = C_1\, e^{-W_N/L_p} + C_2 e^{W_N/L_p} \qquad (3.37a)$$

$$\Delta p_n = C_1 + C_2 \qquad (3.37b)$$

From (3.37a)

$$C_2 = C_1\, e^{-2W_N/L_p}$$

Substituting for C_2 in equation (3.37b)

$$C_1 = \frac{\Delta p_n}{1 - e^{-2W_N/L_p}}$$

$$= \frac{\Delta p_n . e^{W_N/L_p}}{e^{W_N/L_p} - e^{-W_N/L_p}} \quad \text{(multiplying Nr. and Dr. by } e^{W_N/L_p}\text{)}$$

$$= \frac{\Delta p_n e^{W_N/L_p}}{2\sinh\left(\frac{W_N}{L_p}\right)}$$

$$C_2 = -C_1\, e^{-2W_N/L_p} = -\frac{\Delta p_n e^{-W_N/L_p}}{2\sinh\left(\frac{W_N}{L_p}\right)}$$

$$\delta p(x_n) = C_1\, e^{-x_n/L_p} + C_2\, e^{-x_n/L_p} \text{ (from equation (3.29))}$$

$$= \frac{\Delta p_n e^{W_N/L_p} . e^{-x_n/L_p}}{2\sinh\left(\frac{W_N}{L_p}\right)} - \frac{\Delta p_n e^{-W_N/L_p} . e^{-x_n/L_p}}{2\sinh\left(\frac{W_N}{L_p}\right)}$$

$$= \frac{\Delta p_n 2\sinh\left(\frac{W_N - x_n}{L_p}\right)}{2\sinh\left(\frac{W_N}{L_p}\right)} = \frac{\Delta p_n \sinh\left(\frac{W_N - x_n}{L_p}\right)}{\sinh\left(\frac{W_N}{L_p}\right)}$$

$$I_{p\ diff} = -q\ AD_p \frac{d}{dx_n}(\delta p(x_n))$$

$$= qA \frac{D_p}{L_p}\Delta p_n \frac{\cosh\left(\frac{W_N - x_n}{L_p}\right)}{\sinh\left(\frac{W_N}{L_p}\right)} \quad (3.38)$$

Similarly,

$$I_{n\ diff} = -q\ A \frac{D_n}{L_n}\Delta n_p \frac{\cosh\left(\frac{W_P - x_p}{L_n}\right)}{\sinh\left(\frac{W_p}{L_n}\right)} \quad (3.39)$$

Total current

$$I = I_{p\ diff}(x_n = 0) - I_{n\ diff}(x_p = 0)$$

$$= qA\left[\frac{D_p}{L_p} p_{n_o} \coth\left(\frac{W_N}{L_p}\right) + \frac{D_n}{L_n} n_{p_o} \coth\left(\frac{W_p}{L_n}\right)\right]\left(e^{qV_a/kT} - 1\right) \quad (3.40)$$

For long diode, $W_N >> L_p$ and $W_P >> L_n$

$$\therefore \coth\frac{W_N}{L_p} = 1 \quad and \quad \coth\frac{W_p}{L_n} = 1$$

$\therefore$ Equation (3.40) reduces to

$$I = qA\left(\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right)\left(e^{qV_a/kT} - 1\right) \quad (3.41)$$

For short diode, $W_N << L_p$ and $W_p << L_n$

$$\cot hx \Rightarrow \frac{1}{x} \text{ if x is very small}$$

Therefore the diode equation becomes

$$I = qA\left[\frac{D_p}{W_p} p_{n_o} + \frac{D_n}{W_n} n_{p_o}\right]\left(e^{qV_a/kT} - 1\right) \quad (3.42)$$

In short base diodes as reverse voltage increases, width of depletion layer increases so that it completely extends into the neutral region. This phenomenon is called punch through, because the depletion region punches through the neutral region. On punch through, the junction voltage loses control over the current.

3.5 REAL DIODES

The I-V characteristics of a real diode differ from the ideal behaviour due to the following reasons:

(1) generation/recombination in the depletion layer,
(2) voltage drop associated with neutral n and p regions,
(3) current due to leakage across the surface of the junction,
(4) high-level injection.

3.5.1 Generation and Recombination Current

In the ideal diode equation we assume that there is no generation or recombination in the depletion layer. This is actually not true. The current due to carrier generation in the depletion layer may be expressed as

$$I_{gen} = \frac{-qAn_iW}{2\tau_o} = -I_{R_o} \tag{3.43}$$

where τ_o - effective life-time of carriers in the depletion region
W -width of depletion layer
A - area of cross-section

Under forward-bias condition there will be recombination of minority carriers and the resulting current is given by

$$I_{rec} = \frac{-qAn_iW}{2\tau_o}\left(e^{V_a/2V_T} - 1\right)$$
$$= I_{R_o}\left(e^{V_a/2V_T} - 1\right) \tag{3.44}$$

$\therefore$ Total current in the diode may be expressed as

$$I = I_{diff} + I_{rec}$$
$$= I_s\left(e^{V_a/V_T} - 1\right) + I_{R_o}\left(e^{V_a/2V_T} - 1\right) \tag{3.45}$$

3.5.2 Electric Field in the Quasi Neutral N and P Regions

If R_s is the series resistance of the diode neutral region and I is the current through the diode, the voltage drop across the neutral region is IR_s. This voltage drop is proportional to the current through the diode. Therefore, the voltage that appear across the junction is only V_a - IR_s- Therefore, the diode equation modifies to

$$I = I_o\left(e^{(V_a - IR_s)V_T} - 1\right) \tag{3.46}$$

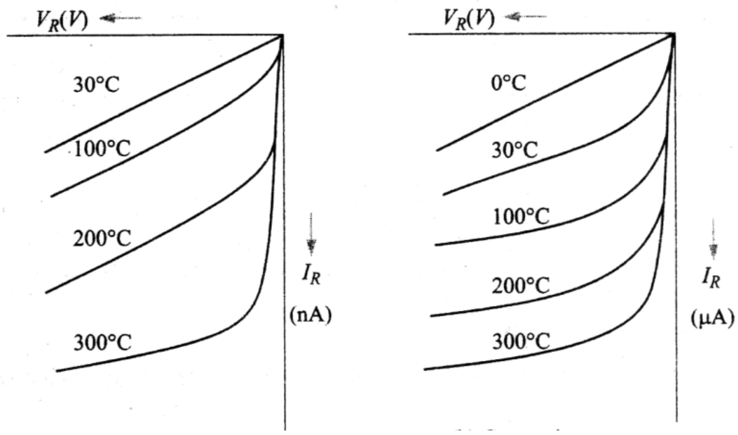
IR_s term is negligible at low currents but becomes significant at high currents and electric field exists in the neutral region.

3.5.3 I-V-characteristics of real diodes

Reverse current is given by I = -(I_s + I_{R_o}). Here Is is independent of bias, but I_{R_o} increases with increase in reverse-bias as it is proportional to width of depletion layer. Thus the diode current fails to saturate.

As I_s α n_i^2 and I_{R_o} α n_i, for a silicon diode, I_{R_o} dominate at low temperature, Is becomes significant only at higher temperature (much higher than room temperature). But for germanium diode Is dominates at room temperature and above. Therefore, Ge diode obeys the ideal behaviour at or above room temperature.

Forward current I = $I_s e^{V_F/V_T} + I_{R_o} e^{V_F/2V_T}$.



(a) Silicon (b) Germanium

Fig. 3.14 Temperature Dependance of Reverse Current of Si and Ge Diodes

For germanium diodes second-term is negligible except for very low temperatures compared to the first-term. In silicon and gallium-arsenide p-n junctions, I_{R_o} is large compared to I_s near room temperature, recombination current dominates the diffusion current. But for higher values of forward-bias, diffusion current dominates over recombination current. The variation of reverse saturation current with temperature for silicon and germanium diodes are shown in Fig. 3.14.

Taking these effects into account the diode equation can be modified as

$$I=I_o\left(e^{V_a/\eta V_T}-1\right) \tag{3.47}$$

where, $I_o = I_s$ when the diffusion process dominates

= I_{R_o} when the generation/recombination process in the depletion layer dominates.

The parameter η known as ideality factor has a value of 1 for diffusion currents and approximately 2 for recombination current. When the two currents are comparable η lies between 1 and 2. (Thus for Ge at room temperature $\eta \cong 1$ and for Si at room temperature $\eta \cong 2$).

The value of I_o and η of a diode can be obtained by plotting its forward characteristics with voltage in linear scale and current in log scale, as in Fig. 3.15.

$I_F = I_o\ e^{V_F/\eta V_T}$ $\quad V_F >> V_T$

$$\ln\ I_F = \ln I_o + \frac{V_F}{\eta V_T} \qquad (3.48)$$

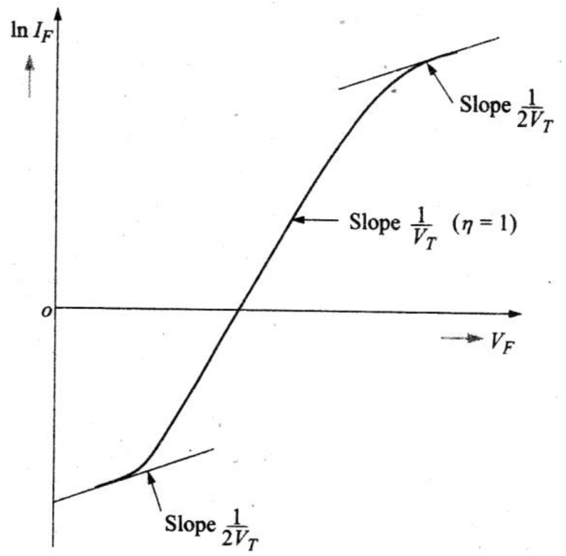


Fig. 3.15 Forward characteristics of real diode

Equation (3.48) shows that Io is the intercept on the y-axis and ηV_T is the reciprocal of the slope of the curve. Plotting the actual characteristics of a diode it can be seen that the slope varies with bias. The ideality factor may be assumed as its value in the linear portion of the curve as shown in Fig. 3.16. At room temperature Io represents saturation current (I_s) for Ge diodes and recombination current (I_{R_o}) for Si diodes. For germanium diodes the reverse saturation current is given by

$$I = -I_s = -qA\, n_i^2 \left[\frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n}\right]$$

In this expression, D_p, D_n, L_p and L_n are temperature dependent, but the temperature dependence of I is dominated by n_i^2

$$n_i(T) = K_2\ T^{3/2} e^{-Eg_o/2kT}$$

Therefore, I_s can be written as

$$I_s = K_3\ T^3 e^{-Eg_o/kT}$$

Taking natural logarithm

$$\ln I_s = \ln K_3 + 3\ln\ T - \frac{E_{go}}{kT}$$

Differentiating with respect to T, $\frac{1}{I_s}.\frac{dI_s}{dT} = 0 + 3 \times \frac{1}{T} + \frac{E_{go}}{kT^2}$

$$\frac{1}{I_s}\cdot\frac{dI_s}{dT}=\frac{1}{T}\left[3+\frac{E_{go}}{kT}\right] \quad (3.49)$$

For Ge, E_{go} = 0.78 eV and this equation predicts 11% increase of I_s for each degree K rise in temperature around 300 K. It also shows that the reverse saturation current doubles for every 9°K rise in temperature for Ge diodes.

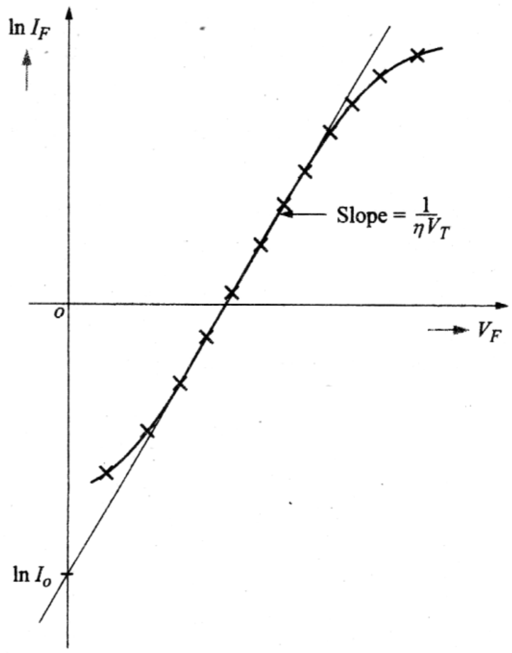


Fig. 3.16 Experimental method to evaluate 77 and Jo

For Si diodes I_{R_o} = $K_4 T^{3/2} e^{-Eg_o/2kT}$

Taking natural logarithm and then differentiating,

We get $$\frac{1}{I_{R_o}}\cdot\frac{dI_{R_o}}{dT}=\frac{3}{2}\times\frac{1}{T}+\frac{E_{go}}{2k}\frac{1}{T^2}$$

$$\frac{1}{I_{R_o}}\cdot\frac{dI_{R_o}}{dT}=\frac{1}{2T}\left[3+\frac{E_{go}}{kT}\right] \quad (3.50)$$

E_{go} = 1.16 eV for Si. Thus I_{R_o} changes by 8% per degree K rise in temperature around 300 K. i.e., the reverse current doubles for every 12.5° K rise in temperature.

Under forward-bias condition,

$$I_F = I_s\ e^{qV_F/kT}$$

$$\ln I_F = \ln I_s + \frac{qV_F}{kT}$$ \nlp

On differentiating

$$\frac{1}{I_F}\cdot\frac{dI_F}{dT}=\frac{1}{Is}\cdot\frac{dI_s}{dT}-\frac{qV_F}{kT^2}+\frac{q}{kT}\cdot\frac{dV_F}{dT}$$

(Q V_F is a temperature dependent quantity)

But $$\frac{1}{Is}\cdot\frac{dI_s}{dT}=\frac{1}{T}\left[3+\frac{E_{go}}{kT}\right]$$

$$\therefore \frac{1}{I_F}\cdot\frac{dI_F}{dT}=\frac{1}{T}\left[3+\frac{E_{go}}{kT}\right]-\frac{qV_F}{kT^2}+\frac{q}{kT}\frac{qV_F}{dT}$$

$$=\frac{3}{T}+\frac{q}{kT^2}\left[\frac{E_{go}}{q}-V_F\right]+\frac{q}{kT}\frac{dV_F}{dT}$$

But, $$\frac{E_{go}}{q} = V_{go}$$

$$\therefore \frac{1}{I_F}\cdot\frac{dI_F}{dT}=\frac{3}{T}+\frac{q}{kT^2}\left(V_{go}-V_F\right)+\frac{q}{kT}\frac{qV_F}{dT} \tag{3.51}$$

$\frac{dV_F}{dT}$ for a given current is obtained by putting $\frac{dI_F}{dT}$ = 0.

$$\frac{dV_F}{dT}=-\frac{kT}{q}\left[\frac{3}{T}+\frac{q}{kT^2}\left(V_{go}-V_F\right)\right]$$

$$=-\left[\frac{3k}{q}+\frac{\left(V_{go}-V_F\right)}{T}\right] \tag{3.52}$$

3.5.4 High-Level Injection Effects

Under high-level injection

(1) minority carrier drift currents are not negligible,

(2) mobility and diffusion constants are different from low-level injection values and

(3) diode equation gets modified.

Assume a p^+n junction with high-level injection in the n-side and low-level injection in the p-side. Since, there is an electric field in the neutral n-region, the electron and hole currents in this region are given by

$$J_p = q\mu_p\, p_n E_{(x)} - qD_p \frac{dp_n}{dx} \tag{3.53}$$

$$J_n = q\mu_n\, n_n E_{(x)} + qD_n \frac{dn_n}{dx} \tag{3.54}$$

where μ_p, D_p, μ_n, D_n etc. are parameters on n-side.

As practically all the current in the depletion region is carried by holes, we can assume J_n =0 at x_n= 0 and 8

$$\therefore\ E(x_n) = \frac{-D_n}{\mu_n}\times\frac{1}{n_n}\cdot\frac{dn_n}{dx_n} \tag{3.55}$$

By quasi neutrality, $\delta_p = \delta_n$ and $\frac{dp_n}{dx_n} = \frac{dn_n}{dx_n}$. Substituting equation (3.55) in equation (3.53) and applying Einstein's relation $\left(\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}\right)$

$$J = J_p(x_n = 0) = -qD_p\left(1 + \frac{p_n}{n_n}\right)\frac{dp_n}{dx_n}\bigg|_{x_n=0} \tag{3.56}$$

For high-level injection $p_n \cong n_n$ at $x_n = 0$

$$\therefore\ J = -2qD_p\,\frac{dp_n}{dx_n}\bigg|_{x_n=0} \tag{3.57}$$

Thus at very high injection level, diode current can be expressed as a diffusion current with D_p replaced by $2D_p$.

At low injection level pn product in the depletion layer is

$$pn = n_i^2 e^{(V_a/V_T)} \qquad \text{(see Problem 3.20)}$$

For a non-degenerate semiconductor the electron and hole concentrations at $x_n = 0$ are expressed as

$$n_n(x_n{=}0) = n_i\, e^{(F_n - E_{in})/kT} \tag{3.58}$$

$$p_n(x_n{=}0) = n_i\, e^{(E_{in} - F_p)/kT} \tag{3.59}$$

Where F_n and F_p represent the quasi Fermi level for electrons and holes as shown in Fig. SP.3.19b.

Multiplying equations (3.58) and (3.59)

$$p_n(x_n = 0)n_n(x_n = 0) = n_i^2 e^{(F_n - F_p)/kT}$$

Under thermal equilibrium, $F_n = F_p$ everywhere and pn product equals n_i^2 everywhere. But when the junction is forward-biased by $V_j = V_a - IR_s$, the quasi Fermi levels will be displaced by qV_j. Since pn product is constant throughout the depletion layer,

$$F_n - F_p = qV_j \quad \text{at} \quad x_n = 0$$

$$\therefore\quad p_n(x_n = 0).\ n_n(x_n{=}0) = n_i^2 e^{V_j/V_T} \tag{3.60}$$

At very high injection level $p_n(x_n = 0) = n_n(x_n = 0)$

$$\therefore\quad p_n(x_n = 0) = n_i\, e^{V_j/2V_T} \quad \text{(See equation (3.60))} \tag{3.61}$$

As in the case of low-level injection,

$$\frac{dp(x_n)}{dx_n}\bigg|_{x_n=0} = -\frac{p(x_n = 0)}{L_p} \qquad \text{(see equation (3.30))}$$

$$\therefore\quad J = 2q\,\frac{D_p}{L_p}\,n_i\, e^{V_j/2V_T} \qquad \text{(See equations (3.57) and (3.61))} \tag{3.62}$$

Due to high level injection, slope of the diode characteristics reduces at high currents as is shown by the above equation.

3.6 DEPLETION LAYER CAPACITANCE

The charge on either side of the depletion layer (Q_D) is given by

$$|Q_D| = qN_AAX_p = qN_DAX_n \tag{3.63}$$

X_n and X_p are functions of bias voltage i.e., the charges in the depletion layer adjusts itself with bias. This situation is similar to charging and discharging of a parallel plate capacitor. But, in the case of p-n junction charges are distributed in the deplection layer and the capacitance value depends on the bias voltage. Capacitance due to variation in depletion layer charges is called depletion capacitance or transition capacitance or junction capacitance (C_j) and is given by

$$C_j = \frac{d|Q_D|}{d(V_o - V_a)} \tag{3.64}$$

$$\begin{aligned}
|Q_D| &= |-qN_AX_pA| \\
&= qAN_AX_p \\
&= qAN_A.W.\frac{N_D}{N_A + N_D} \qquad \left(\because X_p = W.\frac{N_D}{N_A + N_D}\right)
\end{aligned}$$

where

$$W = \sqrt{\frac{2\epsilon}{q}(V_o - V_a)\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

$$\frac{dW}{d(V_o - V_a)} = \frac{1}{2w} \times \frac{2\epsilon}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)$$

$$\begin{aligned}
C_j &= \frac{d|Q_D|}{d(V_o - V_a)} = \frac{d|Q_D|}{dw} \times \frac{dW}{d(V_o - V_a)} \\
&= qAN_A.\frac{N_D}{N_A + N_D} \times \frac{1}{2W} \times \frac{2\epsilon}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)
\end{aligned}$$

i.e.,

$$C_j = \frac{\epsilon A}{W} \tag{3.65}$$

Thus, the junction capacitance can be considered as the capacitance of a parallel plate capacitor with separation between the plates as the depletion layer width W. The expression (3.65) shows that the depletion layer capacitance decreases with increase in reverse bias. This capacitance is dominant under reverse-bias condition and it can be used as a voltage variable capacitor.

3.6.1 Equilibrium Depletion Layer Capacitance

$$C_{jo} = \frac{\epsilon A}{W_o} = \frac{\epsilon A}{\sqrt{\frac{2\epsilon V_o}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}} \tag{3.66}$$

With bias V_a depletion layer capacitance changes to

$$C_j = \frac{\epsilon A}{W} = \frac{\epsilon A}{\sqrt{\frac{2\epsilon(V_o - V_a)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}} \qquad (3.67)$$

$$\frac{C_j}{C_{j_o}} = \sqrt{\frac{V_o}{V_o - V_a}} = \frac{1}{\sqrt{1 - \frac{V_a}{V_o}}}$$

$$\therefore\ C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_a}{V_0}}} \qquad (3.68)$$

From equation (3.67)

$$\frac{C_j}{C_{j_o}} = \frac{2(V_o - V_a)}{q\epsilon A^2}\left(\frac{N_A + N_D}{N_A N_D}\right) \qquad (3.69)$$

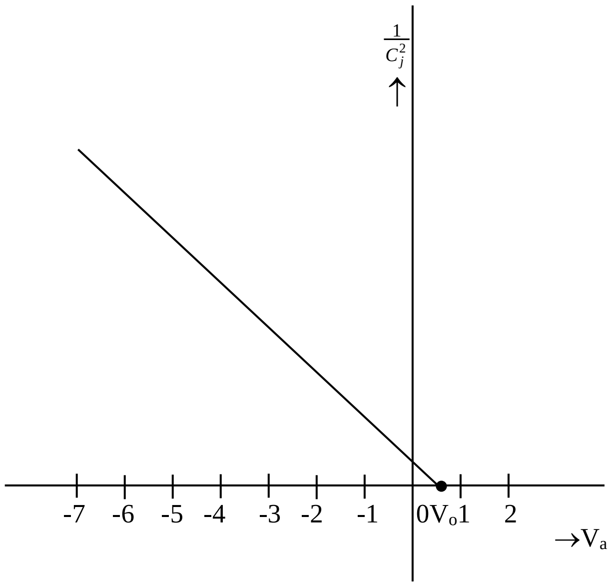


Fig. 3.17 Plot of $1/C_j^2$ as a function of applied bias V_a

The intercept of $\frac{1}{C_j^2}$ versus V_a plot on the voltage axis gives the value of V_o (When $\frac{1}{C_j^2}$ = 0; $V_a = V_o$) as shown in Fig. 3.17.

Example 3.6 A silicon abrupt p-n junction at 300 K has $N_A = 10^{16}$ cm^{-3} on p-side and $N_D = 10^{14}$ cm^{-3} on n-side. Area of cross-section is 10^{-5} cm^2. Calculate the junction capacitance at a. equilibrium b. forward-bias of 0.5 V c. reverse-bias of 1 V and 10V.

Solution

$$n_i = 1.5 \times 10^{10}\ \text{cm}^{-3} \qquad \in_r = 11.8$$

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026\ \ln\left(\frac{10^{16}\times 10^{14}}{(1.5\times 10^{10})^2}\right) = 0.577V$$

$$W_o = \sqrt{\frac{2\epsilon_o\epsilon_r V_o}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2\times 8.854\times 10^{-14}\times 11.8\times 0.577}{1.6\times 10^{-19}}\left(\frac{1}{10^{16}}+\frac{1}{10^{14}}\right)}$$

$$= 2.76 \times 10^{-4}\ \text{cm}$$

a. Equilibrium

$$C_{jo} = \frac{\epsilon A}{W_o} = \frac{\epsilon_o\epsilon_r A}{W_o} = \frac{8.854\times 10^{-14}\times 11.8\times 10^{-5}}{2.76\times 10^{-4}}$$

$$= 0.03785\ \text{pF}$$

b. Forward-bias of 0.5 V (V_a = 0.5 V)

$$C_j = \frac{C_{jo}}{\sqrt{1-\frac{V_a}{V_o}}}$$

$$= \frac{0.03785\times 10^{-12}}{\sqrt{1-\frac{0.5}{0.577}}} = 0.1036\ \text{pF}$$

c. Reverse-bias of 1 V (V_a = -1 V)

$$C_j = \frac{0.03785\times 10^{-12}}{\sqrt{1+\frac{1}{0.577}}} = 0.0228\ \text{pF}$$

Reverse-bias of 10 V (V_a = -10 V)

$$C_j = \frac{0.03785\times 10^{-12}}{\sqrt{1+\frac{10}{0.577}}} = 0.0088\ \text{pF.}$$

p^+n diode

A diode with heavy doping on p-side ($N_A >> N_D$) is designated as a p^+n diode. For a p^+n diode the expressions for depletion layer width, reverse saturation current etc may be approximated as

$$W_o = \sqrt{\frac{2\epsilon V_o}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)} \cong \sqrt{\frac{2\epsilon V_o}{qN_D}} \qquad \left(\because \quad \frac{1}{N_A} << \frac{1}{N_D}\right)$$

$$X_{no} = W_o\frac{N_A}{N_A+N_D} \cong W_o \qquad (\because\ N_A + N_D \cong N_A)$$

$$X_{po} = W_o \cdot \frac{N_A}{N_A + N_D} \cong 0$$

$$I_s = qA\left(\frac{D_p}{L_p} p_{no} + \frac{D_n}{L_n} n_{po}\right)$$

$$= qA \frac{D_p}{L_p} p_{no} \quad \left(\because \quad p_{no} = \frac{n_i^2}{N_D} >> n_{po} = \frac{n_i^2}{N_A}\right)$$

The above results show that the performance of a p^+n diode is decided mainly by the parameters on the n-side (lightly doped side) of the junction.

3.8 ELECTRICAL BREAKDOWN IN P-N JUNCTIONS

When a diode is reverse biased, the ideal diode theory predicts a small saturation current I_s which is independent of the applied reverse-bias. But in diodes where generation current dominates, the reverse current increases with applied reverse-bias. This increase is also negligibly small and do not affect the rectifying property of diode. In any diode, it is observed that if the reverse-bias is gradually increased as shown in Fig. 3.21, the current abruptly increases at a particular voltage. This phenomenon is called breakdown of p-n junction and the reverse voltage at which this happens is called breakdown voltage (V_{Br}).

The breakdown is only electrical and there is no mechanical damage to the diode. The breakdown is reversible and non-destructive if the power dissipation at the junction is limited to a value allowed by thermal considerations. The two different mechanisms of breakdown are Zener breakdown and avalanche breakdown. A diode which is operated in the breakdown region is called Zener diode, irrespective of the breakdown mechanism.

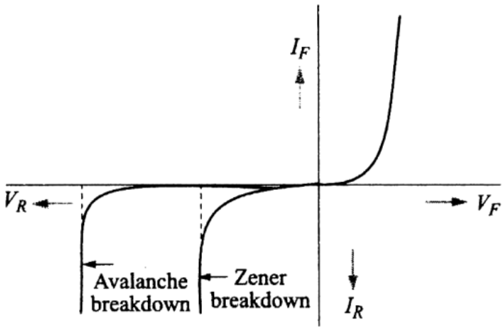
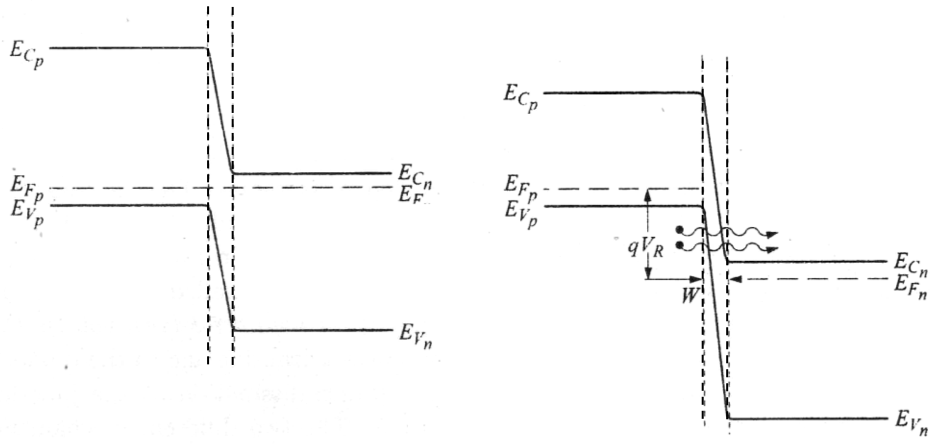


Fig. 3.21 I-V characteristics of a diode including the breakdown region

3.8.1 Zener Breakdown

When a heavily doped junction is reverse-biased, the energy bands get crossed at relatively low voltages, i.e, filled states on p-side (valence band) comes opposite to vacant states on n-side (conduction band). If the barrier separating these two bands is narrow, tunneling takes place from p-side valence band to n-side conduction band as shown in Fig. 3.22(b). This constitutes a reverse current from n to p. The break down of the junction due to this mechanism is called Zener breakdown.

If a large number of electrons are separated from a large number of vacant states by a thin barrier of finite height, tunneling takes place through the barrier. These conditions can be met by an abrupt junction with heavily doped n and p regions at relatively small reverse voltages. If the junction is not abrupt or if either side of the junction is lightly doped, the transition region (W) will be too wide for tunneling.



(a) Equilibrium (b) Reverse-bias

Fig. 3.22 Zener breakdown (EB diagram)

At low reverse voltages the separation between valence band on p-side and conduction band on n-side is small. But if the reverse voltage is higher than a few volts, the separation between bands become large and avalanche breakdown becomes dominant.

In terms of covalent bonding model, Zener effect can be thought of as field ionisation of host atoms at the junction. The reverse-bias across the heavily doped junction causes a heavy electric field within the depletion region. At critical field, electrons in covalent bonds are pulled out from the bonds by the field and accelerated towards the n-side of the junction. Holes are accelerated towards the p-side. The electric field required for this type of ionization is of the order of 10^5 V/cm.

3.8.2 Avalanche Breakdown

For lightly doped junctions electron tunneling is negligible and the breakdown mechanism involves impact ionization of host atoms by highly energetic carriers. If an electron entering the depletion layer has sufficient kinetic energy, it can cause ionizing collision resulting in the generation of EHP and carrier multiplication takes place. The original and generated electrons are swept to the n-side and holes to the p-side. If the width of the depletion layer is large, the carrier multiplication process continues in a cumulative process as shown in Fig. 3.23. This is called avalanche multiplication. Each incoming carrier initiates the creation of large number of new carriers.

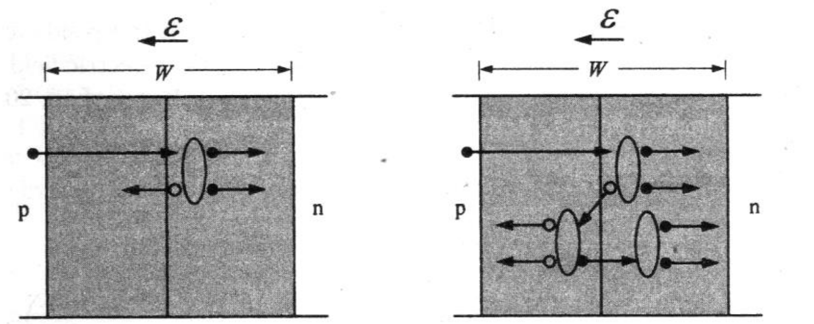
Let P be the probability of ionizing collision for carriers of either type, while accelerated through the depletion layer. Therefore, for n_{in} electrons entering from the p-side there will be $n_{in}P$ collisions and $n_{in}P$ secondary EHP's. The total electron concentration will be $n_{in}(l+P)$.

Similarly, the number of ternary collisions will be ($n_{in}P^2$). After many stages collisions the total number of electrons coming out of the depletion layer is

$$n_{out} = n_{in}\,(1 + P + P^2 + P^3 + \ldots)$$

The electron multiplication factor (M) is given by

$$M = \frac{n_{out}}{n_{in}} = \frac{1}{1-P}$$



(a) Single ionising collision (b) Primary, Secondary and Ternary collisions

Fig. 3.23 EHP generation by impact ionisation

As the probability of ionization P approaches unity, the carrier multiplication and the reverse current increases without limit. The current is limited only by the external circuit. Measurement of carrier multiplication in junctions near breakdown lead to an empirical relation $M = \dfrac{1}{1-\left(\frac{V}{V_{Br}}\right)^n}$, the value of n varies between 3 and 6 depending on the material.

The breakdown voltage increases with increase in band gap of the material, as more energy is required for ionization. The peak electric field within the depletion layer increases with increased doping on the lightly doped side. Therefore, breakdown voltage decreases with increase in doping on lightly doped side.

The peak electric field at which breakdown occurs is called critical electric field (E_{crit})

$$E_m = \frac{-q}{\epsilon} N_A X_p = -\frac{q}{\epsilon}\frac{N_A N_D}{N_A + N_D} W$$

$$= \frac{-q}{\epsilon}\frac{N_A N_D}{N_A + N_D}\sqrt{\frac{2\epsilon(V_o - V_a)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

$$= -\sqrt{\frac{2q(V_o - V_a)}{\epsilon}\frac{N_A + N_D}{N_A N_D}} \tag{3.86}$$

The breakdown voltage ($V_a = -V_R = -V_{Br}$) is the voltage at which $E_{max} = E_{crit}$

$$E_{crit} = -\sqrt{\frac{2q(V_o - V_{Br})}{\epsilon}\frac{N_A + N_D}{N_A N_D}} \tag{3.87}$$

As $V_{Br} >> V_o$, $V_o + V_{Br} \cong V_{Br}$

$$\therefore\ V_{Br} = \frac{\epsilon E_{crit}^2}{2q}\frac{N_A + N_D}{N_A N_D} \tag{3.88}$$

For $p^+ n$ junction $N_A >> N_D \;\therefore\; N_A + N_D \cong N_A$

$$V_{Br} = \frac{\epsilon E_{crit}^2}{2qN_D} \tag{3.89}$$

Example 3.8 A Si abrupt p-n junction has $N_A = 10^{18}$ cm^{-3} on the p-side and $N_D = 10^{16}$ cm^{-3} on the n-side. Determine the breakdown voltage if critical electric field is 3×10^5 V/cm. Determine the avalanche multiplication factor at reverse voltages of 10, 20, 29, 29.2 and 29.6 V. Assume n = 3.

Solution

$$V_{Br} = \frac{\epsilon E_{crit}^2}{2q} \frac{(N_A + N_D)}{N_A N_D} \qquad \text{By equation (3.88)}$$

$$= \frac{11.8 \times 8.854 \times 10^{-14} \times (3 \times 10^5)^2}{2 \times 1.6 \times 10^{-19}} \left(\frac{10^{18} \times 10^{16}}{10^{18} \times 10^{16}} \right)$$

$$= 29.67 \text{ V}$$

$$M = \frac{1}{1 - \left(\dfrac{V}{V_{Br}} \right)^n}$$

V = 10V, $\quad M = \dfrac{1}{1 - \left(\dfrac{10}{29.67} \right)^3} = 1.0398$

V = 20V, $\quad M = \dfrac{1}{1 - \left(\dfrac{20}{29.67} \right)^3} = 1.441$

V = 29V, $\quad M = \dfrac{1}{1 - \left(\dfrac{29}{29.67} \right)^3} = 15.09$

V = 29.2V, $\quad M = \dfrac{1}{1 - \left(\dfrac{29.2}{29.67} \right)^3} = 21.38$

V = 29.6V, $\quad M = \dfrac{1}{1 - \left(\dfrac{29.6}{29.67} \right)^3} = 141.6$

Note: The results indicate that the multiplication factor suddenly increase near the breakdown voltage.

3.8.3 Comparison Between Zener and Avalanche Breakdown

The characteristics near breakdown is smooth in the case of Zener breakdown and sharp in the case of avalanche breakdown.

Breakdown voltage decreases with increase in temperature for Zener breakdown while it increases with temperature for avalanche breakdown.

If breakdown voltage is less than $4V_g \left(V_g = \frac{E_g}{q}\right)$ the breakdown mechanism is usually Zener breakdown and if $V_{Br} > 8V_g$, the mechanism is avalanche breakdown. Between these limits breakdown is due to combination of these mechanisms.

3.8.5 Applications of Zener Diodes

Diodes operated in breakdown region are known as Zener diodes irrespective of the break-down mechanism. Zener diodes are used as voltage regulators. They are also used as a reference voltage source. Diodes with Zener breakdown has negative temperature coefficient of breakdown voltage and with avalanche breakdown has positive temperature coefficient of breakdown voltage. Two such diodes with properly matched (opposite), temperature coefficients provide a constant reference voltage independent of temperature.

3.9 Switching Characteristics

So far we have discussed only equilibrium and steady-state conditions of p-n junctions. But most of the solid state devices are used for switching or for processing a.c signals. The complete analysis of time dependent process involves equations in two simultaneous variables, space and time.

In this section we investigate some of the special cases involving time variations. The effect of excess carriers on transient response, the switching of diode from forward state to reverse state, capacitance due to stored charge, etc.. are discussed.

3.9.1 Time Variation of Stored Charge

In a p-n junction any change in forward current leads to a change in the stored charge in the excess minority carrier distribution. The stored charge lags behind the current. This is due to the capacitive effect of stored charge as explained in Section 3.9.4.

The time dependent continuity equation can be written as

$$\frac{-\partial J_p(x,t)}{\partial x} = \frac{q\delta p(x,t)}{\tau_p} + q.\frac{\partial p(x,t)}{\partial t} \tag{3.90}$$

To obtain the instantaneous current density, integrate equation (3.90) at time t so that

$$J_p(0) - J_p(x) = \int_0^x \left(\frac{\delta p(x,t)}{\tau_p} + \frac{\partial p(x,t)}{\partial t} \right) dx$$

For a p+n junction with $W_N >> L_p$, the current at $X_n = 0$ can be considered due to holes only. As $x \to \infty (W_N)$; $J_p(x) = 0$

$$\therefore \quad A(J_p(0) - J_p(x)= i(t)$$

$$i(t) = i_p(x_n = 0,t)$$

$$= \frac{qA}{\tau_p}\int_0^\infty \delta p(x,t)dx + qA\frac{\partial}{\partial t}\int_0^\infty \delta p(x,t)dx \quad \left(\because \ \frac{\partial}{\partial t} = \frac{\partial \delta p}{\partial t}\right)$$

$$= \frac{Q_p(t)}{\tau_p} + \frac{d}{dt}Q_p(t)\left(\because \ qA\int_0^\infty \delta p(x,t)dx = Q_p\right) \tag{3.91}$$

Equation (3.91) shows that the hole current injected across a p+n junction (approximately, total current) is determined by two components-two charge storage effects.

(1) the recombination term $\frac{Q_p}{\tau_p}$ in which excess carrier distribution is replaced every τ_p seconds and

(2) a charge build up term $\frac{dQ_p}{dt}$ which shows that the distribution of excess carriers can be increasing or decreasing. For steady-state $\frac{dQ_p}{dt}$ term is zero.

This shows that the hole current injected at any time t must supply minority carriers for recombination and for whatever variation that occurs in the total stored charge.

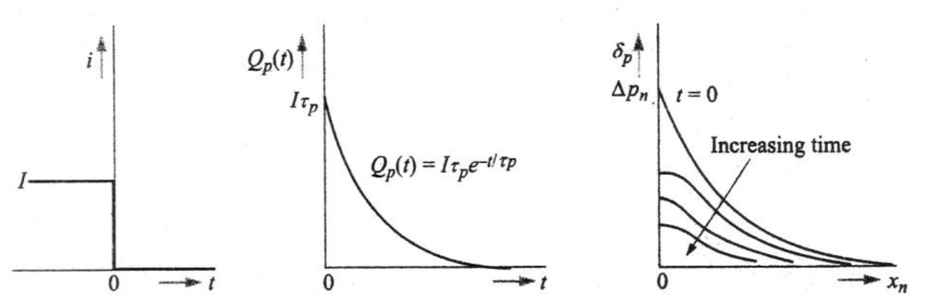
For a given current transient, the stored charge can be obtained as a function of time. Consider the turn off transient in which the current is suddenly removed at t = 0. This leaves the diode with stored charge. This charge die out by recombination with electrons in the n-region which takes some time.

Equation (3.91) can be solved to get the time variation of stored charge.

$$i_{(t>0)} = 0 \qquad \text{and} \qquad Q_p(0) = I\tau_p$$

Therefore, equation (3.91) becomes

$$0 = \frac{Q_p}{\tau_p} + \frac{dQ_p}{dt}$$



(a) Current through the diode

(b) Decay of stored charge in the n-region

(c) Excess hole distribution in the n-region as a function of time during the transient

Fig. 3.25 Effects of a step tum-off transient in a p+n diode

Taking Laplace transform

$$0 = \frac{Q_p(s)}{\tau_p} + sQ_p(s) - Q_p(0)$$

$$= \frac{Q_p(s)}{\tau_p} + s\,Q_p(s) - I\tau_p$$

$$Q_p(s) \quad = \frac{I\tau_p}{s + \frac{1}{\tau_p}}$$

Taking inverse Laplace transform

$$Q_p(t) \quad = I\tau_p e^{-t/\tau_p} \tag{3.92}$$

The stored charge dies out exponentially from its initial value Irp with a time constant equal to the hole lifetime in the n-region as shown in Fig. 3.25(b).

Even though the current is terminated, the presence of stored charge cause a junction voltage $V_{(t)}$ until the stored charge is completely removed. Since, the current for t > 0 is zero, the slope of the minority carrier distribution is zero at x = 0 for all t > 0. Even though δp does not change instantaneously, the slope of the distribution at x = 0 must change to zero immediately as shown in Fig. 3.25 c. This can occur in a small region near the junction with negligible re-distribution of charge. Since, the slope at x = 0 remains zero, the distribution deviates from exponential form and it is difficult to solve for exact distribution of δp(x,t).

An approximate solution for V(t) can be obtained by assuming an exponential distribution of δp at every instant during the decay of stored charge. This neglects the distortion due to slope requirement at x = 0 and the diffusion during decay (quasi steady-state). Thus, we have

$$\delta p(x_n, t) = \Delta p_n(t)\; e^{x_n/L_p}$$

The stored charge at any instant of time is given by

$$Q_p(t) \quad = qA \int_0^\infty \Delta p_n(t) e^{-x_n/L_p}\, dx = qAL_p \Delta p_n(t)$$

But $$\therefore\ \Delta p_n(t) = \frac{Q_p}{qAL_p} = \frac{I\tau_p e^{-t/\tau_p}}{qAL_p}$$

$$\Delta p_n(t) \quad = p_{no}\left(e^{qV(t)/kT} - 1\right)$$

i.e., $$p_{no}\left(e^{qV(t)/kT} - 1\right) = \frac{I\tau_p e^{-t/\tau_p}}{qAL_p}$$

$$\therefore V_{(t)} \quad = \frac{kT}{q} \ln\left(\frac{I\tau_p}{qAL_p p_{no}} e^{-t/\tau_p} + 1\right) \tag{3.93}$$

Equation (3.93) shows the variation of junction voltage during turn off transient with quasi steady-state approximation. It indicates that the voltage across a p-n junction cannot be Aanged instantaneously. This also shows that stored charge will have an adverse effect when the diode is used for switching.

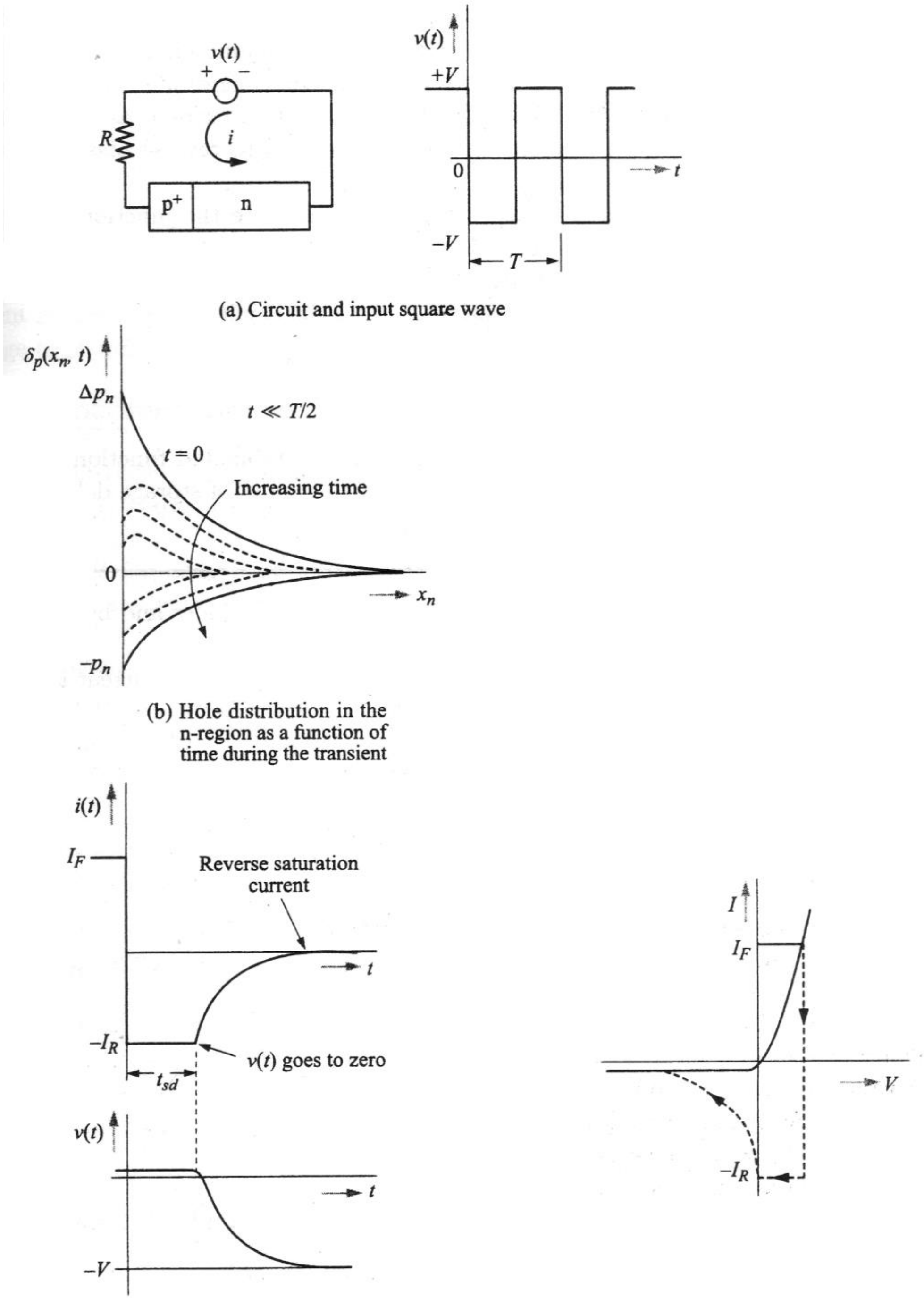
The problems due to stored charge can be reduced by the use of p+n junction with $W_n << L_p$. In this case, very little charge is stored in the minority carrier distribution. Thus, only little time is required to turn ON and OFF the diode. Such narrow base diode makes switching faster. The switching speed can be improved by introducing recombination centres. Gold (Au) in Si acts as efficient recombination centre. Therefore, addition of Au in Si improves the switching speed.

3.9.2 Reverse Recovery Transient

In most switching applications, diode is switched from forward-bias to reverse-bias or vice versa. When a diode is switched from ON state to OFF state, a reverse current much greater than the reverse saturation current can flow through the diode.

For example, consider the situation when the voltage across a p+n junction diode suddenly changes from +V to -V as shown in Fig. 3.26(a). While the diode is forward-biased the current through the diode is $\frac{V}{R}$ (as the forward drop across the diode is negligible compared to R). As the source voltage is reversed to -V at t = 0, the current must initially reverse to $i = I_R = \frac{-V}{R}$. This is because, the stored charge and junction voltage cannot be changed instantaneously. Thus, at t = 0, the current is just reversed to $\frac{-V}{R}$ and voltage drop across the junction remains at the small forward drop before t = 0. This implies that the slope of the distribution must be positive at x = 0 since the current is negative as shown in Fig. 3.26(b).

As the stored charge dies out, the junction voltage can be evaluated using equation (3.93). As long as Apn is positive, the junction voltage is positive and small. Therefore, $i = \frac{-V}{R}$ until Δp_n reduces to zero. When the stored charge is removed and Apn becomes



(a) Circuit and input square wave

(b) Hole distribution in the n-region as a function of time during the transient

c) Variation of current and voltage with time

(d) Sketch of transient current and voltage on the device characteristics

Fig. 3.26 Storage delay time in a p+n diode

negative, the junction exhibits a negative voltage. Now, the source voltage divides between the junction and external resistance R as the resistance of the junction increases. As more and more voltage drop across the reverse biased junction, the magnitude of reverse current goes on decreasing until it reaches the small reverse saturation current, as shown in Fig. 3.26(c). The switching current trajectory from forward bias to reverse bias is shown a Fig. 3.26(d) .

The time (t_{sd}) required for the stored charge (and therefore the junction voltage) to become zero is called storage delay time. This is an important figure of merit for dioda used for switching applications. The carrier life-time is the critical parameter deciding the value of tsd- An exact analysis of the situation leads to the result

$$t_{sd} = \tau_p \left[erfc\left(\frac{I_F}{I_F + I_R} \right) \right]^2 \tag{3.94}$$

where erfc refers to error function complement which is a tabulated function. Thus, the minority carrier life-time can be determined from measurement of storage delay time.

3.9.3 Measurement of t_{sd}

Storage delay time (t_{sd}) can be reduced by using narrow base diodes and by introducing recombination centeres.

Assume a p+n diode forward-biased with current I_F. At t = 0, the current is switched to $-I_R$. By equation (3.91), i(t) is given by

$$i(t) = \frac{Q_p(t)}{\tau_p} + \frac{qQ_p(t)}{dt} \tag{3.95}$$

For $t < 0$, $Q_p = I_F\tau_p$

$t > 0$, $i(t) = -I_R$

Taking Laplace transform of equation (3.95), for t > 0

$$\frac{-I_R}{s} = \frac{Q_p(s)}{\tau_p} + Q_p(s) - I_F\tau_p$$

$$Q_p(s) = \frac{I_F\tau_p}{s + \frac{1}{\tau_p}} - \frac{I_R}{s\left(s + \frac{1}{\tau_p}\right)}$$

Taking inverse Laplace transform.

$$Q_p(t) = I_F\tau_p\ e^{-t/\tau_p}\left(e^{-t/\tau_p} - 1\right)$$

$$= \tau_p\ [-I_R + (I_F + I_R)\ e^{-t/\tau_p}\ ]$$

Assuming $Q_p(t) = gAL_p\Delta p_n(t)$

$$\Delta p_n(t) = \frac{\tau_p}{qAL_p}\ [-I_R + (I_F + I_R)\ e^{-t/\tau_p}\ ]$$

At $t = t_{sd}$, $\Delta p_n(t) = 0$

$$\therefore \quad t_{sd} = -\tau_p \ln\left(\frac{I_R}{I_F + I_R}\right)$$

$$= \tau_p \ln\left(1 + \frac{I_F}{I_R}\right) \qquad (3.96)$$

Equation (3.96) shows that storage delay is less if forward current is less and reverse current is more. Storage delay of a given device can be reduced by a using a large negative votlate to switch off the device.

3.9.4 Capacitance Due to Charge Storage (C_s)

Under forward-bias the capacitance due to stored charge dominates the capacitance due to depletion layer. Capacitance due to stored charge is called storage capacitance or diffusion capacitance.

To evaluate C_s, consider p^+n junction forward-biased with a steady current I_F. The stored charge in the injected hole distribution is

$$Qs = I_F\tau_p$$

But $\quad I_F = qA \dfrac{D_p}{L_p} \Delta p_n \quad$ and $\quad D_p\tau_p = L_p^2$

$$\therefore \quad Q_s = qA \frac{\Delta D_p}{L_p} D_p\tau_p = qA \frac{L_p^2}{L_p} \Delta p_n$$

$$= qA\, L_p\Delta p_n$$

$$= qA\, L_p p_{no}\, e^{qV_F/kT} \quad \text{for } V_F >> V_T$$

Alternate method

$$Q_s = qA \int_0^\infty \delta p(x)dx$$

$$= qA \int_0^\infty \Delta p_n(t) e^{-x/L_p} dx$$

$$= qAL_p\Delta p_n$$

The capacitance due to small changes in stored charge is

$$C_s = \frac{dQ_s}{dV_F} = \frac{q^2}{kT} AL_p p_{no}\, e^{qV_F/kT} = \frac{q}{kT} Q_s$$

$$= \frac{q}{kT} I_F\tau_p \qquad (3.97)$$

Similarly, the ac conductance is given by

$$G_s = \frac{dI_F}{dV_F} = \frac{qAL_p p_n \frac{d}{dV_F}(e^{qV_F/kT})}{\tau_p}$$

$$= \frac{q}{kT} . I_F \qquad = \frac{I_F}{V_T}$$

$$i_{(ac)} = G_s v_{(ac)} + C_s \frac{dv_{(ac)}}{dt}$$

where $G_s = \frac{qI_{ac}}{kT}$

and $C_s = G_s \tau_p = \frac{I_F}{V_T} \tau_p$ (3.98)

The stored charge capacitance is a serious limitation to a p-n junction at high - frequency.
Note: An alternate method for evaluating Q_s is given in Problem 3.25.

3.9.5 Small Signal Equivalent Circuit

Fig. 3.27 shows the small signal equivalent circuit of diode under forward-bias and reverse bias.

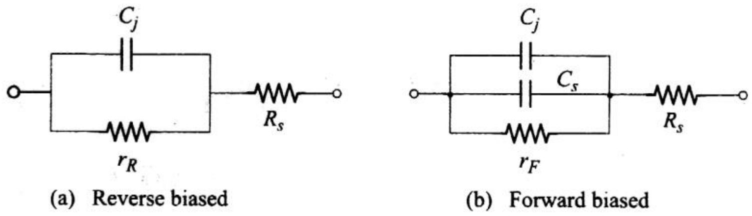


Fig. 3.27 Equivalent circuit of diode

The different parameters of the circuit are:

C_j - depletion layer capacitance
C_s - storage capacitance
R_s - resistance of the neutral region (bulk)
r_R - dynamic reverse resistance
r_F - dynamic forward resistance.

$$IF = I_o \left(e^{V_F/V_T - 1}\right) \cong I_o e^{V_F/V_T} \quad \text{if} \quad V_F << V_T$$

$$\frac{1}{rF} = \frac{dI_F}{dV_F} = \frac{d(I_o e^{V_F/kT})}{dV_F}$$

$$= \frac{I_o e^{V_F/V_T}}{V_T} = \frac{I_F}{V_T}$$

or $r_F = \frac{V_T}{I_F}$ (3.99)

Example 3.10 For a p$^+$s. Si diode at 300 K $N_D = 10^{15}$ cm^{-3} on the n-side. Cross-sectional area is 10^{-3} cm^2 on n-side and minority carrier life-time is 0.1 μs. If the diffusion constant is 16 cm^2/s, calculate dynamic forward resistance and storage capacitance at forward-bias of 0.6 V.

Solution

For a p$^+$n junction,

$$I_o = qA \frac{D_p}{L_p} p_{no}$$

D_p = 16 cm²/s, $\tau_p = 0.1\ /\mu s = 10^{-7}$ s

L_p = $\sqrt{D_p \tau_p} = \sqrt{16 \times 10^{-7}} = 1.265 \times 10^{-3}$ cm

p_{no} = $\frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5$

= 4.553×10^{-13} A

when V_F = 0.6 V, $I_F = I_o e^{V_F/V_T}$

i.e., I_F = $4.553 \times 10^{-13} \times e^{0.6/0.026} = 4.79$ mA

Dynamic conductance G_s = $\frac{I_F}{V_T}$

= $\frac{4.79 \times 10^{-3}}{0.026} = 0.184$ S

Dynamic forward resistance r_F = $\frac{V_T}{I_F} = \frac{0.026\ \ 26}{4.79 \times 10^{-3}} = 5.43\Omega$.

Storage capacitance C_s = $\frac{I_F}{V_T} \tau_p = 18.4$ nF

3.10 METAL SEMICONDUCTOR CONTACT

A junction between a metal and semiconductor may behave like a diode or it may be an ohmic contact which conduct well in both directions. A rectifying metal semiconductor contact is called a Schottky diode.

3.10.1 Ideal Metal Semiconductor Contact

An ideal metal semiconductor contact is one in which the interface is free from any charge or defects. The behaviour of an ideal metal semiconductor contact depends on the relative values of work functions of metal (ϕ_m) and semiconductor (ϕ_{sc}) used as listed in Table 3.1. Work function is the energy difference between vacuum level and Fermi level. Vacuum level is always continuous.

Table 3.1

Type of contact	Relative work functions	
	$\phi_m > \phi_{sc}$	$\phi_m < \phi_{sc}$
metal n-type semiconductor	rectifying	ohmic
metal p-type semiconductor	ohmic	rectifying

Tips to draw energy band diagrams

(1) Draw the energy bands 'of Isolated metal and semiconductor with common vacuum level.

(2) Under thermal equilibrium (when contact is made between metal and semiconductor) the Fermi levels align, so that it is a common horizontal line.

(3) The Fermi level in metal is taken as reference.

(4) The location of E_C, E_i and E_V at the interface remain the same as that in isolated energy band diagram with respect to metal Fermi level.

(5) The energies remain fiat in neutral region.

(6) Fermi level in semiconductor side moves up if negative potential is applied to semiconductor and move down if positive potential is applied to semiconductor with respect to metal.

Metal n-type Semiconductor Schbttky Contact

When a contact is made between metal and n-type semiconductor with $\phi_m > \phi_{sc}$, the Fermi level in semiconductor is above that in metal. The average electron energy in semiconductor is more than that in metal, so that electrons move from semiconductor to metal until the Fermi levels align and equilibrium condition is attained. As electrons move away from n-type semiconductor, the semiconductor near the metal get depleted of mobile charge carriers.

The energy Band diagram1 in the neutral region can be drawn by knowing the position of Fermi level with respect to band edge (E_C), which is a function of doping. The position of E_C, E_V and E_i at interface remain unchanged with respect to metal Fermi level in the separate energy 'band diagram" with common vacuum level. Connecting these points at the interface with that at the edge of the depletion layer completes the equilibrium energy band diagram as shown in Fig. 3.28(b).

From Figs. 3.28(a) and (b) under equilibrium,

$$q\phi_{Bm} \text{ - Barrier for electron flow from metal to semiconductor} = q(\phi_m - \psi_s) \tag{3.100}$$

where ψ_s is the electron affinity of the semiconductor.

$$\phi_{sc} = \psi_s + \frac{(E_C - E_{F_s})}{q}$$

The energy barrier for electron movement from semiconductor to metal is qV_o, as shown in Fig. 3.28 b, which is equal to the built in potential of schottky diode, given by

$$\begin{aligned} qV_o &= q\phi B_m - (E_C - E_{F_s}) \\ &= q(\phi_m - \psi_s) - (E_C - E_{F_s}) \\ &= q\phi_m - [q\psi_s + (E_C - E_F) \\ &= q\phi_m - q\phi_{sc} \\ &= q(\phi_m - \phi_{sc}) \end{aligned} \tag{3.101}$$

Also, from Fig. 3.28 (a) and (b),

$$q\phi B_m = qV_o + E_C - E_F \tag{3.102}$$

This energy barrier from metal to semiconductor is called Schottky barrier and is independent of bias voltage. The measured barrier heights of metals on different semi conductors is shown in Table 3.2.

The barrier for electron movement from semiconductor to metal changes with change in bias as shown in Fig. 3.28(c) and (d). A forward bias (applying negative potential to n type semiconductor) decreases the barrier, increasing current from metal to semiconductor. Reverse bias increases the barrier so that, injection of electrons from semiconductor to metal reduces to zero, so that Jms approaches zero as shown in Fig. 3.28(d).

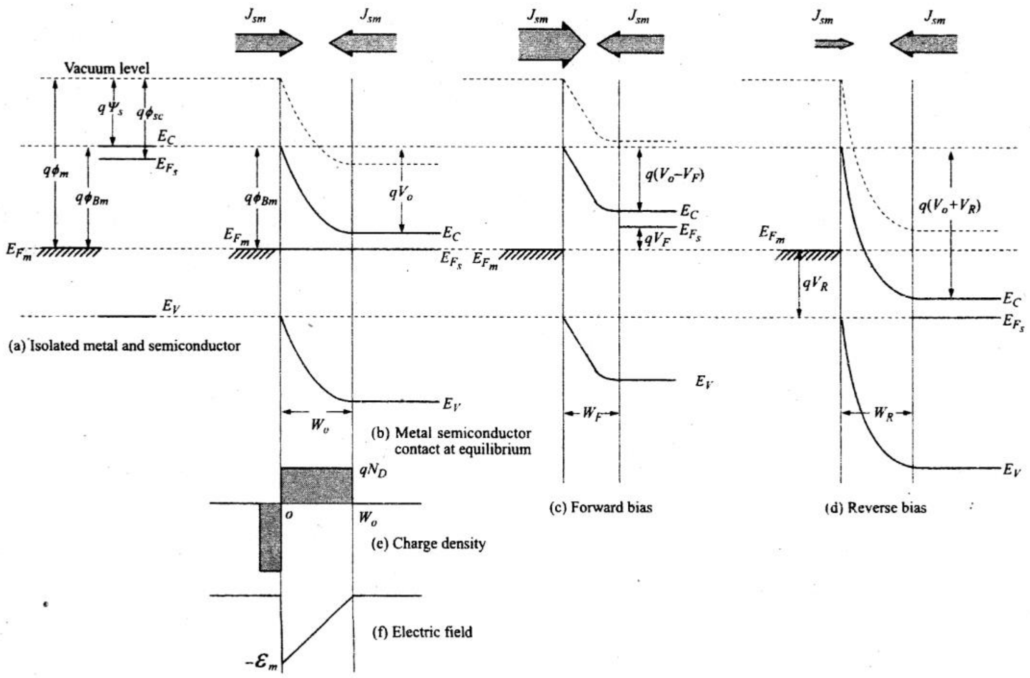


Fig. 3.28 Energy band diagram of Schottky diode

Table 3.2 Measured barrier heights of various metals on some elemental and compound semiconductors (in eV) at 300 K

Semiconductor	Type	Metal				
		Ag	Al	Au	Pt	W
Ge	N	0.54	0.48	0.59	-	0.48
Gc	P	0.50	-	0.30	-	-
Si	N	0.78	0.72	0.80	0.90	0.67
Si	P	0.54	0.58	0.34	-	0.45
GaAs	N	0.88	0.80	0.90	0.84	0.80
GaAs	P	0.63	-	0.42	-	-

The expression for width of depletion layer is similar to that for a p+n junction and is given by

$$W_o = \sqrt{\frac{2\epsilon V_o}{qN_D}} \tag{3.103}$$

From Fig. 3.28(f), the potential drop across the depletion layer is $V_o = \frac{1}{2} E_m W_o$

Or $$E_m = \frac{2V_o}{W_o} \tag{3.104}$$

Table 3.2 shows measured barrier heights of various metals on some elemental and compound semiconductors.

Example 3.11 A contact between tungsten and n-type silicon with ND : 10^{15} cm^{-3} is made at 300 K. Calculate the contact potential, equilibrium depletion layer width maximum electric field at equilibrium. Given $\phi_m = 4.5V$, $\psi_s = 3.84$ V, $N_C = 2.8 \times 10^{19}$ cm^{-3}, $\in_r = 11.8$.

Solution

Contact potential $V_o = \phi_m - \psi_s - \frac{E_C - E_F}{q}$

$$E_C - E_F = kT \ln\left(\frac{N_C}{N_D}\right)$$

$$= 0.026 \ln\left(\frac{2.8\times 10^{19}}{10^{15}}\right)$$

$$= 0.266 \text{ eV}$$

$$\frac{E_C - E_F}{q} = 0.266\text{V}$$

$$V_o = 4.5 - 3.84 - 0.266$$

$$= 0.394 \text{ V}$$

Depletion layer width $W_o = \sqrt{\frac{2 \in V_o}{qN_D}}$

$$= \frac{\sqrt{2\times 8.854\times 10^{-14}\times 11.8\times 0.394}}{1.6\times 10^{-19}\times 10^{15}}$$

$$= 7.173 \times 10^{-5} \text{ cm}$$

$$= 0.717\ \mu\text{m}$$

Maximum electric filed $E_m = \frac{2V_o}{W_o}$

$$= \frac{2\times 0.394}{7.173\times 10^{-5}}$$

$$= 10990.2 \text{ V/cm}$$

3.10.2 Depletion Layer Capacitance

$$C_j = \frac{dQ_D}{d(V_o - V_a)}$$

$$|Q_D| = qN_DWA$$

$= qN_DA \sqrt{\frac{2\in}{qN_D}(V_o - V_a)}$ (The expression for W is same as that for a p+n junction)

$$= A\sqrt{2\grave{o}N_D(V_o - V_a)}$$

$$C_j = A\,(2\epsilon q N_D)^{\frac{1}{2}} \times \tfrac{1}{2}(V_o - V_a)^{\frac{-1}{2}}$$

$$= A\sqrt{\frac{\epsilon q N_D}{2(V_o - V_a)}} = \frac{\in A}{W}\left(\because \quad W = \sqrt{\frac{2\in (V_o - V_a)}{qN_D}}\right)$$

3.10.3 Forward Characteristics of Metal Semiconductor Junction

The forward-bias does not change the barrier from metal to semiconductor. Therefore, emission of electrons from metal to semiconductor remains unchanged. The barrier from semiconductor to metal reduces by qV_F, which causes an increased rate of emission of electrons from semiconductor to metal producing a net current from metal to semiconductor as shown in Fig. 3.28(c).

It can be observed that in Schottky diode the increase in forward current is not due to the diffusion of minority carriers, but is due to the increased emission of majority carriers from semiconductor to metal. Therefore, there is no minority carrier storage in Schottky diode.

The work function of a metal is of the order of several volts. But formation of metal semiconductor contact reduces the barrier from metal to semiconductor as well as that from semiconductor to metal. Therefore, the current in metal semiconductor contact may be evaluated from the expression for thermionic emission current with appropriate values of barrier.

Thermionic emission current is given by Richardson-Dushmann equation

$$I = ART^2\, e^{-q\phi_m/kT} \tag{3.105}$$

where

R - Richardson constant = $\dfrac{4\pi q m_n k^2}{h^3}$

A - surface area of emission

T - absolute temperature

ϕ_m - metal work function

k - Boltzmann constant

m_n - mass of electron

I_{sm} is the current due to electrons crossing from metal to semiconductor over the barrier

$$I_{sm} = AR^*T^2\, e^{-q\phi_m/kT} \tag{3.106}$$

Similarly, the current due to electrons crossing from semiconductor to metal with bias V_a applied is

$$I_{ms} = AR^*T^2\, e^{-q(\phi_{Bm} - V_a)/kT} \tag{3.107}$$

where R^* is the effective Richardson constant with rrin replaced by effective mass of charge carriers.

Measured values of Richardson constant for electrons and holes in silicon are 110 and 32 $A/K^2\ cm^2$.

The net current through the junction may be computed as the difference between the two.

i.e., $$I = I_{ms} - I_{sm} = AR^*T^2\, e^{-q\phi_{Bm}/kT}\left[e^{V_a/V_T} - 1\right]$$

$$= AJ_o\left(e^{V_a/V_T} - 1\right) \tag{3.108}$$

where $J_o = R^*T^2\, e^{-q\phi_{Bm}/kT}$

This shows that the current equation is similar to that of a p-n junction diode. The difference lies in the reverse saturation current. For Schottky diode,

$$I_o = AJ_o = AR^*T^2\ e^{-q\phi B_m/kT} \qquad (3.109)$$

which is much larger than that in p-n junction diode. Fig. 3.29 shows the characteristics of a Si p-n junction and a metal-silicon Schottky diode.

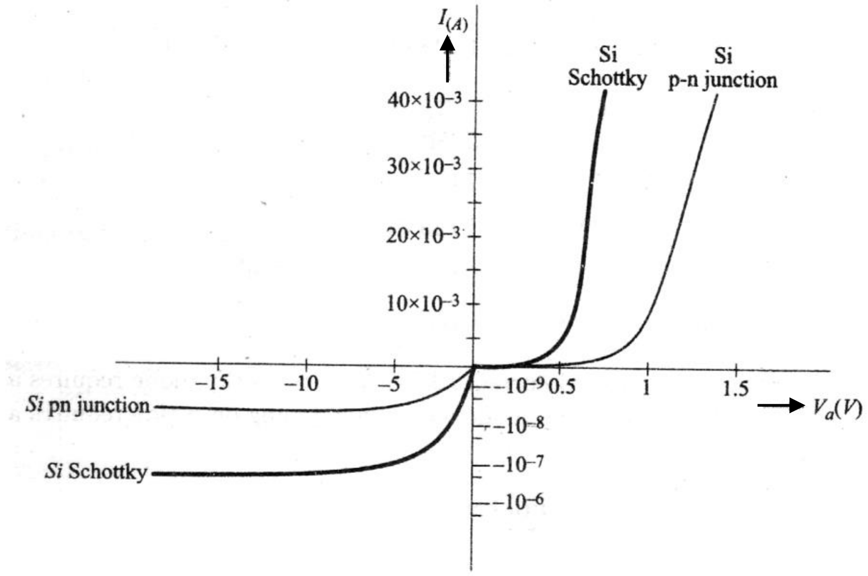


Fig. 3.29 J-V Characteristics of Si p-n junction and Si Schottky diode

Example 3. 12 A Schottky diode between tungsten and silicon doped with 10^{15} As atoms/cm3 has a junction area of 10^{-3} cm^2. R^* = 110 A/K^2 cm^2 ϕ_{Bm} = 0 66 V at 300 V.

a. Determine the current through the diode at 300 K for forward-bias of 0 3 V
b. Consider a p+n junction diode with equal doping on n-side. What is the current at the same forward-bias. Take D_{pn} = 12 cm^2/s, τ_{pn} = 1 µs.
c. What is the forward voltage required for the same forward current as that in part (a)?

Solution

a. $I = AR^*T^2\ e^{-q\phi_{Bm}/kT}\left[e^{V_a/V_T} - 1\right]$

$= 10^{-3} \times 110 \times 300^2 \times e^{-0.66/0.026}\ [e^{0.3/0.026} - 1]$

$= 9.6 \times 10^{-3} = 9.6$ mA

b. $L_{pn} = \sqrt{D_{p_n}\tau_{p_n}}$

$= \sqrt{12\times10^{-6}} = 3.46 \times 10^{-3}$ cm

$n_i = 1.5 \times 10^{10}$ cm^{-3}

$I = qA\left(\dfrac{D_{p_n} n_i^2}{L_{p_n} N_D}\right)\left(e^{V_a/V_T} - 1\right)$

$= 1.6 \times 10^{19} \times 10^{-3}\left(\dfrac{12}{3.46\times10^{-3}} \times \dfrac{(1.5\times10^{10})^2}{10^{15}}\right)(e^{0.3/0.026} - 1)$

$= 1.2485 \times 10^{-13}\ (e^{0.3/0.026} - 1)$

$= 1.28 \times 10^{-8}$ A

c. $9.6 \times 10^{-3} = 1.2485 \times 10^{-13} \left(e^{V_F/V_T} - 1\right)$

$$e^{V_F/V_T} - 1 = \frac{9.6\times10^{-3}}{1.2485\times10^{-13}}$$

$= 7.69 \times 10^{10}$

$\therefore \quad V_F = V_T \ln 7.69 \times 10^{10}$

$= 0.026 \times \ln 7.69 \times 10^{10}$

$= 0.652$V.

Remark: For the same forward current (9.6 mA) Si Schottky diode requires a forward- bias of only 0.3 V whereas a Si p+n diode with same doping on n-side requires a forward-bias of 0.652 V.

3.10.4 Metal p Type Semiconductor Schottky Diode

A contact between metal and p type semiconductor behaves like rectifying contact if $\phi_{sc} > \phi_m$. The energy band diagram of metal p type semiconductor under different conditions are shown in Figure 3.29.

Barrier for hole movement from semiconductor to metal under equilibrium is $qV_o = q(\phi_{sc} - \phi_m)$ as shown in Figure 3.29(a). With forward bias this barrier reduces to $q(V_o - V_F)$ and with reverse bias the barrier increases to $q(V_o + V_R)$.

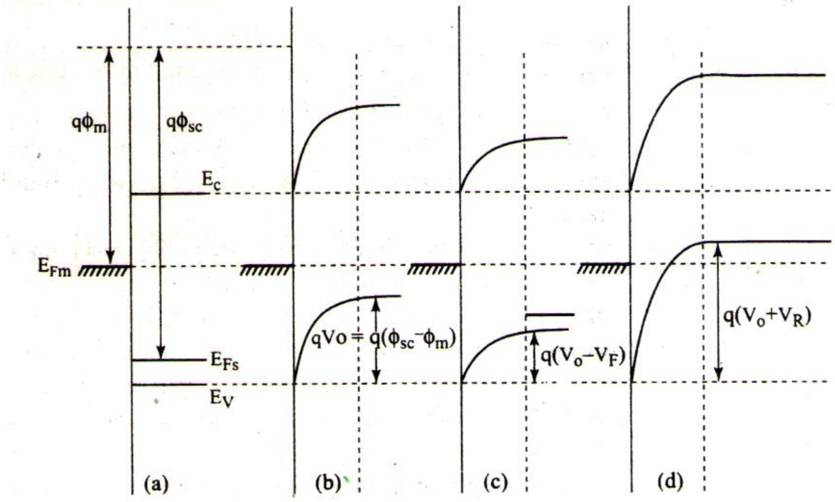


Fig. 3.29(a) Energy band diagram of metal p-type semiconductor: (a) isolated metal and semiconductor (b) at equilibrium (c) forward bias (d) reverse bias

3.10.5 Comparison between Schottky diode and p-n junction diode

(1) The reverse saturation current of Schottky diode is many orders higher than that of p-n junction of same material. Therefore, the forward voltage drop for a given current is much less than that of p-n junction diode. Hence, Schottky diode is preferred in low voltage high current rectifiers.

(2) There is no storage capacitance as there w no storage of minority carriers in Schottky diode. Its reverse recovery time is decided only by the depletion layer capacitance and series resistance of the diode (CjR,). Thus, it can be used for high speed switching.

The disadvantage is its high reverse saturation current and poor reproducibility.

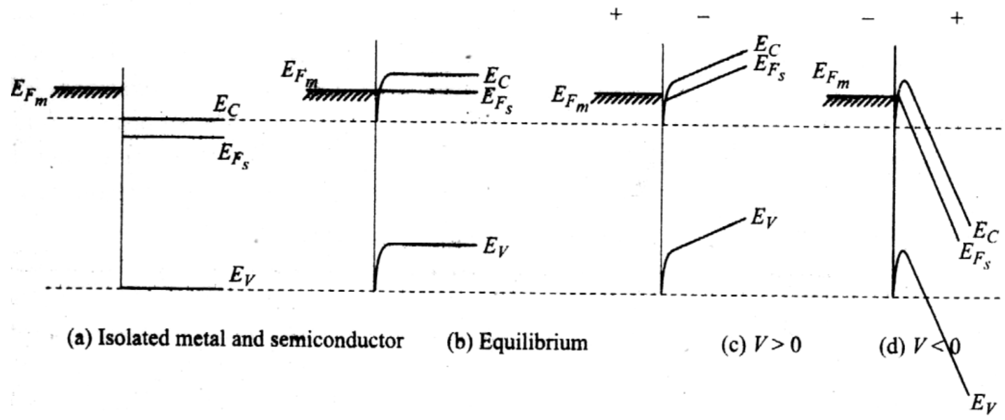


Fig. 3.30 Energy band diagram of an ohmic metal n-type semiconductor contact

Ohmic Contacts

Semiconductor devices need electrodes (connecting leads), which are made by metallisation over the semiconductor. This metal semiconductor contact should not behave like rectifying diodes. It must be a low resistance contact which conducts well in both directions. Such a metal semiconductor contact is called ohmic contact. A metal n-type semiconductor contact acts as an ohmic contact if $\phi_m < \phi_{sc}$ Fig. 3.30 shows the energy band diagram of a metal semiconductor contact.

It can be observed from Fig. 3.30(b) that there is no barrier for electron movement from metal to semiconductor or from semiconductor to metal. At equilibrium, as E_{Fm} is above E_{Fs} electrons move from metal to semiconductor, which increases the electron concentration near the interface in semiconductor i.e., electrons are accumulated near the interface. There is no depletion layer near the junction. On applying a bias, the potential drops across the neutral region (relatively high resistance) and the energy bands in the neutral regions bend upward or downward.

In either case, there a no depletion layer and no energy barrier for electron movement. Therefore, this junction conducts well in both directions. Similarly, an ohmic contact on p-type semiconductor can be formed by a metal with $\phi_m > \phi_{sc}$.

If a semiconductor is very heavily doped, a metallic contact will behave as an ohmic contact, irrespective of the work function. This is due to the very small depletion layer width and very high electric field present at the junction. Under this condition tunneling of charge carriers takes place in both directions with very small bias itself. During fabrication of devices, region below contacts are usually heavily doped to make the contacts ohmic.

3.11 PHOTODIODE

If light (photon) of energy hv > E_g falls on a semiconductor, it is absorbed by the semiconductor. The minimum frequency of light that is absorbed by a given material is called cutoff frequency.

$$E_{min} = hv_{min} = \frac{hc}{\lambda_{max}} = E_g$$

Cutoff wavelength is the maximum wavelength of light that is absorbed by a material. The absorption of photon by the semiconductor results in the generation of EHP, increasing the minority as well as majority carrier concentration. The increase in minority carrier concentration causes an increase of drift current across the junction. Let I_L be the current due to optically generated minority carriers. Now, the diode equation gets modified to

$$I = I_s \left(e^{V_a/V_T} - 1\right) - I_L \tag{3.110}$$

This equation represents the I-V characteristics of photodiode which is plotted for different values of I_L in Fig. 3.31.

From equation (3.110) the open circuit voltage (voltage when current is zero) and short-circuit current (current when voltage is zero or if terminals are short-circuited) are given by

$$V_{oc} = \frac{kT}{q} \ln\left(1 + \frac{I_L}{I_s}\right) \tag{3.111}$$

$$I_{sc} = -I_L \tag{3.112}$$

V_{oc} and I_{sc} are zero for an ordinary p-n junction. Equations (3.111) and (3.112) show that when an illuminated p-n junction with hv > E_g is open-circuited, a voltage develops across its terminals. This is called photovoltaic effect. This is the basic principle on which a solar cell works.

3.11.1 Solar Cell

From the characteristics shown in Fig. 3.31 it is seen that power is positive in 1st and 3rd quadrants and negative in the 4th quadrant. Negative power represents power delivered by the device to external circuit whereas positive power represents power dissipated by the device from the external source. Thus, when a photodiode is operated in the 4th quadrant it acts as a cell which delivers power to external circuit.

Fig. 3.32 shows the portion of photodiode characteristics in 4th quadrant redrawn in 1st quadrant.

The power delivered by a solar cell is maximum when I-V is maximum. The values of V and I when the power is maximum are denoted as Vm and I_m. The values of V_m and I_m are in such a way that it represent a square of maximum area as shown in Fig. 3.32.

A figure of merit of solar cell is defined by fill factor,

$$FF = \frac{V_m I_m}{V_{oc} I_{sc}} \tag{3.113}$$

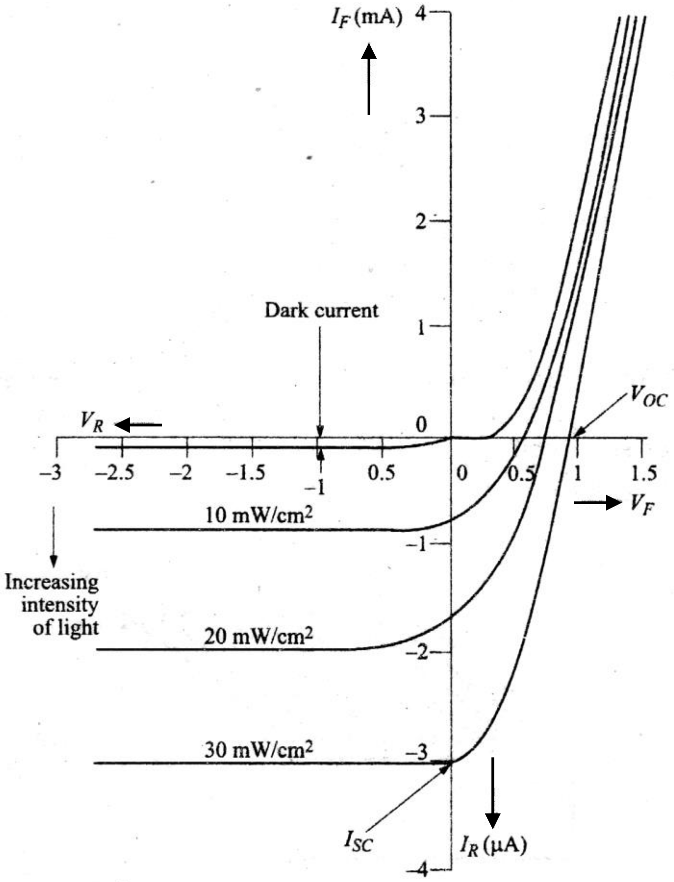


Fig. 3.31 I-V characteristics of illuminated photodiode

The efficiency of a solar cell is

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_m I_m}{P_{in}}$$

$$= FF \frac{V_{oc} I_{sc}}{P_{in}} \tag{3.114}$$

The open-circuit voltage increases with increase in IL and decrease in I_s . I_s decreases with band gap of the material. I_L can be increased by using special structures, light concentrators etc.

The n + p structure provides a large value for I_L, giving large V_{oc}. The top surface is coated with anti-reflection material to improve the efficiency. A well defined solar cell has a FF of 0.7 to 0.8 and efficiency around 15. Solar cells provide maintenance free service for a long time. Solar cells are used to power satellites, for remote and rural electrification etc.

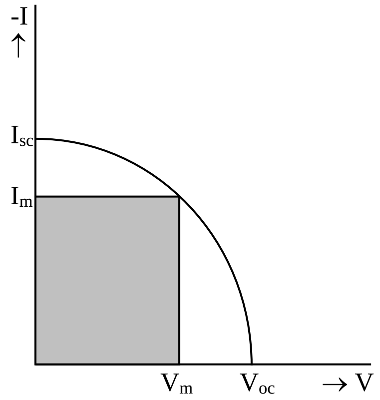


Fig. 3.32 Characteristics of a Solar Cell drawn in the first quadrant

Construction of soter cell

Simplified structure of a solar cell is shown in Fig. 3.33.

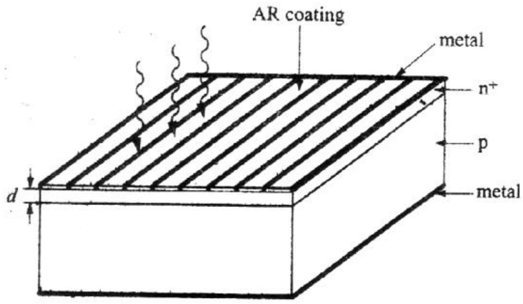


Fig. 3.33 Simplified structure of a solar cell

A Solar cell consists of an n-p junction with very thin n+ region and thick p-region Metal contacts are taken out from the top and bottom surfaces. An interdigitated structure (as shown in Fig. 3.33) is used for top surface contact so that area of exposure to sunlight is maximum and series resistance is minimum. The thickness of n-region d must be much less than the hole diffusion length Lp. The doping on p-side is small so that the diffusion length is large and minority carriers cross the junction without recombination maximising I_L.

Example 3.13 A. silicon solar cell has an area of 10 cm^2 with n+p structure with $N_A = 10^{15}$ cm^{-3} on the p-side. If μ_n = 1350 cm^2, τ_n = 1 μ_s on the p-side and I_L = 200 mA, compute I_{sc}, V_{oc} and maximum power output. FF = 0.75. If the incident optical power is 70 mW/cm^2, what is the efficiency of the solar cell?

Solution

For n_p^+ diode,

$$I_s = qA \frac{D_n}{L_n} n_{p_o}$$

$$D_n = \mu_n \frac{kT}{q} \quad \text{by Einstein relation}$$

$$= 1350 \times 0.026 = 35.1\ cm^2/s$$

$$L_n = \sqrt{D_n \times \tau_n} = \sqrt{35.1 \times 10^{-6}} = 5.92 \times 10^{-3}\,\text{cm}$$

$$n_{po} = \frac{n_i^2}{N_A} = \frac{\left(1.5 \times 10^{10}\right)}{10^{10}} = 2.25 \times 10^5\, cm^{-3}$$

$$I_s = 1.6 \times 10^{-19} \times 10 \times \frac{35.1}{5.92 \times 10^{-3}} \times 2.25 \times 10^5$$

$$= 2.13 \times 10^{-9}\ A = 2.13\ nA$$

$$I_{sc} = -I_L = -200mA = -0.2A$$

$$V_{oc} = \frac{kT}{q} \ln\left(1 + \frac{I_L}{I_s}\right)$$

$$= 0.26 \ln\left(1 + \frac{0.2}{2.13 \times 10^{-9}}\right) = 0.477V$$

Maximum output power $= V_m I_L = FFV_{oc}\, I_{sc}$

$$= 0.75 \times 0.477 \times 0.2$$

$$= 71.55\ mW$$

Total incident power = power/unit area x area of cell

$$= 70 \times l0 = 700mW$$

Maximum efficiency of solar cell,

$$\eta = \frac{P_{\max}}{P_{in}} \times l00$$

$$= \frac{71.55}{700} \times l00$$

$$= 10.22\%$$

Photodetector

When photodiode is operated in itNmrd quadrant of characteristics, the reverse current increases with increase in intensity of light falling on it. Thus it acts a. a Photodetector. It can be used to measure light intensity,, to convert optical signal to electrical signals (demodulation in optical communication).

In a photodetector it is desirable that the width of-depletion region (W) be large enough so that most of the photons are absorbed within W rather than in neutral p and n regions to have high sensitivity. When the carriers are generated primarily within the depletion layer W, the detector is called a depletion layer photodiode.

The speed of response and sensitivity are critical in most of the optical detection applications. The appropriate width of depletion layer is chosen as a compromise between sensitivity and speed of response. When the width is more, the incident photons are mostly absorbed in the depletion layer. It also results in a small junction capacitance. I hereby reducing the RC time constant of the detector circuit. One side of the junction may he lightly doped so that W can be made large. But a too wide depletion region can increase the carrier transit time through it, reducing speed.

In p-i-n diode, an intrinsic region is sandwiched between p and n regions as shown in Fig. 3.34. The intrinsic region may be grown epitaxially on the n-type substrate and p-region can be obtained by diffusion. The resistivity of the intrinsic region is high compared to p and n regions. Therefore, when this device is reverse biased, the applied voltage appears almost, entirely across the i-region. If the drift time is less compared to the carrier lifetime within the i-region, most of the photogenerated carriers are collected by p and n regions. The intrinsic region is made long enough that most of the incident radiation is absorbed within that region.

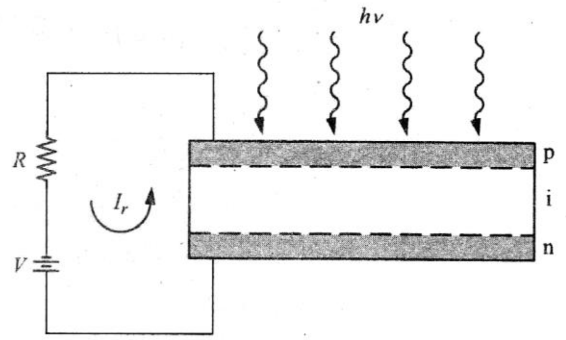


Fig. 3.34 Schematic representation of a p-i-n photodiode

The sensitivity of p-i-n photodetector is very low. Therefore, it is not suitable for detecting weak optical signals. Low-level optical signals can be detected by operating the photodiode in the avalanche region of its characteristics. This enables avalanche multiplication of photogenerated carriers resulting in a higher current than the primary current generated by incident radiation. Avalanche photodiode (APD) makes use of this principle.

Fig. 3.35 shows the schematic representation of a typical avalanche photodiode. APD is biased in the avalanche multiplication region. Large reverse-bias is required for this. In Fig. 3.35, the region denoted by π represents lightly doped p-region.

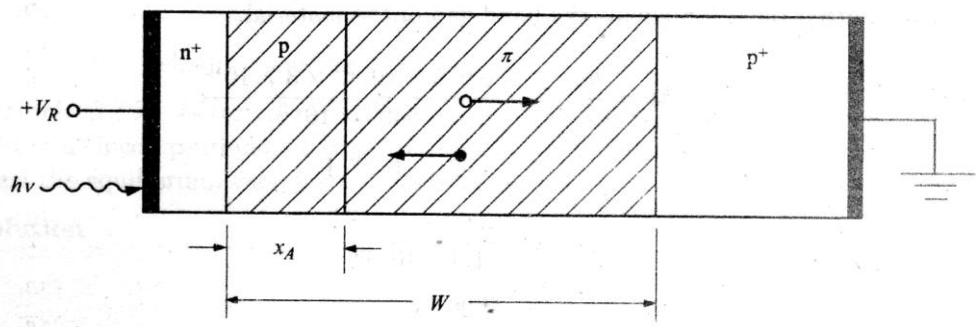


Fig. 3.35 Schematic cross-section of a typical avalanche photodiode

3.12 LIGHT EMITTING DIODES

When charge carriers are injected across a p-n junction (under forward-bias), the current is due to recombination in the depletion region and in the neutral region near the junction. For indirect recombination, the energy will be released as heat. If the recombination is direct, the energy will be released as photons. Therefore, if a p-n junction made of direct band gap semiconductor is forward-biased, the injected charge carriers recombine radiatively emitting light (photons) of energy equal to the band gap of the semiconductor. This phenomenon is called injection electro luminescence. LEDs work on this principle.

LEDs can emit radiations in ultra violet, visible or infrared regions. They are used as generators of light in different, colours. They are widely used in digital displays, instrument panels and traffic signal lighting. LEDs are also used in fibre optic communication and in opto isolators.

The colour of emitted radiation depends on the energy of the emitted photon which is equal to the band gap of the semiconductor. Therefore, LEDs of different colours may be realized by using materials of suitable band gaps.

3.12.1 LED Materials

Visible LEDs are made of materials, which have band gap between 1.8 eV and 2.8 eV (see optical spectrum in Appendix 5). The band gap of Ga $As_{1-x}P_x$ varies from 1.4 eV to 2.3 eV as the percentage composition of phosphoros (a:) is increased from 0 to 1. For value of x upto 0.45 the material has direct band gap. The composition used for red LEDs has x ≅ 0.4. If we can introduce localized states in the band gap through which radiative recombination is possible, LEDs may be fabricated using indirect band gap semiconductors also. GaP is an indirect band gap semiconductor. But, if nitrogen is added to both n and p sides of p-n junction made of GaP, it emits green radiation (λ = 0.565 μm). If zinc and oxygen are added to GaP it produces red emission (λ = 0.69μm).

Example 3.14 A particular green LED emits light of wavelength 5490 A. Determine the energy band gap of the material used.

Energy of emitted radiation equals the band gap of the material.

$$\therefore \quad E_g = hv = \frac{hc}{\lambda} \frac{6.626\times10^{-34}\times3\times10^{10}}{5490\times10^{-8}}$$

$$= 3.62 \times 10^{-19} \text{ J}$$

$$= \frac{3.62\times10^{-19}}{1.6\times10^{-19}}$$

$$= 2.26 \text{ eV.}$$

3.13 VARACTOR DIODE

Varactor diode (variable reactor diode) is used as a voltage variable capacitor. The depletion layer capacitance of the diode is used as the voltage variable capacitor.

For a p-n junction depletion capacitance $C_j \propto V_R^{-n}$ (neglecting V_o)

where n = $\frac{1}{m+2}$

The values of m and n for different diodes are:

m - 0 for abrupt p-n junction M = 0, n = $\frac{1}{2}$

m - 1 for linearly graded junction, M = 1, n = $\frac{1}{3}$

m - $\frac{-3}{2}$ for hyper abrupt junction, M=l - $\frac{3}{2}$, n=2

Varactor diodes are used for electronic tuning.

Resonant frequency

$$f \, \alpha \, \frac{1}{\sqrt{LC}}$$

If a hyper abrupt junction is used for tuning along with an inductor L (which is kept constant)

$$f \, \alpha \, \frac{1}{\sqrt{C}}$$

$$\alpha \, \frac{1}{\sqrt{V_R^{-2}}} \, \alpha V_R$$

Thus the resonant frequency is directly proportional to the applied reverse voltage. Hence; varactor diodes find use in radio and TV receivers for frequency tuning. Other applications are in harmonic generation, parametric amplification, frequency modulation etc.

3.14 TUNNEL DIODE

Example 3.15 helps to understand the principle of a tunnel diode.

Example 3.15 Determine the built-in potential width of depletion layer and peak electric field of a silicon p-n junction at 300 K with p_{po} = 2 × 10^{19} and n_{no} 2 × 10^{19} cm^{-3} and draw the equilibrium energy band diagram. Assume E_g = 1.1 eV.

Solution

$$V_o = \frac{kT}{q} \ln\left(\frac{2\times10^{19}\times2\times10^{19}}{(1.5\times10^{10})^2}\right)$$

$$= 1.09\text{V}$$

$$E_{F_n} - E_i = \text{kT ln } \frac{n_{n_o}}{n_i}$$

$$= 0.026 \text{ ln } \frac{2\times10^{19}}{1.5\times10^{10}}$$

$$= 0.546 \text{ eV}$$

$$E_i - E_{F_p} = \text{kT ln } \frac{p_{p_o}}{n_i} = 0.546 \text{ eV}$$

$$W_o = \sqrt{\frac{2 \in V_o}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$= 2.386 \times 10^{-6} \text{ cm}$$

$$\varepsilon_{m_o} = \frac{2V_o}{W_o} = 9.137 \times 10^5 \text{ V/cm}$$

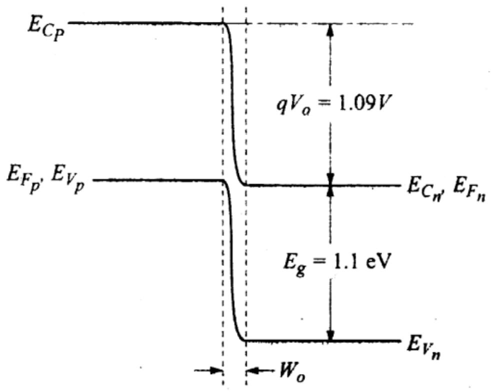


Fig. Ex.3.14

At this doping, Fermi-level position in p-material is at E_V and in n-material is at E_C. If the doping is increased further the Fermi-level in p-material moves down, below E_{V_p} and that in n-mateial moves up, above E_{C_n}. Such very heavily doped semiconductor with Fermi-level lying inside valence band or conduction band is called a degenerate semiconductor. For a degenerate semiconductor doping density is higher than density of states. A tunnel diode is made of this type of semiconductors. The VI characteristics of tunnel diode is shown in Fig. 3.36. The equilibrium energy band diagram of a tunnel diode is shown in Fig. 3.37(a).

The barrier potential is high for this diode and depletion layer is very thin. In a semi-conductor most of the states are vacant above E_F (probability of occupancy < $\frac{1}{2}$) and most of the states are filled below E_F (probability of occupancy > $\frac{1}{2}$). Filled portion of the energy band is shaded in the energy band diagram.

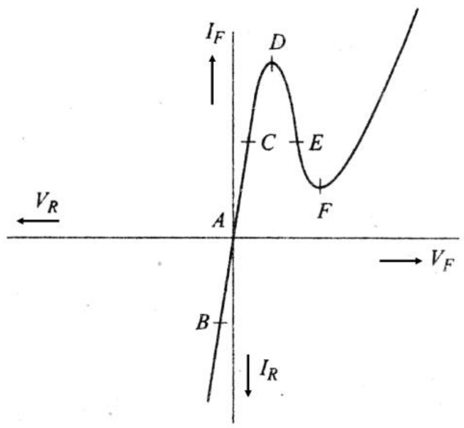


Fig. 3.36 I-V Characteristics of tunnel diode

Reverse-Bias

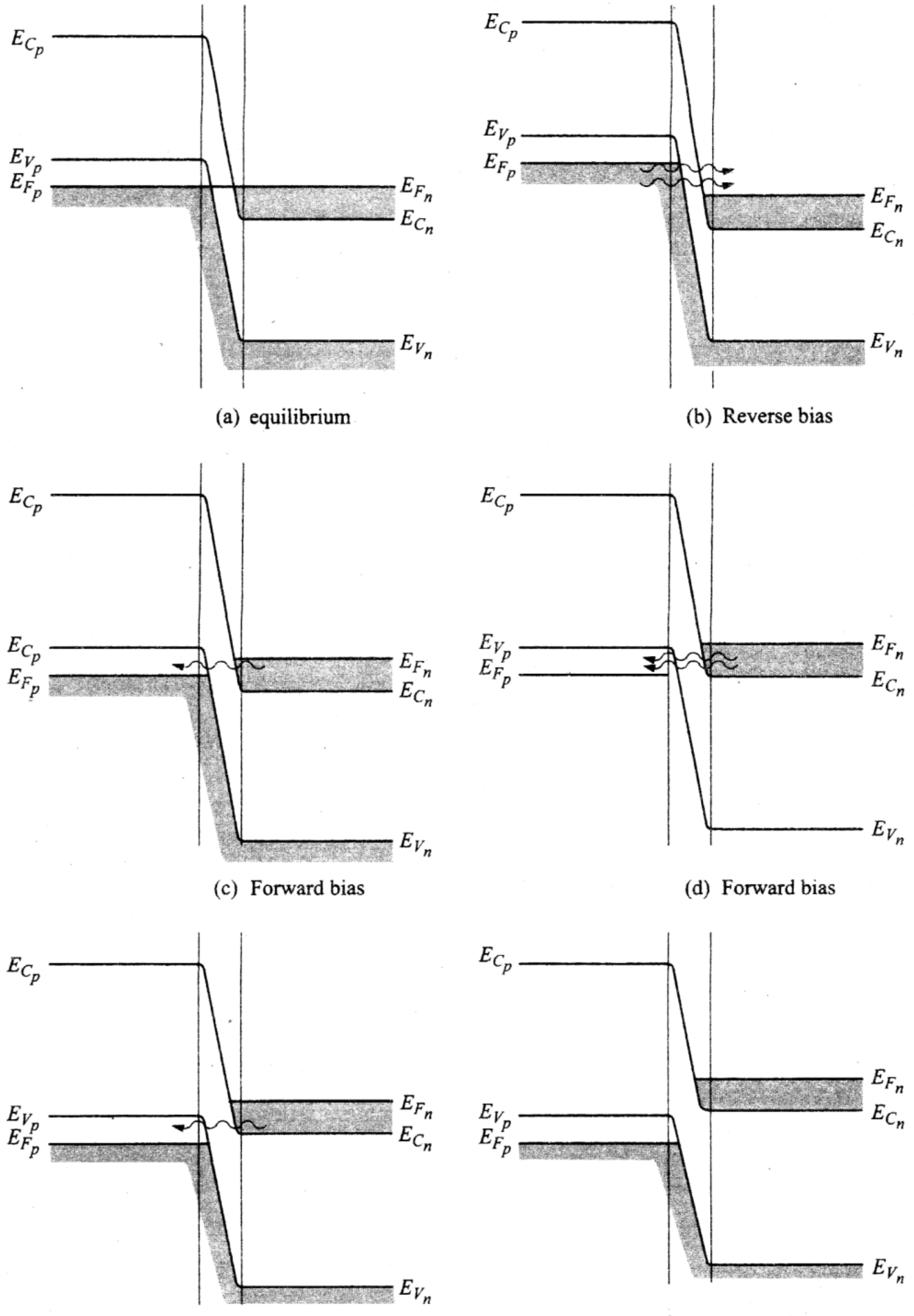
On applying .a very small reverse-bias itself, filled states in the valence band on p-side and vacant states on n-side conduction band appear in same energy range so that electrons tunnel from filled to vacant states as in Fig. 3.37(b). (Tunneling is possible because of the very thin depletion layer and very high electric field as observed in Example 3.15.) This is represented by point B in Fig. 3.36. Tunnel current increases with increase in overlapping energy of filled and vacant states. Therefore tunnel current increases with increase in reverse-bias.

Forward-Bias

When a very small forward-bias is applied, filled states in the conduction band on p-side and vacant state on p-side comes at same energy level as shown in Fig. 3.37(c). Tunneling takes place through this energy range. This is represented by point C in the characteristics.

As forward-bias increases, the overlapping energy range between filled states in conduction band on n-side and vacant states on p-side increases and tunnel current increases. The current reaches maximum when the overlapping is maximum as in Fig. 3.37(d) and point D in the characteristics. As forward-bias is increased further, overlapping between filled and vacant states decreases. Tunneling also decreases, decreasing the forward current as shown in Fig. 3.36(e) and point E in the characteristics. On further increasing the forward-bias the overlap between filled and vacant states vanishes and the tunnel current reduces to zero as shown in Fig. 3.37(f). The current reaches a minimum value at this condition as shown by point F. At this forward-bias, the current is due to the crossing of majority carriers over the potential energy barrier. If forward-bias is increased further, forward current increases exponentially as in an ordinary p-n junction.

Tunnel diode has zero breakdown voltage and zero cut-in voltage as shown by point A in Fig. 3.36. It also consists of a negative resistance region represented by region D-F in the characteristics. Because of small depletion layer width its switching speed is high. Because of these properties a tunnel diode is used for high-frequency switching, amplification, oscillation etc.



Solved Problems

Problem 3.1

A long abrupt Si p-n junction at 300 K has $N_A = 10^{14}$ cm^{-3} and $N_D = 10_{15}$ cm^{-3} on the p and n sides respectively and junction cross-section is 1 mm^2. To attain equilibrium, how many electron-hole pairs might have recombined to form the depletion layer?

Solution

The number of EHPs recombined equals the number of charges on one side of the depletion layer. The number of charges on one side of depletion layer equals doping concentration on one side, multiplied by the volume of depletion layer on that side.

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \text{-0.026 ln } \frac{10^{14}\times 10^{15}}{\left(1.5\times 10^{10}\right)^2} = 0.518 \text{ V}$$

Number of EHP recombined

$$= N_A X_{p_o} A$$

$$= \frac{N_D}{N_A + N_D} W_o A \quad \left[\because X_{p_o} = W_o \cdot \frac{N_D}{N_A+N_D}\right]$$

$$= \frac{N_A N_D A}{N_A + N_D}\sqrt{\frac{2\in V_o}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)}$$

$$= A\sqrt{\frac{2\in V_o}{q}\frac{N_A N_D}{N_A+N_D}}$$

$$= 10^{-2}\times\sqrt{\frac{2\times 8.854\times 10^{-14}\times 11.8\times 0.518}{1.6\times 10^{-19}}\times\left(\frac{10^{14}\times 10^{15}}{10^{14}+10^{15}}\right)}$$

$$= 2.479 \times 10^8.$$

Problem 3.2

Compute contact potential V_o of a Si abrupt p-n junction with $N_D = 10^{15}$ cm^{-3} and $N_A = 10^{17}$ cm^{-3} at (a) 300 K (b) 450 K. Compare the values and justify.

Solution

At 300 K, $\frac{kT}{q} = 0.026$ V; $n_i = 1.5 \times 10^{10}$ cm^{-3}

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026 \times \ln\left[\frac{10^{17}\times 10^{15}}{\left(1.5\times 10^{10}\right)^2}\right] = 0.697 \text{ V}$$

At 450 K, $\frac{kT}{q} = 0.026 \times 1.5 = 0.039$ V

$$n_i(450\text{K}) = 3.88 \times 10^{16}\,(450)^{3/2}\, e^{(-7000/450)}$$

$$= 6.5 \times 10^{13} \text{ cm}^{-3}$$

$$V_o = 0.039 \ln\left[\frac{10^{17}\times 10^{15}}{\left(6.5\times 10^{13}\right)^2}\right]$$

$$= 0.393 \text{ V}$$

As explained in Section 3.2.3, as temperature increases V_o decreases.

Problem 3.3

Two identical Si junction diodes D_1 and D_2 with $\eta = 1$ are connected back-to-back as shown in Fig. 3.30. The reverse saturation current I_s of the diode is 10^{-8} A and breakdown voltage V_{Br} is 50 V. Calculate the voltages V_1 and V_2 droped across the diodes D_1 and D_2 assuming $\frac{kT}{q} = 26$ mV.

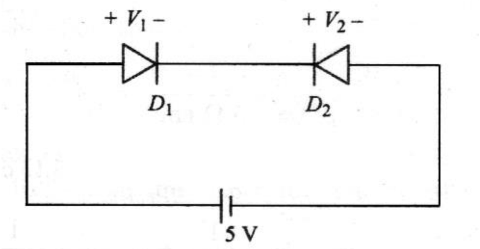


Fig. Sp.3.3

Solution

$$I_s\left(e^{V_1/V_T}-1\right)+I_s\left(e^{V_2/V_T}-1\right)=0$$

$$I_s\left(e^{V_1/V_T}+e^{V_2/V_T}\right)=2I_s$$

$$e^{V_1/V_T}+e^{-V_2/V_T}=2$$

As the two diodes are connected back-to-back, current through the forward-biased diode is limited to I_s.

i.e.,
$$I_s\left(e^{V_1/V_T}-1\right)=I_s$$

$$I_s e^{V_1/V_T}=2I_s$$

$$V_1 = V_T \ln 2$$
$$= 0.026 \ln 2$$
$$= 0.018 \text{ V}$$
$$V_2 = 5 - 0.018$$
$$= 4.982 \text{ V.}$$

Problem 3.4

Assuming that a GaAs junction doped to equal concentrations on n and p side, would you expect electron or hole injection to dominate in forward-bias? Explain.

Solution

For GaAs $\mu_n \cong 8500$, $\mu_p \cong 400$ $\left[\frac{D_p}{D_n}=\frac{\mu_p}{\mu_n}=0.047\right]$

Therefore, under forward-bias the current due to electrons injected from n to p dominates if n and p regions are equally doped.

Problem 3.5

At 300 K for a Si abrupt p-n junction, the resistivity is 5 Ω cm on the p-side and 1Ω cm on the n-side. Assume $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_n = 1350$ cm^2 /Vs, $\mu_p = 480$ cm^2/Vs on both the sides. Determine the built-in voltage.

Solution

On p-side

$$\rho = 5\ \Omega\ \text{cm}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{qp_{p_o}\mu_p} = 5\ \Omega\ \text{cm}$$

$$\therefore \quad p_{p_o} = \frac{1}{q\rho\mu_p} = \frac{1}{1.6\times10^{-19}\times5\times480}$$

$$= 2.6 \times 10^{15}\ \text{cm}^{-3}$$

On n-side

$$n_{n_o} = \frac{1}{q\mu_n\rho} = \frac{1}{1.6\times10^{-19}\times1350\times1}$$

$$= 4.63 \times 10^{15}\ \text{cm}^{-3}$$

Built-in voltage $V_o = \frac{kT}{q}\ln\left(\frac{n_{n_o}p_{p_o}}{n_i^2}\right)$

$$= 0.26\ln\left(\frac{4.63\times10^{15}\times2.6\times10^{15}}{(1.5\times10^{10})^2}\right)$$

$$= 0.642\ \text{V}.$$

Problem 3.6

For a p+n Si abrupt junction diode, determine the effect of increase of N_D on the following. Briefly explain each. (a) E_m (b) C_j (c) C_s (d) τ_p in n-side (e) τ_n in p-side (f) depletion layer width.

Solution

a. With increase in N_D, E_m increases because the width of depletion layer decreases and V_o increases with increase in N_D.

$$V_o = \frac{1}{2}E_{m_o}W \qquad \therefore \qquad E_{m_o} = \frac{V_o}{2W}$$

b. $C_j = \frac{\epsilon A}{W}$ increases; with increase in N_D.

c. C_s decreases; With increase in N_D diffusion length and lifetime decrease which reduces the storage capacitance.

d. τ_p decreases; with increases in N_D recombination rate increases reducing lifetime.

e. τ_n on p-side remains unchanged.

f. Depletion layer width decreases with increase in doping.

Problem 3.7

For an abrupt p-n junction, show that

$$\frac{dV_o}{dT} = \frac{V_o - V_g}{T} + \frac{dV_g}{dT} - \frac{3k}{q}$$

where V_g is the voltage corresponding to the band gap of the semiconductor. For a Si p-n junction with $N_D = 10^{15}$ cm^{-3} and $N_A = 10^{18}$ cm^{-3}, calculate $\frac{dV_o}{dT}$ at 300 K and V_o at 303 K. Assume

Solution

$$\frac{dV_g}{dT} = -2.75 \times 10^{-4}\ \text{V/}^{\circ}\text{K}$$

$$E_g = 1.12\ \text{eV} \quad \text{at} \quad 300\ \text{K}.$$

$$V_g = \frac{E_g}{q}$$

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = \frac{kT}{q}\left[\ln N_A N_D - \ln\left(K_1^2 T^3 e^{-E_g/kT}\right)\right]$$

$$= \frac{kT}{q}\ln(N_A N_D) - \frac{kT}{q}\ln K_1^2 - \frac{3kT}{q}\ln T + \frac{E_g}{q}$$

$$V_o - \frac{E_g}{q} = \frac{kT}{q}\ln(N_A N_D) - \frac{kT}{q}\ln K_1^2 - \frac{3kT}{q}\ln T$$

i.e., $$V_o - V_g = \frac{kT}{q}\ln(N_A N_D) - \frac{kT}{q}\ln K_1^2 - \frac{3kT}{q}\ln T \qquad \text{(A)}$$

$$\frac{dV_o}{dT} = \frac{k}{q}\ln(N_A N_D) - \frac{k}{q}\ln K_1^2 - \frac{3k}{q}\ln T - \frac{3k}{q} + \frac{dV_g}{dT}$$

$$= \frac{V_o - V_g}{T} - \frac{3k}{q} + \frac{dV_g}{dT} \text{ (using equation (A))}$$

At T $= 300$ K,

$$V_o = 0.026\ln\left(\frac{(10^{18}\times 10^{15}}{(1.5\times 10^{10})^2}\right) = 0.757\ \text{V}$$

$$V_g = 1.12\ \text{V}$$

$$\frac{dV_o}{dT} = \frac{0.757 - 1.12}{300} - \frac{3\times 1.38\times 10^{-23}}{1.6\times 10^{-19}} - 2.75\times 10^{-4}$$

$$= -1.743 \times 10^{-3}\ \text{V/K}$$

Therefore at T = 303 K,

$$V_o(303) = V_o(300) + \frac{dV_o}{dT}\times 3$$

$$= 0.757 - 1.743 \times 10^{-3} \times 3$$

$$= 0.751\ \text{V}.$$

Problem 3.8

A Ge p-n junction has a donor concentration of 10^{-8} N on the n-side and an acceptor concentration of 2×10^{-6} N on the p-side where N denotes the number of Ge atoms/cm^3. Calculate the built in voltage at 290 K and determine the temperature at which V_o decreases by 4 percent. Assume

$E_g = 0.67$ eV and $\frac{dE_g}{dT} = 0$, n_i (290 K) = 1.34×10^{13} cm^{-3}

Solution

$$\text{Number of Ge atoms / cm}^3 = \frac{\text{Avogadro number} \times \text{density}}{\text{atomic weight}}$$

$$N = \frac{6.023 \times 10^{23} \times 5.33}{72.59} = 4.42 \times 10^{22} \text{ cm}^{-3}$$

$N_D = 4.42 \times 10^{14}$ cm^{-3}

$N_A = 2 \times 10^{-6}$ N = 8.84×10^{16} cm^{-3}

$n_i = 1.34 \times 10^{13}$ cm^{-3}

$$\frac{kT}{q} \text{ at 290 K} = 0.026 \times \left(\frac{290}{300}\right) = 0.02513$$

$$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$= 0.02513 \ln \left(\frac{4.42 \times 10^{14} \times 8.84 \times 10^{16}}{(1.34 \times 10^{13})^2}\right)$$

$= 0.308$ V

$$\frac{dV_o}{dT} = \frac{V_o - V_g}{T} - \frac{3k}{q} + \frac{dV_g}{dT}$$

$$V_g = \frac{E_g}{q} = \frac{0.67eV}{q} = 0.67 \text{ V}$$

$$\frac{dV_g}{dT} = 0 \left(\because \frac{dE_g}{dT} = 0\right); \frac{k}{q} = 8.63 \times 10^{-5}$$

$$\therefore \quad \frac{dV_o}{dT} = \frac{0.308 - 0.67}{290} - 3 \times 8.63 \times 10^{-5} + 0$$

$= -1.507 \times 10^{-3}$ V/K

ΔV_o = 4% of V_o

$$= \frac{4}{100} \times 0.308 = 1.232 \times 10^{-2}$$

$$\text{Change in temperature } \Delta T = \frac{-\Delta V_o}{\frac{dV_o}{dT}}$$

$$= \frac{-1.232 \times 10^{-2}}{1.507 \times 10^{-3}} = 8.2 \text{ K}$$

Therefore, temperature at which V_o decreases by 4%

= 290 + 8.2 = 298.2 K.

Problem 3.9

Two ideal p-n junction diodes are connected in series across a 1 V battery such that both of them are forward-biased. One diode (D_1) has $I_{s_1} = 10^{-5}$ A and the other (D_2) has $I_{s_2} = 10^{-8}$ A. Calculate the current through the circuit and voltage drop across each diode.

Solution

As the diodes are connected in series currents through them are equal. As the diodes are ideal,

$$I_{s_1} e^{V_1/V_T} = I_{s_2} e^{V_2/V_T}$$

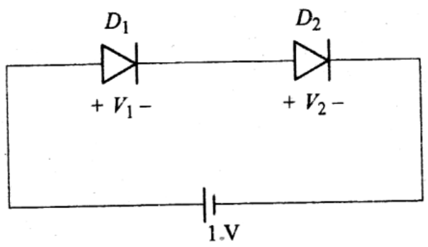


Fig. Sp.3.9

$$\frac{I_{s_1}}{I_{s_2}} = e^{V_2 - V_1/V_T}$$

$$(V_2 - V_1) = \ln\left(\frac{I_{s_1}}{I_{s_2}}\right) \times V_T$$

$$= \ln\left(\frac{10^{-5}}{10^{-8}}\right) \times 0.026 = 0.1796 \text{ V} \qquad \text{(B)}$$

Also, $V_2 + V_1 = 1\text{V}$

By equations (A) and (B)

$$V_2 = \frac{1 + 0.1796}{2} = 0.5898V$$

$$V_1 = 1 - 0.5898 = 0.4102 \text{ V}$$

Current through the circuit = $I_{s_1} e^{V_1/V_T}$ = 71.09 A.

Problem 3.10

The ratio of current due to holes injected from p-side to n-side at $x_n = 0$ to the total diode current is called the injection efficiency. Calculate the injection efficiency of a p-n diode as a function of N_A/N_D.

Solution

$$I_p\,(x_n = 0) = qA\, \frac{D_p}{L_p} p_{n_o} \left(e^{qV_a/kT} - 1\right)$$

$$I = qA\left(\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right)\left(e^{qV_a/kT} - 1\right)$$

$$\text{Injection efficiency } \gamma = \frac{I_p(x_n = 0)}{I} = \frac{1}{\frac{I}{I_p}}$$

$$= \frac{1}{1+\frac{D_n}{D_p}\cdot\frac{L_p}{L_n}\cdot\frac{n_{p_o}}{p_{n_o}}}$$

$$= \frac{1}{1+\frac{D_n}{D_p}\cdot\frac{L_p}{L_n}\cdot\frac{N_D}{N_A}}$$

$$= \frac{1}{1+\frac{\mu_n}{\mu_p}\frac{L_p N_D}{L_n N_A}}$$

Problem 3.11

For a forward-biased p+n abrupt silicon diode $W_N >> L_p$, $L_p = 1$ μm, the ratio of hole current to electron current at $x_n = 0$ is 100 in the steady-state. Determine $\frac{I_p}{I_n}$ at $x_n = 1$μm.

Solution

At $x_n = 0$, $\frac{I_p(0)}{I_n(0)} = 100$

$I_n(0) = 0.01\, I_p(0)$

Total diode current $I = I_p(0) + I_n(0)$

$= 1.01\, I_p(0)$

At $x_n = 1$ μm,

$I_p(x_n) = I_p(0)\, e^{-x_n/L_p}$

$= I_p(0)\, e^{-1\mu m/+1\mu m} = \frac{I_p(0)}{e}$

$I_n(x_n) = I - I_p(x_n) = I - \frac{I_p(0)}{e}$

$$\frac{I_p}{I_n}\text{ (at } x_n = 1\ \mu\text{m)} = \frac{\frac{I_p(0)}{e}}{1-\frac{I_p(0)}{e}}$$

$$= \frac{\frac{I_p(0)}{e}}{1.01 I_p(0)-\frac{I_p(0)}{e}} = \frac{1}{1.01e-1}$$

$= 0.5729.$

Problem 3.12

A germanium p-n junction diode has $N_D = 10^{16}$ cm^{-3} on the n-side and $N_A = 10^{19}$ cm^{-3} on the p-side. Calculate the forward voltage at which injected hole concentration at the edge of the depletion region on the n-side become equal to the majority carrier concentration. Assume T = 300 K, $D_p = 40$ cm^2/s, $\tau_p = 1$ ps. Calculate the current density at this voltage and compare with thermal equilibrium diffusion current density ($n_i = 2.5 \times 10^{13}$ cm^{-3}).

Solution

$n_{n_o} = N_D = 10^{16}$ cm^{-3}, $\quad p_{p_o} = N_A = 10^{19}$ cm^{-3}

$$p_{n_o} = \frac{n_i^2}{N_D} = \frac{\left(2.5\times10^{13}\right)^2}{10^{16}} = 6.25 \times 10^{10}\ \text{cm}^{-3}$$

$$p_n(x_n=0) = e^{qV_a/kT}$$

The injected hole concentration ($\Delta p_n = p_{no}\left(e^{qV_a/kT}-1\right)$ equals the majority carrier concentration when

$$p_n\left(e^{qV_a/kT}-1\right) = n_{n_o}$$

$$e^{qV_a/kT} = \frac{n_{n_o}}{p_{n_o}}+1$$

$$V_0 = \frac{kT}{q}\ln\left[\frac{n_{n_o}}{p_{n_o}}+1\right]$$

$$= 0.026\ln\left[\frac{10^{16}}{6.25\times10^{10}}+1\right] = 0.311\text{V}$$

At $V_a = V_F = 0.311$ V,

$$L_p = \sqrt{D_p\tau_p} = \sqrt{40\times1\times10^{-6}} = 6.32 \times 10^{-3}\ \text{cm}$$

This is a p+n jnuction ($N_A >> N_D$)

$$\therefore \quad J_F = q\frac{D_p}{L_p}p_{n_o}\left(e^{V_F/V_T}-1\right)$$

$$= 1.6\times10^{-19}\times\frac{40}{6.32\times10^{-3}}(e^{0.311/0.026}-1)$$

$$= 1.586\times10^{-10}\ \text{A/cm}^2$$

To find thermal equilibrium diffusion current density

$$V = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

$$= 0.026\ln\left(\frac{10^{19}\times10^{16}}{(2.5\times10^{13})^2}\right) = 0.491\ \text{V}$$

$$W_o = \sqrt{\frac{2\in V_o}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2\times8.854\times10^{-14}\times16\times0.491}{1.6\times10^{-19}}\times\left(\frac{1}{10^{19}}+\frac{1}{10^{16}}\right)}$$

$$= 2.95\times10^{-5}\ \text{cm}$$

$$J_{diff} = -qD_p\frac{dp(x)}{dx} = -qD_p.\frac{p_{n_o}-p_{p_o}}{W_o}$$

$$= -1.6\times10^{-19}\times40\times\frac{\left(6.25\times10^{10}\text{-}10^{19}\right)}{2.95\times10^{-5}}$$

$$= 2.169\times10^{6}\ \text{A/cm}^2$$

Note : Even at high-level injection; the forward current is very small compared to thermal equilibrium diffusion current.

Problem 3.14

A Si abrupt p-n junction has $N_A = 3 \times 10^{18}$ cm^{-3} on the p-side and an area of 1.6×10^{-3} cm^2. The junction capacitance is 18 pF at reverse-bias of 3.2 V and 12 pF at 8.2 V. Calculate the built in voltage and donor concentration on the n-side.

Solution

$$C_j = \frac{C_{jo}}{\sqrt{1-\frac{V_a}{V_o}}}$$

$$18\text{pF} = \frac{C_{jo}}{\sqrt{1+\frac{3.2}{V_o}}}$$

$$12\text{ pF} = \frac{C_{jo}}{\sqrt{1+\frac{8.2}{V_o}}}$$

$$\frac{18}{12} = \frac{\sqrt{1+\frac{8.2}{V_o}}}{\sqrt{1+\frac{3.2}{V_o}}} \quad i.e., \left(\frac{3}{2}\right)^2 = \frac{1+\frac{8.2}{V_o}}{1+\frac{3.2}{V_o}}$$

$$Vo = 0.8\text{ V}$$

$$N_A = 3 \times 10^{18}\text{ cm}^{-3}$$

$$C_j = \frac{\grave{o}A}{\sqrt{\frac{2\grave{o}(V_o+V_R)}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

$$18\times10^{-12} = \frac{8.854\times10^{-14}\times11.8\times1.6\times10^{-3}}{\sqrt{\frac{2\times8.854\times10^{-14}\times11.8\times(0.8+3.2)}{1.6\times10^{-19}}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

$$\frac{1}{N_A}+\frac{1}{N_D} = 1.65 \times 10\text{-}16$$

$$\therefore N_D = 6.07 \times 10^{15}\text{ cm}^{-3}$$

Problem 3.15

A silicon p+n junction diode has $A = 10^{-2}$ cm^2. The junction capacitance vary with the applied bias as

$$\frac{1}{C_j^2} = 7.5 \times 10^{18}\,(7 - 10\,V_a)$$

Determine (a) V_o (b) doping densities N_A and N_D.

Solution

The built-in potential is obtained by setting $\frac{1}{C_j^2}$ to zero (see Section 3.6.1 and Fig. 3.17).

$$\frac{1}{C_j^2} = 7.5 \times 10^{18} (7 - 10\, V_o) = 0$$

i.e., $(7 - 10\, V_o) = 0 \therefore V_o = \frac{7}{10} = 0.7V$

$$\frac{1}{C_j^2} = 7.5 \times l0^{18} (7 - 10\, V_a) \text{ (given)}$$

$$\frac{1}{C_j^2} = \frac{2(V_o + V_a)}{A^2 q\grave{o}}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) \quad \text{from equation (3.69)}$$

Differentiate the above equations $\frac{d}{dV_a}\left(\frac{1}{C_j^2}\right)$ and equate.

$$-7.5 \times 10^{18} \times 10 = \frac{-2}{A^2 q\grave{o}} \times \left(\frac{1}{N_A} + \frac{1}{N_D}\right)$$

$$\frac{1}{N_A} + \frac{1}{N_D} = \frac{7.5 \times 10^{18} \times 10 \times 10^{-4} \times 1.6 \times 10^{-19} \times 8.854 \times 10^{-14} \times 11.8}{2}$$

$$= 6.26 \times 10^{-16}$$

Also, $$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$N_A N_D = n_i^2\, e^{qV_a/kT}$$

$$= (1.5 \times 10^{10})^2\, e^{0\text{-}7/0\text{-}026}$$

$$= 1.108 \times 10^{32}$$

$$N_D = \frac{1.108 \times 10^{32}}{N_A} \; 1.108 \text{ x } 1032$$

$$\frac{1}{N_A} + \frac{1}{N_D} = 6.26 \times 10^{-16}$$

$$\therefore \frac{N_A}{1.108 \times 10^{32}} + \frac{1}{N_A} = 6.26 \times 10^{-16}\; 0.902 \times 10^{-33} N_A^2 - 6.26 \times 10^{-16} N_A + 1 = 0$$

$$\therefore N_A = 6.924 \therefore 10^{17} \text{ cm}^{-3}$$

$$N_D = \frac{1.108 \times 10^{32}}{6.924 \times 10^{17}} = 1.6 \times 10^{14} \text{ cm}^{-3}.$$

Problem 3.16

1300 K, for a diode current of 1 mA a Ge diode requires a forward-bias of 0.1435 V whereas a silicon diode requires a forward-bias of 0.718 V. Determine the approximate ratio of reverse saturation currents.

Solution

For silicon diodes, recombination current (η = 2) dominates at room temperature whereas for Ge diodes at room temperature the diffusion currents dominates (η= 1) (see Section 3.5).

Let I_{o1}, be reverse saturation current of Ge diode and I_{o2} be the reverse saturation current in Si diode.

1mA = $I_{o1}\,(e^{0.1435/0.026} - 1)$ for Ge diode

1mA = $I_{o2}\,(e^{0.718/2 \times 0.026} - 1)$ for Si diode

$$\therefore \quad \frac{I_{01}}{I_{02}} = \frac{\left(e^{0.718/2 \times 0.026} - 1\right)}{\left(e^{0.1435/0.026} - 1\right)} = 3.99 \times 10^3.$$

Problem 3.17

A p+n Si diode has $N_A = 10^{17}$ cm^{-3} and $N^D = 10^{15}$ cm^{-3}. A = 10^{-3} cm^2. Let the lifetime in the n and p regions be 1 ps. Determine ideal and total (diffusion and recombination) diode currents for (a) V_a = 0.1 V (b) V_a = 0.5 V; D_p = 10 cm^2/s, D_n = 36 cm^2/s.

Solution

$$V_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.026 \ln \frac{10^{17} \times 10^{15}}{\left(1.5 \times 10^{10}\right)^2} = 0.697 \text{ V}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{10 \times 1 \times 10^{-6}} = 3.16 \times 10^{-3} \text{ cm}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{36 \times 1 \times 10^{-6}} = 6 \times 10^{-3} \text{ cm}$$

$$p_{no} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

$$n_{no} = \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

$$I_s = qA\left(\frac{D_p}{L_p} p_{n_o} + \frac{D_n}{L_n} n_{p_o}\right)$$

$$I_s = 1.6 \times 10^{-19} \times 10^{-3}\left(\frac{10}{3.16 \times 10^{-3}} \times 2.25 \times 10^5 + \frac{36}{6 \times 10^{-3}} \times 2.25 \times 10^3\right)$$

$$= 1.16 \times 10^{-13} \text{ A}$$

a. V_a = 0.1 V

$$W = \sqrt{\frac{2\in}{q}(V_o - V_a)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2 \times 8.854 \times 10^{-14} \times 11.8}{1.6 \times 10^{-19}}(0.697 - 01)\left(\frac{1}{10^{17}} + \frac{1}{10^{15}}\right)}$$

$$= 8.873 \times 10^{-5} \text{ cm}$$

$$I_{R_o} = \frac{qAn_iW}{2\tau_o}$$

$$= \frac{1.6 \times 10^{-19} \times 10^{-3} \times 1.5 \times 10^{10} \times 8.873 \times 10^{-5}}{2 \times 1 \times 10^{-6}}$$

$$= 1.06 \times 10^{-10} \text{ A}$$

Ideal diode current $= I_s \left(e^{V_a/V_T} - 1\right)$

$= 1.16 \times 10^{-13} (e^{0.1/0.026} - 1)$

$= 5.31 \times 10^{-12}$ A

Total diode current I $= I_s \left(e^{V_a/V_T} - 1\right) + I_{R_o} \left(e^{V_a/2V_T} - 1\right)$

$= 5.31 \times 10^{-12} + 1.06 \times 10^{-10} (e^{0.1/2 \times 0.026} - 1)$

$= 6.245 \times 10^{-10}$ A

b. $V_a = 0.5V$

$$W = \sqrt{\frac{2 \times 8.854 \times 10^{-14} \times 11.8}{1.6 \times 10^{-19}}(0.697 - 0.5)\left(\frac{1}{10^{17}} + \frac{1}{10^{15}}\right)}$$

$= 5.097 \times 10^{-5}$ cm

$$= \frac{1.6 \times 10^{-19} \times 10^{-3} \times 1.5 \times 10^{10} \times 5.097 \times 10^{-5}}{2 \times 1 \times 10^{-6}}$$

$= 6.116 \times 10^{-11}$ A

Ideal diode current $= 1.16 \times 10^{-13} (e^{0.5/2 \times 0.026} - 1)$

$= 9.59 \times 10^{-6}$ A

Total diode current $= 9.59 \times 10^{-6} + 6.116 \times 10^{-11} (e^{0.5/2 \times 0.026} - 1)$

$= 1.05 \times 10^{-5}$ A

The above results show that for Si diode at low forward voltages recombination current dominates, at room temperature.

Problem 3.18

A p-n junction diode has a reverse saturation current (I_s) of 1 µA. The resistivity of p-region is 0.01 Ω cm and n-region is 0.1 Ω, cm. Each is 1 mm long and has area A = 0.5 mm^2. Include the IR drops in the p and n-regions and find the forward voltage V_a for (a) I = 1 mA (b) I = 10 mA. Assume $\mu_n = 1350$ cm^2/Vs, $\mu_p = 480$ cm^2/Vs, $n_i = 1.5 \times 10^{10}$cm^{-3}.

Solution

$$\rho_p = \frac{1}{q p_{po} \mu_p}$$

$$\therefore \quad p_{po} = \frac{1}{\rho_p q \mu_p} = \frac{1}{0.01 \times 1.6 \times 10^{-19} \times 480}$$

$= 1.3 \times 10^{18}$ cm^{-3}

$$n_{no} = \frac{1}{\rho_n q \mu_n} = \frac{1}{0.1 \times 1.6 \times 10^{-19} \times 1350}$$

$= 4.63 \times 10^{16}$ cm^{-3}

Resistance of p-region $= \rho_p \dfrac{L}{A} = \dfrac{0.01 \times 0.1}{0.5 \times 10^{-2}} = 0.2\Omega$

Total resistance of neutral regions $R_s = 2.2\ \Omega$

$$I = I_s\, e^{(V_a - IR_s)/VT}$$

a. $I = 1$ mA $= 10^{-3}$ A

$I_s = 1\mu A = 10^{-6}$ A

$10^{-3} = 10^{-6}\ e^{(V_a - IR_s)/VT}$

$V_a - IR_s = V_T \ln 10^3$

$V_a = 0.026 \ln 10^3 \times 2.2 = 0.182$ V

b. $I = 10mA = 10^{-2}$ A

$I_s = 10^{-6}$ A

$10^{-2} = 10^{-6}\ e^{(V_a - IR_s)/VT}$

$V_a - IR_s = V_T \ln 10^4$

$V_a = 0.026 \ln 10^4 + 10^{-2} \times 2.2$

$V_a = 0.261V.$

Problem 3.19

Draw the energy band diagram of a p-n junction under forward and reverse-bias conditions. Show the quasi-Fermi levels.

Solution

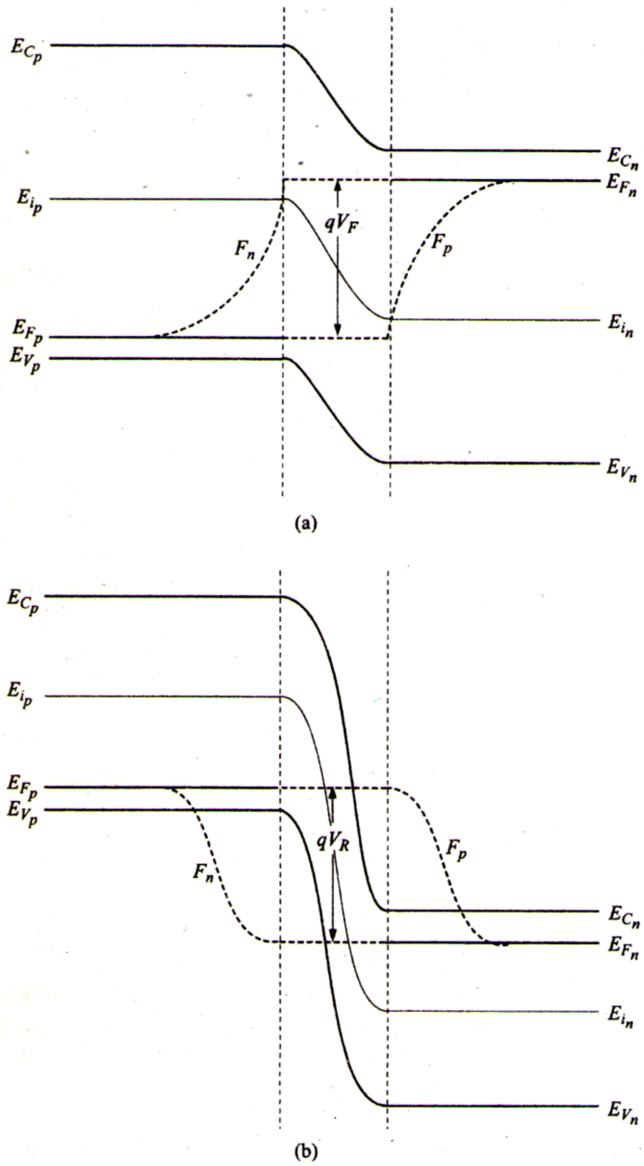


Fig. Sp.3.19 EB diagram of p-n junction under (a) Forward bias (b) Reverse bias

Note: The position of quasi Fermi levels in Fig. Sp-3.19 indicates the increase of minority carriers under forward-bias and decrease of minority carriers under reverse-bias from thermal equilibrium values.

Problem 3.20

Show that the electron hole product is constant and equal to n_i^2 throughout the space charge region of a p-n junction in thermal equilibrium. How will this product change when the junction is biased? Find out its value for a Ge p-n junction at (a) $V_a = 0.2$ V (b) $V_a = -1$ V. Take T = 300 K (see figure of Problem 3.19).

Solution

$$p(x) = n_i\, e^{[E_i(x) - F_p(x)]/kT}$$

$$n(x) = n_i\, e^{[F_n(x) - E_i(x)]/kT}$$

$$p(x)\, n(x) = n_i^2\, e^{[F_n(x) - F_p(x)]/kT}$$

$$= n_i^2 \quad (\because\ F_n = F_p \text{ at equilibrium})$$

When the junction is biased by V_a

$$F_n(x) - F_p(x) = qV_a$$

$$\therefore\ p(x)n(x) = n_i^2\, e^{qV_a/kT}$$

For Ge at 300 K,

$$n_i = 2.5 \times 10^{13}\ cm^{-3}$$

a. At $V_a = 0.2$ V

$$p(x)n(x) = (2.5 \times 10^{13})^2\, e^{0.2/0.026}$$

$$= 1.37 \times 10^{30}$$

b. At $V_a = -1$ V

$$p(x)n(x) = (2.5 \times 10^{13})^2\ cm^{-3} \times e^{-1/0/026}$$

$$= 1.24 \times 10^{10}.$$

Problem 3.21

It is required to have an avalanche breakdown voltage ($V_{B\tau}$) of 75 V for an abrupt Si p+n junction with $N_D = 10^{15}$ cm^{-3}. What is the minimum thickness of the n-region (between the metallurgical junction and the ohmic contact) required to ensure avalanche breakdown rather than punch through.

Solution

$$E_{crit} = \frac{qN_D W}{\in}$$

where W is depletion layer width at breakdown.

$$V_{Br} = \frac{1}{2} E_{crit} \times W \quad \therefore \quad E_{crit} = \frac{2VB_r}{W}$$

$$\frac{qN_D W}{\grave{o}} = \frac{2V_{Br}}{W}$$

$$W^2 = \frac{2V_{Br}\grave{o}}{qN_D}$$

$$W = \sqrt{\frac{2 \times 75 \times 8.854 \times 10^{-14} \times 11.8}{1.6 \times 10^{-19} \times 10^{15}}}$$

$$= 9.896 \times 10^{-4} cm = 9.89 \mu m$$

Minimum thickness of n-region to ensure avalanche breakdown is 9.89 μm.

Problem 3.22

A symmetrical abrupt Ge p-n junction has an impurity concentration of 10^{16} atoms/cm^3 on both sides. If the critical field is 3×10^5 V/cm, (a) determine the breakdown voltage (b) what will be the breakdown voltage if the impurity concentration on n-side remains unchanged and that on the p-side changes to 10^{19} cm^{-3}, (c) what will be the breakdown voltage if doping on n-side is changed to 10^{15} cm^{-3} keeping the doping on p-side as 10^{16} cm^{-3}.

Solution

a. By equation (3.88)

$$V_{Br} = E_{crit}^2 \frac{\grave{o}}{2q} \frac{N_A + N_D}{N_A N_D}$$

$$= (3\times10^5)^2 \times \frac{8.854\times10^{-14}\times16}{2\times1.6\times10^{-19}} \times \frac{10^{16}+10^{16}}{10^{16}\times10^{16}}$$

= 79.686 V

b. $$V_{Br} = (3\times10^5)^2 \times \frac{8.854\times10^{-14}\times16}{2\times1.6\times10^{-19}} \times \frac{10^{19}+10^{16}}{10^{19}\times10^{16}}$$

= 39.87 V

c. $$V_{Br} = (3\times10^5)^2 \times \frac{8.854\times10^{-14}\times16}{2\times1.6\times10^{-19}} \times \frac{10^{16}+10^{15}}{10^{16}\times10^{15}}$$

= 438.27 V

Note: As the doping concentration increases the breakdown voltage decreases.

Problem 3.23

A Schottky barrier is formed by depositing tungsten on n-type silicon. Determine at 300 K if $N_D = 10^{15}$ cm^{-3}, $q\phi_m$ = 4.9 eV, $q\psi_s$ = 4.15 eV, (a) V_o (b) W_o (c) E_{mo}.

Solution

$$E_{Fs} - E_i = kT\ln\left(\frac{N_D}{n_i}\right)$$

$$= 0.026 \ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = 0.289 \text{ eV}$$

$$E_C - E_{Fs}, \quad = \frac{E_g}{2} - (E_{Fs} - E_i)$$

= 0.55 - 0.289 = 0.261 eV

$q\phi_{sc}$ = $q\psi_s$ + (E_C - E_{Fs})

= 4.15 + 0.261 = 4.411 V

V_o = ϕ_m - ϕ_{sc}

= 4.9 - 4.411 = 0.489 V

$$W_o = \sqrt{\frac{2\grave{o}V_o}{qN_D}} = \sqrt{\frac{2\times8.854\times11.8\times10^{-14}\times0.489}{1.6\times10^{-19}\times10^{15}}}$$

= 8 × 10^{-5} cm

$$\varepsilon_{m_o} = \frac{2V_o}{W_o} = \frac{2\times0.489}{8\times10^{-5}} = 1.22\times10^4 V/cm$$

Problem 3.24

A Schottky barrier is formed from n-type Si having a doping of 10^{16} cm^{-3} and area 10^{-3} m^2. A silicon p-n junction has same area and $N_A = 10^{19}$, $N_D = 10^{16}$, $\tau_p = \tau_n = 1\mu s$.

a. Calculate the Schottky barrier diode current at 0.4 V and 300 K at 300K.
b. Calculate the value of forward-bias to obtain same current for a p-n junction.
c. Compute equilibrium depletion layer capacitance (C_j) and storage capacitance (C_s) at 0.4 V forward-bias for both diodes. Electron affinity of Si is 4.15 eV, metal work function is 4.9 eV, $R^* = 110$ A/K^2 cm^2, $D_{pn} = 12$ cm^2 /s.

Solution

a. $I = AR^*T^2 \, e^{-q\phi_{Bm}/kT}\left[e^{V_a/V_T} - 1\right]$

$q\phi_{Bm} = q\phi_{Bm} - q\psi_s$
$= 4.9 - 4.15 = 0.75$ eV

$I = 10^{-3} \times 110 \times 3002 \times e^{-075/0.026} (e^{0.4/0.26} - l)$
$= 14$mA

b. $L_{pn} = \sqrt{D_{pn}\tau_n} = \sqrt{12\times10^{-6}} = 3.46 \times 10^{-3}$ cm

$$I = qA\left(\frac{D_{pn}}{L_{pn}}.\frac{n_i^2}{N_D}\right)\left(e^{V_a/V_T}-1\right)$$

$$14\times 10^{-3} = 1.6 \times 10^{-19} \times 10^{-3}\left(\frac{12}{3.46\times10^{-3}}\times\frac{1.5\times10^{10}}{10^{16}}\right)\left(e^{V_a/0.026}-1\right)$$

$$= 1.2485 \times 10^{-14}\left(e^{V_a/0.026}-1\right)$$

$$e^{V_a/0.026} = \frac{14\times10^{-3}}{1.2485\times10^{-13}}+1$$

$\therefore\ V_a = 0.026 \ln 1.21\times 10^{12} = 0.721$ V

c. For Schottky diode $q\phi_m = 4.9$ eV ; $q\psi_s = 4.15$ eV.

$$E_F - E_i = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

$$= 0.026 \ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.349 \text{ eV}$$

$$E_C - E_F = \frac{E_g}{2} - (E_F - E_i) = 0.55 - 0.349 = 0.201 \text{ eV}$$

$q\phi_s = q\psi_s + E_C - E_F$
$= 4.15 + 0.201 = 4.351$ eV

$$W_o = \sqrt{\frac{2\in V_o}{qN_D}} = \sqrt{\frac{2\times8.854\times10^{-14}\times11.8\times0.549}{1.6\times10^{-19}\times10^{16}}}$$

$= 2.678 \times 10^{-5}$ cm

$$C_j = \frac{\grave{o}A}{W_o}$$

$$= \frac{8.854\times10^{-14}\times11.8\times10^{-3}}{2.678\times10^{-5}}$$

$= 39 \times 10^{-12}$ F $= 39$ pF

$C_s \quad = 0$

For p-n junction diode

$$V_o = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

$$= 0.026 \ln\left(\frac{10^{19}\times10^{16}}{(1.5\times10^{10})^2}\right) = 0.877 \text{ V}$$

$$= \sqrt{\frac{2\times8.854\times10^{-14}\times11.8\times0.877}{1.6\times10^{-19}}\left(\frac{1}{10^{19}}+\frac{1}{10^{16}}\right)}$$

$$= 3.386\times10^{-5} \text{ cm}$$

$$C_j = \frac{\grave{o}A}{W} = \frac{8.854\times10^{-14}\times11.8\times10^{-3}}{3.386\times10^{-5}}$$

$$= 30.856\times10^{-12} \text{ F} = 30.856 \text{ pF}$$

$$C_s = \frac{I_F\tau_p}{V_T}$$

$$= \frac{14\times10^{-3}\times1\times10^{-6}}{0.026}$$

$$= 5.38\times10^{-7} \text{ F.}$$

Problem 3.25

A p-n junction diode has $N_A = 10^{17}$ cm^{-3} on p-side and $N_D = 10^{15}$ cm^{-3} on n-side. If $\tau_n = \tau_p = 1\mu$s, A = 10^{-3} cm^2, D_p = 10 cm^2/s on n-side, D_n = 36 cm^2/s on p-side and n, = 1.5×10^{10} cm^{-3}.

a. Determine the voltage at which high injection is reached.
b. Under this condition compute the stored charges on the n and p sides of the junction (neutral region).

Solution

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10\times1\times10^{-6}} = 3.16\times10^{-3} \text{ cm}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{36\times1\times10^{-6}} = 6\times10^{-3} \text{ cm}$$

$$p_{no} = \frac{n_i^2}{N_{AD}} = \frac{(1.5\times10^{10})}{10^{15}} = 2.25\times10^5 \text{ cm}^{-3}$$

$$n_{po} = \frac{n_i^2}{N_D} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3 \text{ cm}^{-3}$$

a. At the onset of high injection, the excess carrier concentration on the lightly dope side equals $\frac{1}{10}$ of the majority carrier concentration.

i.e., $\quad \Delta p_n = \dfrac{N_D}{10} = \dfrac{10^{15}}{10} = 10^{14}$

i.e, $p_{no}\left(e^{qV_F/kT}-1\right) = 10^{14}$

$2.25\times10^5\left(e^{qV_F/kT}-1\right) = 10^{14}$

$$e^{qV_F/kT} = \frac{10^{14}}{2.25\times10^{5}} + 1$$

$$\therefore \quad V_F = \frac{kT}{q}\ln\left(\frac{10^{14}}{2.25\times10^{5}} + 1\right) = 0.518 \text{ V}$$

b. On n-side, the charge stored in the minority carrier distribution in the neutral region, assuming long diode is given by

$$|Q_{s_n}| = qA\int_0^\infty \Delta p e^{-x_n/L_p}\,dx_n$$

$$= \left.\frac{qA\Delta p e^{-x_n/L_p}}{-\frac{1}{L_p}}\right|_0^\infty$$

$= qAL_p\Delta p_n$

$= 1.6 \times 10^{-19} \times 10^{-3} \times 3.16 \times 10^{-3} \times 10^{14}$

$= 5.056 \times 10^{-11}$ C.

Similarly on the p-side,

Stored charge $|Q_{s_p}| = qAL_n\Delta n_p$

$\Delta n_p = \left(e^{qV_F/kT} - 1\right)$

$= 2.25 \times 10^{3} (e^{0.518/0.026} - l)$

$= 1.01 \times 10^{12}$

$|Q_{s_p}| = 1.6 \times 10^{-19} \times 10^{-3} \times 6 \times 10^{-3} \times 1.01 \times 10^{12}$

$= 9.696 \times 10^{-13}$ C.

Problem 3.26

A Si p-n junction at 300 K has $N_A = 10^{16}$ and $N_D = 10^{15}$ cm^{-3}, $\tau_n = \tau_p = 1\mu$s, A= 10^{-3} cm^2. At 300 K determine:

a. Junction capacitance at zero bias C_{jo},
b. The junction capacitance at $V_a = -5$ V,
c. The storage capacitance at $V_a = 0.5$ V.

Solution

Given $\mu_p = 480$ cm^2/Vs on n-side $\mu_n = 1300$ cm^2 /Vs on p-side.

$$V_o = \frac{kT}{q}\ln\frac{N_AN_D}{n_i^2}$$

$$= 0.026 \ln \frac{10^{19}\times10^{15}}{(1.5\times10^{10})^2} = 0.637 \text{ V}$$

$$W_o = \sqrt{\frac{2\grave{o}V_o}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$= \sqrt{\frac{2\times 8.854\times 10^{-14}\times 11.8\times 0.637}{1.6\times 10^{-19}}\left(\frac{1}{10^{16}}+\frac{1}{10^{15}}\right)}$$

$= 9.566 \times 10^{-5}$ cm

a. C_{j0} $= \frac{\epsilon_o \epsilon_R A}{W_o} = \frac{8.854\times 10^{-14}\times 11.8\times 10^{-3}}{9.566\times 10^{-5}}$

$= 10.92$ pF

b. Junction capacitance at $V_a = -5$ V

C_j $= \frac{C_{jo}}{\sqrt{1-\frac{V_a}{V_o}}} = \frac{10.92\times 10^{12}}{\sqrt{1-\frac{5}{0.637}}}$

$= 3.67$ pF

c. Storage capacitance $C_s = G_s\tau_p = \frac{I_F \tau_p}{V_T}$

D_p $= \mu_p . \frac{kT}{q} = 480 \times 0.026 = 12.48$ cm^2/s

D_p $= \mu_n . \frac{kT}{q} = 1300 \times 0.026 = 33.8$ cm^2/s

L_p $= \sqrt{D_p \tau_p} = \sqrt{12.48\times 0.1\times 10^{-6}} = 1.117 \times 10^{-3}$

L_n $= \sqrt{D_n \tau_n} = \sqrt{33.8\times 0.1\times 10^{-6}} = 1.838 \times 10^{-3}$ cm

I_s $= qA\left(\frac{D_p}{L_p}.p_{n_o} + \frac{D_n}{L_n}.n_{p_o}\right)$

$$= 16 \times 10^{-19} \times 10^{-3}\left(\frac{12.48}{1.117\times 10^{-3}}.\frac{(1.5\times 10^{10})^2}{10^{15}}\right)$$

$$+ \frac{33.8}{1.838\times 10^{-3}}.\frac{(1.5\times 10^{10})^2}{10^{16}}$$

$= 4.684 \times 10^{-13}$ A

I_F $= I_s\left(e^{qV_F/kT} - 1\right)$

$= 4.684 \times 10^{-13} (e^{0.5/0.026} - 1) = 1.053 \times 10^{-4}$ A

C_s $= \frac{I_F}{V_T}\tau_p$

$= \frac{1.053\times 10^{-4}}{0.026}\times 0.1\times 10^{-6} = 4.05 \times 10^{-10}$

$= 0.405$ nF.

Problem 3.27

An ideal long silicon abrupt p-n junction has $N_A = 10^{16}$ cm^{-3} and $N_D = 10^{14}$ cm^{-3} $\tau_n = \tau_p = 1\mu$s, A = 10^{-3} cm^2. At 300 K, $D_p = 10$ cm^2 /s, $D_n = 24$ cm^2/s. Determine dynamic forward resistance at 0.1 V, 0.5 V and 0.7 V.

Solution

$$I_s = qA\left(\frac{D_p}{L_p}p_{n_o} + \frac{D_n}{L_n}n_{p_o}\right)$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10\times 0.1\times 10^{-6}} = 10^{-3} \text{ cm}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{24\times 0.1\times 10^{-6}} = 1.55 \times 10^{-3} \text{ cm}$$

$$p_{no} = \frac{n_i^2}{N_{AD}} = \frac{(1.5\times 10^{10})}{10^{16}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

$$n_{po} = \frac{n_i^2}{N_A} = \frac{(1.5\times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$I_s = 1.6 \times 10^{19} \times 10^{-3}\left(\frac{10}{10^{-3}}\times 2.25\times 10^6 + \frac{24\times 2.25\times 10^4}{1.55\times 10^{-3}}\right)$$

$$= 3.6 \times 10^{-12} \text{ A}$$

Dynamic forward resistance,

$$r_F = \frac{V_T}{I_F} = \frac{V_T}{I_s\left(e^{V_F/V_T} - 1\right)}$$

When $V_F = 0.1$V,

$$r_F = \frac{0.026}{3.6\times 10^{-12}(e^{0.1/0/026} - 1)} = 157 \times 10^8\ \Omega$$

When $V_F = 0.5$V,

$$r_F = \frac{0.026}{3.6\times 10^{-12}(e^{0.5/0/026} - 1)} = 32.1\ \Omega$$

When $V_F = 0.7$V,

$$r_F = \frac{0.026}{3.6\times 10^{-12}(e^{0.7/0/026} - 1)} = 1.466 \times 10^{-2}\ \Omega.$$

Problem 3.28

Referring to Fig. Sp. 3.28(a) the switch is in position 1 initially and steady-state condition exists from $t = 0$ to $t = t_0$. At $t = t_0$ switch is suddenly thrown to position 2. Sketch the current through 10 kΩ resistor from $t = 0$ to $t = \infty$.

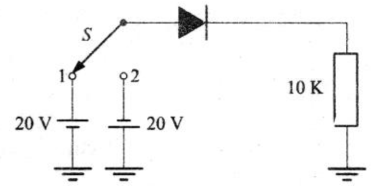


Fig. Sp. 3.28(a)

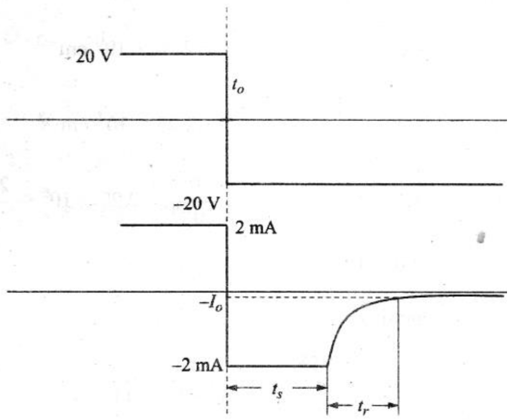


Fig. Sp.3.28(b)

Problem 3.29

Plot the characteristics of Si, Ge and GaAs diodes at room temperature on the same graph and explain.

Solution

The cut in voltage is minimum for Ge diode and maximum for GaAs diode. The reverse saturation current is minimum for GaAs diode and maximum for Ge diode. At room temperature the reverse current increase with increase in reverse bias for GaAs and Si diodes. But it remains almost constant for Ge diode.

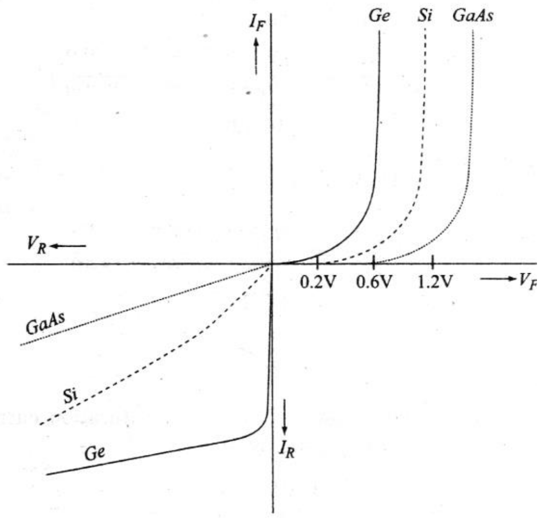


Fig. Sp.3.29

Problem 3.30

a. Assuming that the effect of changes in bandgap and mobility are negligible, show that the revers saturation current of a diode is given by I_s = $KT^3\ e^{-E_g/kT}$ where K is a constant that depends on diode properties.

b. Calculate the factor by which Is increases as the temperature increases from 27° C to 100 ° C for (i) Ge diode (ii) Si diode.

Solution

By equation (3.35 a),

$$I_s = qA\left(\frac{D_p n_i^2}{L_p N_A} + \frac{D_n n_i^2}{L_n N_D}\right)$$

$$\therefore \quad I_s \ \alpha \ n_i^2$$

By equation (1.57), n_i^2 = $K_1T^3 e^{-E_g/kT}$

$$I_s = KT^3 e^{-E_g/kT}$$

(b) (i) For Ge diode:

$$T_1 = 27^oC = 300\ K$$
$$T_2 = 100^oC = 373K$$
$$k = 8.62 \times 10^{-5}\ eV/K$$
$$E_g = 0.67\ eV$$

$$\frac{I_s(T_2)}{I_s(T_1)} = \frac{T_2^3 e^{(-0.67/8.62\times10^{-5}\times373)}}{T_1^3 e^{(-0.67/8.62\times10^{-5}\times300)}}$$

$$= 306.12$$

(ii) For Si diode, Eg = 1.11 eV

$$\frac{I_s(T_2)}{I_s(T_1)} = \frac{T_2^3 e^{(-1.11/8.62\times10^{-5}\times373)}}{T_1^3 e^{(-1.11/8.62\times10^{-5}\times300)}}$$

$$= 8552$$

Problem 3.31

(a) Show that the fractional change in the reverse saturation current of a diode per unit change in temperature is given by

$$\frac{1}{I_s}\cdot\frac{dI_s}{dT} = \frac{3}{T} + \frac{E_g}{kT^2}$$

(b) Determine the fractional change in the reverse saturation current at T = 300 K for a germanium diode and a silicon diode.

Solution

(a) $I_s = kT^3\ e^{(-E_g/kT)}$

$$\frac{dI_s}{dT} = kT^3 e^{-E_g/kT}\ \frac{E_g}{kT^2} + e^{-E_g/kT}\ 3kT^2$$

$$= kT^3 e^{-E_g/kT}\left[\frac{E_g}{kT^2} + \frac{3}{T}\right]$$

$$\frac{1}{I_s}\cdot\frac{dI_s}{dT}=\frac{3}{T}+\frac{E_g}{kT^2}$$

(b) At T = 300 K, For Ge diode:

$$\frac{1}{I_s}\cdot\frac{dI_s}{dT}=\frac{3}{300}+\frac{0.67}{0.026\times 300}$$

$$= 0.095\ K^{-1}$$

For Si diode:

$$\frac{1}{I_s}\cdot\frac{dI_s}{dT}=\frac{3}{300}+\frac{1.11}{0.026\times 300}$$

$$=0.152\ K^{-1}$$

Problem 3.32

A Schottky barrier diode is made by depositing tungsten on n-type silicon. If n_i = 1.5 × $10^1$0 cm^{-3}, ϕ_{Bm} = 0.6.7 V, determine at T = 300 K, and for $N_D = 10^{15}\ cm^{-3}$

(a) Built in voltage
(b) The depletion region width at equilibrium
(c) The maximum electric field intensity at equilibrium.

Solution

(a) $V_o = \phi_{Bm} - \frac{E_C - E_F}{q}$

$$E_C - E_F = \frac{E_g}{2} - \frac{kT}{q}\ln\frac{N_D}{n_i}$$

$$= 0.55 - 0.026 \ln \frac{10^{15}}{1.5\times 10^{10}} = 0.261\ eV$$

$$V_o = 0.67 - 0.261 = 0.409V$$

(b) $W_o = \sqrt{\frac{2\grave{o}V_o}{qN_D}}$

$$= \sqrt{\frac{2\times 8.854\times 10^{-14}\times 11.8\times 0.409}{1.6\times 10^{-19}\times 1\times 10^{15}}} = 7.3 \times 10^{-5}\ cm$$

(c) $E_m = \frac{-q}{\grave{o}} N_D W_0$

$$= \frac{1.6\times 10^{-19}}{8.854\times 10^{-14}\times 11.8}\times 10^{-15} \times 7.3 \times 10^{-5}$$

$$= 1.11 \times 10^4\ V/cm$$

Problem 3.33

A Schottdy barrier diode formed on n-silicon is operating at 300 K. If the electron affinity of silicon is 4.15 V, the metal work function is 4.9 V and $N_D = 2 \times 10^{15}\ cm^{-3}$. Determine: (a) the built-in voltage (b) the barrier height and (c) the width of the depletion layer at equilibrium.

Solution

Assume

$n_i = 1.5 \times 10^{10}\ cm^{-3}$

$\phi_m = 4.9V$

$\psi = 4.15\ V$

$$E_C - E_F = kT \ln\left(\frac{N_D}{n_i}\right)$$

$$= 0.026 \ln\left(\frac{2\times10^{15}}{1.5\times10^{10}}\right) = 0.30eV$$

$E_C - E_F = 0.55 - 0.30 = 0.25\ eV$

$\phi_{sc} = 4.15 + 0.25 = 4.4V$

$V_o = \phi_m - \phi_{sc} = 4.9 - 4.4 = 0.5V$

$\phi_{Bm} = \phi_m - \psi_s$

$= 4.9 - 4.15 = 0.75V$

$$W_o = \sqrt{\frac{2\grave{o}V_o}{qN_D}}$$

$$= \sqrt{\frac{2\times8.854\times10^{-14}\times11.8\times0.5}{1.6\times10^{-19}\times2\times10^{15}}}$$

$= 5.71 \times 10^{-5}$ cm

$= 0.57\ \mu m$

Problem 3.34

A metal with a work function of 4.3 V is deposited on n-type silicon. Determine the doping density, which results in absence of a space-charge region, at equilibrium.

Solution

Assume T = 300 K

$n_i = 1.5 \times 10^{10}\ cm^{-3}$

ψ_s for Si = 4.05 eV

In the absence of space-charge region,

$V_o = 0$

$\phi_m = \phi_{sc} = 4.3V$

$$E_F - E_i = \frac{E_g}{2} - (E_C - E_F) = \frac{E_g}{2} - (\phi_{sc} - \psi_s)$$

i.e., $E_F - E_i = 0.55 - (4.3 - 4.05) = 0.55 - 0.25 = 0.3$ eV

$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$$

$N_D = n_i\ e^{(E_F - E_i)/kT}$

$= 1.5 \times 10^{10}\ e^{(0.3/0.026)} = 1.5\times10^{15} cm^{-3}$

▪ Points to Remember

- A diode with constant doping on the p and n sides is called abrupt or step graded diode.
- The region of immobile charges in a p-n junction is called depletion region or space charge region.
- The potential appearing across the depletion layer of a p-n junction under thermal equilibrium is called built-in potential. It is also known as contact potential and diffusion potential.
- Depletion approximation: Depletion layer is completely depleted of mobile charges and region outside the depletion region is neutral.
- Built-in potential is related to the energies as

$$qV_o = E_{C_p} - E_{C_n} = E_{i_p} - E_{i_n} = E_{V_p} - E_{V_n}$$

In a p-n junction

- $N_A X_p = N_D X_n$ i.e., product of doping and depletion layer width is equal on both sides.

$$W = X_n + X_p$$

$$X_p = W.\frac{N_D}{N_A + N_D}$$

$$X_n = W.\frac{N_A}{N_A + N_D}$$

$$V_o = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}$$

$$W = \sqrt{\frac{2\varepsilon(V_o - V_a)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

- Maximum electric field in a p-n junction

$$E_m = \frac{-q}{\varepsilon} N_A X_p = \frac{-q}{\varepsilon} N_D X_n$$

- Potential drop is the area under electric filed

$$V = \frac{1}{2} E_m W$$

- Forward-bias: Applying positive potential to p-side with respect to that on n-side.
- Reverse-bias: Applying negative potential to p-side with respect to that on n-side.
- Charge density on p-side = $-qN_A$
- Charge density on n-side = $+qN_D$
- Charge on p-side of depletion layer, $Q_D = -qAN_A X_p$
- Reverse saturation current in a p-n junction is the current flowing through a reverse biased diode. In an ideal diode it remains constant for a given temperature. It is the current due to thermally generated minority carriers.
- The ideal diode equation $I = I_s \left(e^{V_a/V_T} - 1\right)$

$$I_s = qA\left(\frac{D_p}{L_p}p_{n_o} + \frac{D_n}{L_n}n_{p_o}\right)$$

- Under forward-bias, diode current increases exponentially with increse in applied bias.
- Under reverse-bias, current remains constant at I_s.
- General form of diode equation with neutral region widths W_P and W_N is given as

$$I = qA\left[\frac{D_p}{L_p}p_{n_o}\coth\left(\frac{W_N}{L_p}\right) + \frac{D_n}{L_n}n_{p_o}\coth\left(\frac{W_P}{L_n}\right)\right]\left(e^{qV_a/kT} - 1\right)$$

- For short-diode $W_N << W_P << L_n$

$$I = qA\left[\frac{D_p}{W_N}p_{n_o} + \frac{D_n}{W_P}n_{p_o}\right]\left(e^{qV_a/kT} - 1\right)$$

- Real diode is a diode with non ideal behaviour like generation / recombination in the depletion layer, high level injection etc.,
- Under reverse-bias

$$I_{gen} = \frac{-qAn_iW}{2\tau_0}$$

- Current due to recombination under forward-bias

$$I_{rec} = \frac{-qAn_iW}{2\tau_0}\left(e^{qV_a/2kT} - 1\right)$$

- Total diode forward current

$$I = I_{diff} + I_{rec}$$
$$= I_s\left(e^{V_a/V_T} - 1\right) + I_{R_o}\left(e^{V_a/2V_T} - 1\right)$$

Where $I_{R_o} = \dfrac{-qAn_iW}{2\tau_0}$

- Ideality factor: The I-V characteristics of a real diode may be expressed as $I = I_o\left(e^{V/\eta V_T} - 1\right)$ where η is called ideality factor. Its value is 1 if current is by diffusion only and 2 if the current is due to recombination in the depletion region. It may vary between 1 and 2 if the current is due to the combination of both mechanisms.
- For Ge diode at room temperature $\eta \cong 1$
- For Si diode at room temperature $\eta \cong 2$
- For a Ge diode at room temperature the reverse saturation current (I_s) increases by around 11% for each degree K rise in temperature. For Si diode at room temperature (I_s is negligible) I_{R_o} increases by 8% for each degree K rise in temperature.
- The slope of diode characteristics decrease at high currents due to high-level injection.
- Depletion layer capacitance (junction capacitance) of a p-n junction is given by

$$C_j = \frac{\grave{o}A}{W}$$

- Also $$C_j = \frac{C_{jo}}{\sqrt{1-\frac{V_a}{V_o}}}$$

C_{jo} - equilibrium depletion layer capacitance.

C_j - depletion layer capacitance at bias V_a.

- The intersection of $\frac{1}{C_j^2}$ vs V_a curve on the voltage axis gives the value of built-in potential V_o
- p+n diode is a diode with very heavily doped p-side ($N_A >> N_D$).
- Linearly graded p-n junction is a p-n junction in which the doping concentration increases linearly with distance from the junction.
- Breakdown is the phenomenon by which the current in a reverse biased diode increase sharply at a particular reverse voltage called breakdown voltage. Zener breakdown is the breakdown by field ionisation or tunneling. Avalanche breakdown is the breakdown by secondary emission (impact ionisation) and avalanche multiplication.
- The breakdown voltage of a p+n diode is

$$V_{Br} = \frac{\varepsilon E_{crit}^2}{2qN_D}$$

- The peak electric field at which breakdown occurs is called critical field, (ε_{crit}).
- Punched through diode is a diode in which depletion layer punches through the neutral region.
- Storage delay time (t_{sd}) of a diode is the time required for the stored charge to become zero after the applied voltage is reduced to zero or negative value.

$$t_{sd} = \ln\left[\frac{I_F}{I_R}\right]$$

- Storage capacitance or diffusion capacitance Cs is the capacitance due to charges stored in the neutral region of a diode under forward-bias.

$$C_s = G_s\,\tau_p = \frac{I_F}{V_T}\,\tau_p$$

- Schottky diode: A metal semiconductor rectifying contact is called Schottky diode. I-V characteristics of a Schottky diode is given by

$$I = AR^*T^2\,e^{-q\phi Bm/kT}$$

$$= I_o\left(e^{V_a/V_T} - 1\right) \quad \text{where } I_o = AR^*T^2\,e^{-q\phi Bm/kT}$$

- Schottky diode has low forward voltage drop. It has no storage capacitance. But its leakage current is more.
- Photo diode works on the principle of absorption of light.
- When an illuminated photo diode is open circuited, voltage develops across its terminals is

$$V_{oc} = \frac{kT}{q}\ln\left(1 + \frac{I_L}{I_o}\right)$$

This is called photo-voltaic effect. This is the basic principle of operation of a solar cell.

- A reverse biased photo diode works as a photodetector as the current through it is propotional to the intensity of light falling on it.
- Light emitting diodes (LEDs) are usually made of direct band gap semiconductors. When a p-n junction made of direct band gap semiconductor is forward-biased, the injected charge carriers recombine radiatively-emitting light.

▪ Exercise Problems

1. For long abrupt p-n junction $N_A = 10^{17}$ cm^{-3} and $N_D = 10^{14}$ cm^{-3}. Calculate the built-in voltage at 300 K for
   (a) silicon diode (b) germanium diode
   Ans: (a) 0.637 V (b) 0.252 V.
2. Determine the built-in voltage of a silicon abrupt p-n junction with $N_A = 10^{17}$ cm^{-3} and $N_D=10^{14}$ cm^{-3},
   (a) at 300 K (b) at 450 K
   Ans: (a) 0.637 (b) 0.303 V.
3. A p+n silicon diode has $N_D = 10^{15}$ cm^{-3}, $\tau_p= 1$ μs, $D_p = 12$ cm^2/s and $A = 10^{-4}$ cm^2. At 300 K determine,
   (a) the reverse saturation current,
   (b) current when Va = 0.7 V (forward-bias) and
   (c) current when Va = -0.7 V (reverse-bias).
   Hint: current in a p+n diode = $I_{p\ diff}(x_n = 0)$
   Ans: (a) 1.27×10^{-14} A (b) (b) 6.257 mA (c) -1.27×10^{-14} A.
4. Show that the maximum electric field in a p-n junction is twice the average field. A Si p-n junction has $N_A = 10^{15}$ cm^{-3} on the p-side and $N_D = 2 \times 10^{16}$ cm^{-3} on the n-side. Using depletion approximation, determine the values of X_n and X_p and the maximum electric field E_m at a reverse-bias of 10 V. Assume T = 300 K.
   Ans: $X_n = 0.182$ μm, $X_p = 3.64$ μm, $E_m = 5.57 \times 10^4$ V/cm.
5. Two ideal and identical (η = 1) junction diodes are connected in series as shown in Fig. SP.3.3
   (a) Show that $e^{qV_1/kT} + e^{qV_2/kT} = 2$ where V_1 and V_2 are the voltage drop across the diodes
   (b) Assuming that the current due to reverse-bias diode is saturated at I_o, calculate the voltage drop across the forward-biased diode.
6. Draw the energy band diagram of a p-n junction
   (a) under equilibrium (b) forward-bias (c) reverse-bias.
   Show the variation of quasi-Fermi levels if exist.
7. A n^+p silicon diode has the following properties. $N_D = 10^{18}$ cm^{-3}, $N_A = 10^{14}$ cm^{-3}, A = 10^{-3}, cm^2, $W_N = 2$ μm, $W_P = 200$μm, τ_n - τ_p = 0.1 p.s, $D_n = 30$ cm^2/s, $D_p = 10$ cm^2/s. At 300 K, plot electron and hole currents in the p-region as a function of distance for V_a = 0.6 V.
8. Sketch the electron and hole currents as a function of position in a forward-biased (V_F = 0.65 V) Si p-n junction diode with $N_D = 5 \times 10^{16}$ cm^{-3}, $\tau_p = 10^{-6}$ s and $D_p = 12$ cm^2/s on the n-side and $N_A = 10^{17}$ cm^{-3} and $\tau_n = 6 \times 10^{-7}$ s and $D_n = 36$ cm^2/s on the p-side. The length of the neutral region on each side is half of the minority carrier diffusion length. Assume T = 300 K and a forward-bias of 15 VT. Neglect carrier recombination in the depletion region. Repeat the calculations for a reverse-bias of 2 V.
9. A Ge p-n junction diode has $N_D = 2 \times 10^{16}$ cm^{-3} on the n-side and $N_A = 3 \times 10^{19}$ cm^{-3} on the p-side.

(a) Calculate the forward voltage at which the injected hole concentration at the edge of the depletion region on the n-side becomes equal to the majority carrier concentration. Assume T = 300 K, $D_p = 42$ cm^2/Vs, $\tau_p = 3 \times 10^{-7}$ s.

(b) Calculate the current density at this voltage and compare it with the thermal equilibrium diffusion current density.

Ans: (a) 0.3476 V (b) 47.2 mA/cm2.

(10) A long base Ge p-n junction diode has an abrupt, junction with uniformly doped regions. The p-side has a resistivity of 1 Ωcm and the n-side has a resistivity of 0.2 Ω cm.

(a) Calculate the concentrations of minority carriers at the edge of the depletion region with a forward-bias of 0.207 V, and sketch the majority and minority carrier current densities as functions of distance from the edges of the depletion region on each side of the junction.

(b) Calculate locations of the planes at which majority and minority carrier currents arc equal in magnitude. Assume $\tau_p = 10^{-7}$ s, $\tau_n = 10^{-5}$ s, T = 300 K, $D_{pn} = 40$ cm^2, $D_{np} = 100$ cm^2/s, $\mu_{nn} = 2\mu_{pp} = 3800$ cm^2/Vs.

Ans: (a) $\Delta n_p = 5.448 \times 10^{14}$ cm^{-3}, $\Delta p_n = 2.179 \times 10^{14}$ cm^{-3} (b) $x_n = 7.2$ μm.

(11) Show the fractional change in reverse saturation current for unit change in temperature is given by

$$\frac{1}{Is}.\frac{dIs}{dT} = \frac{1}{T}\left(3 + \frac{E_{go}}{kT}\right) \text{ for Ge diodes and}$$

$$\frac{1}{I_{R_o}}\frac{dI_{R_o}}{dT} = \frac{1}{2T}\left[3 + \frac{E_{go}}{kT}\right] \text{for Si diodes}$$

(12) Determine the percentage change in reverse saturation current for 10°C rise in temperature at 300 K for Si and Ge assuming $\frac{dV_g}{dT} = 0$.

Ans: 110%, 80%.

(13) (a) Plot on semilog sheet the ideal diode characteristics at T = 300 K as V_a is increased from 0 to 0.75 V in steps of 0.05 V, if $I_s = 10^{-15}$ A.

(b) Plot. the characteristics assuming that there is constant recombination current of 10^{-13} A, which is independent of the bias voltage.

(14) A long base silicon abrupt p-n junction with an area of 10^{-3} cm^2 has $N_D = 10^{18}$ cm^{-3}, $N_A = 10^{16}$ cm^{-3}, $\tau_p = 5 \times 10^{-7}$ s, $\tau_n = 5 \times 10^{-6}$ s, $D_p = 6.0$ cm^2/s, $D_n = 20$ cm^2/s.

(a) At 300 K calculate the diode current with a forward-bias of 0.5 V, and then with a reverse-bias of 5 V.

(b) Repeat part (a) including the generation recombination currents if $\tau_o = 10^{-7}$ s.

Ans: (a) 1.6×10^{-6} A, -7.32×10^{-15} A (b) (b) 5.24×10^{-6} A, -1.032×10^{-9} A.

(15) For a p + n Ge diode $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{14}$ cm^{-3}.

(a) Determine the excess hole concentration at the edge of the depletion layer on n-side at $x_n = 0$ for $V_a = 80$ mV.

(b) At what value of V_a is the limit of low-level injection reached (low-level injection condition become invalid as the injected carrier concentration on the lightly doped side become $\frac{1}{10^{th}}$ of the doping).

Ans: (a) 1.293×10^{14} cm^{-3} (b) 0.0248 V.

(16) An ideal silicon abrupt long p-n junction has $N_A = 10^{16}$ cm^{-3}, $N_D = 10^{14}$ cm^{-3}, $\tau_n = \tau_p = 0.1$ μs, $D_n = 36$ cm^2/s. $D_p = 12$ cm^2/s and $A = 10^{-3}$ cm^2. At 300 K determine C_s and C_j

(a) At forward-bias of 0.1 V, 0.4 V and 0.7 V.

(b) At reverse-bias of 0.1 V and 10 V.

Ans: (a) (i) $C_s = 7.05 \times 10^{-16}$ F $C_j = 4.162 \times 10^{-12}$ F
(ii) $C_s = 73.8 \times 10^{-12}$ F $C_j = 6.826 \times 10^{-12}$ F
(iii) $C_s = 3.26 \times 10^{-8}$ F $C_j = 17.32 \times 10^{-12}$ F
(b) (i) $C_s = 0$ $C_j= 3.49 \times 10^{-12}$ F
(ii) $C_s = 0$ $C_j = 0.884 \times 10^{-12}$ F.

(17) A silicon abrupt p-n junction has a junction capacitance of 10 pF at reverse-bias of 8.8 V and 20 pF at reverse-bias of 1.6 V. Determine the built-in voltage and the doping on the n-side if the doping on the p-side, $N_A = 10^{16}$ cm^{-3} at 300 K.

Ans: (a) Va = 0.8 V (b) $N_D = 5.19 \times 10^{17}$ cm^{-3}.

(18) If the critical field strength for avalanche breakdown is 3×10^5 V/cm in a silicon abrupt p+n junction, what should be the doping for a breakdown voltage of 100 V?

(ϵ_r= 11.8, q = 1.6×10^{-19} C).

Ans: $N_D = 2.938 \times 10^{15}$ cm^{-3}.

(19) An abrupt p+n silicon junction has breakdown voltage of 500 V. The critical electric field is 3×10^5 V/cm. At 300 K determine

(a) N_D (b) depletion layer width at breakdown.

Ans: (a) $N_D = 5.88 \times 10^{14}$ cm^{-3} (b) 33.33 μm.

(20) A Schottky barrier diode is made by depositing tungsten on p-type silicon having $N_A = 10^{16}$ cm^{-3}. Electron affinity of Si = 4.15 eV, work function of tungsten is 4.9 eV. Calculate

(a) the built-in voltage,
(b) depletion layer width and
(c) maximum electric field. Draw the equilibrium energy band diagram.

Ans: (a) 0.149 V (b) 1.395×10^{-5} cm (c) 2.14×10^4 V/cm.

(21) A Schottky barrier diode formed on n-type Si at 300 K has $N_D = 2 \times 10^{15}$ cm^{-3}, electron affinity of silicon ψ_s is 4.15 V and metal work function is 4.9 eV. Determine

(a) the built-in voltage,
(b) the barrier height ϕ_{Bm} and
(c) the width of depletion layer at equilibrium.

Ans: (a) 0.507 V (b) 0.75 V (c) 5.75×10^{-5} cm.

(22) A metal with work function of 4.3 eV is deposited on n-type Si. Determine the doping density required at 300 K, so that there is no space charge region at equilibrium. Electron affinity of Si = 4.15 eV. Draw the energy band diagram.

Ans: 7.2×10^{16} cm^{-3}.

(23) A Schottky barrier diode is made up of Al deposited on N-type GaAs. The diameter of junction is 100 μm. Plot the I-V characteristics of the diode from a reverse-bias of 1 V to forward-bias of 0.55 V. Assume R* = 8 A cm^{-2} K^{-2} at 300 K.

(24) Draw the energy band diagram of a metal p-type silicon contact. ϕ_m = 4.6 V ψ_s = 4.1, V, $N_A = 10^{16}$ cm^{-3}. Determine W_o, V_o andE_{mo}.

Ans: $W_o = 2.28 \times 10^{-5}$ cm
$V_a = 0.3987$ V
$E_{mo} = 3.497 \times 10^4$ V/cm.

(25) Show that in an aprupt p-n junction, qVo = $E_{C_p} - E_{C_n}$

- **Review Questions**

(1) What is meant by graded p-n junction?
(2) Define built-in potential. Why is it known as diffusion potential?
(3) What is meant by depletion region of a p-n junction? How is it formed?
(4) List the properties of depletion region of a p-n junction.
(5) Draw the equilibrium energy band diagram of a p-n junction.
(6) Define Fermi potentials. How is it related to built-in potential of a p-n junction.
(7) Plot the distribution of potential, electric field and charge density across an abrupt p-n junction.
(8) Derive the expression for built-in potential of an abrupt p-n junction.
(9) Explain the temperature dependance of built-in potential. How is it related to
  (a) dopings and
  (b) band gap of a material
(10) Derive expression for maximum electric field across an abrupt p-n junction.
(11) Express electric field as a function of distance in the depletion region of an abrupt p-n junction.
(12) Express the potential in a p-n junction as a function of distance.
(13) Derive expression for depletion layer width of a p-n junction.
(14) Plot the potential distribution across an abrupt p-n junction under
  (a) equilibrium
  (b) forward-bias
  (c) reverse-bias
(15) Explain qualitatively the different current components in a p-n junctions, their directions and variation with bias.
(16) Plot the energy band diagram of a p-n junction under
  (a) forward-bias
  (b) reverse-bias
(17) What are the approximations made in the derivation of ideal diode equation?
(18) Derive the ideal diode equation for a long base diode.
(19) Plot the minority and majority carrier currents as a function of distance across a long abrupt p-n junction.
(20) Plot the characteristics of a diode. Explain.
(21) Plot the minority carrier distribution across an abrupt p-n junction under
  (a) forward-bias
  (b) reverse-bias
(22) Derive generalised form of a diode-equation. How is it modified for a short diode?
(23) What is real diode? What are the factors that deviate the characteristics of a real diode from that of the ideal diode?
(24) Explain generation and recombination currents.
(25) Explain the temperature dependance of reverse characteristics of Si and Ge diodes.
(26) What is ideality factor of a diode? How does it vary with temperature? What is its value at room temperature for Si and Ge diodes?
(27) Explain the methods to experimentally determine reverse saturation current and ideality factor of a diode.
(28) Show that the forward current in a p+n junction with high-level injection is given by

$$I = 2qA\frac{D_p}{L_p}n_i e^{V_j/2V_T}$$

(29) How do resistance of the bulk of the diode affects its characteristics?
(30) Why do the potential applied across a diode drops across the depletion region alone?
(31) What is depletion layer capacitance? Derive an expression for it.
(32) Show that $C_j = \dfrac{C_{jo}}{\sqrt{1-\frac{V_a}{V_o}}}$
(33) What is p+n diode? Write the approximate expression for W_o and I_o of a p+n diode.
(34) Derive expression for
 (a) Built-in potential
 (b) Electric field distribution
 (c) Potential distribution and
 (d) Depletion layer capacitance of a linearly graded p-n junction.
(35) What is meant by breakdown in p-n junction? Does break-down damage a p-n junction? Why?
(36) What is Zener breakdown?
(37) What is avalanche breakdown?
(38) What are the differences between Zener breakdown and avalanche breakdown?
(39) What is meant by critical field?
(40) Derive expression for breakdown voltage of a p-n junction.
(41) What is meant by avalanche multiplication? How is the multiplication factor related to reverse voltage?
(42) What is a punched through diode? What is its advantage?
(43) What are the applications of diode breakdown?
(44) What Is meant by stored charge in a p-n junction?
(45) Derive an expression for the time variation of stored charge in a p-n junction.
(46) Derive expression for the time variation of voltage across a p-n junction as it is switched from forward-bias to reverse-bias condition.
(47) What is meant by storage delay time? How is it related to the current through the diode? How is it related to life time?
(48) What is storage capacitance or diffusion capacitance? How is it related to forward current?
(49) How do the junction capacitance vary with bias?
(50) The capacitance of a forward-biased diode is due to the stored charges. Is this statement correct? Why?
(51) Why is there no storage capacitance under reverse-bias?
(52) Derive the expression for conductance of diode. How does it vary with variation in forward-bias?
(53) Draw the small signal equivalent circuit of an abrupt p-n junction
 (a) under forward-bias
 (b) under reverse-bias
(54) Define electron afinity and workfunction of a semiconductor.

(55) Draw the energy band diagram of a metal p-type semiconductor contact with

(a) $\phi_m < \phi_{sc}$

(b) $\phi_m < \phi_{sc}$ under equilibrium and with bias.

(56) Write expressions for V_o and ϕ_m for metal n-type semiconductor contact in terms of ϕ_m and ϕ_{sc}.

(57) Derive expressions for depletion layer capacitance of Schottky diode?

(58) What is the equilibrium depletion layer width of a Schottky diode?

(59) Derive expression for the I-V relationship of a Schottky diode.

(60) What are the advantages of a Schottky diode over an abrupt p-n junction?

(61) Compare the forward and reverse characteristics of a p-n junction diode and Schottky diode made of same semiconductor material.

(62) What is an ohmic contact?

(63) Is it possible to make ohmic contact on lightly doped semiconductor?

(64) Draw the energy band diagram of a silicon n-type semiconductor ohmic contact.

(65) Draw the characteristics of a photo diode and explain. Write the current voltage relationship.

(66) What is photovoltaic effect?

(67) Explain the principle of operation of a solar cell. Describe its structure. What are its application?

(68) Explain the principle of operation of PIN photo detector and avalanche photo diode detector.

(69). Explain the principle of operation of LED. What are the materials used for fabrication of LEDs? Explain.

(70) What is varactor diode? What are its applications?

(71) Draw the characteristics of a tunnel diode and explain.

(72) Explain principle of operation of a tunnel diode.

(73) Explain the difference between I-V characteristics of Si and GaAs diodes at room temperature.

(74) Why is Ge not used for making solar cells?

(75) Under what conditions are the energy bands in a metal semiconductor contact flat?