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Semiconductor Materials and Physics

INTRODUCTION

The growth in Electronics and Communication industry has been tremendous with the invention of transistors and integrated circuit technology. The major cause of reduction in size, delay, power, weight and cost in all areas of Electronics and Communication is the result of improved devices and technology. Semiconductors, metals and insulators are required for fabricating electronic devices, but the crucial role is played by semiconductors at present.

1.1 SEMICONDUCTOR MATERIALS

Semiconductors are materials with electrical conductivity between that of conductors and insulators. Semiconductors, in general, have conductivity between $10^{-8}\Omega^{-1}cm^{-1}$ and $10^{3}\ \Omega^{-1}cm^{-1}$ (resistivity between $10^{8}\ \Omega cm$ and $10^{-3}\ \Omega cm$). The conductivity of a semiconductor can be varied over a wide range, by the process of doping. Conductivity of doped semiconductors remain almost independent of temperature over a wide range of temperature. These properties make semiconductors suitable for fabricating electronic devices.

1.1.1 Elemental and Compound Semiconductors

Semiconductors which are constituted by single species of atoms are called elemental semiconductors. Fourth group elements, Silicon (Si) and Germanium (Ge) are elemental semiconductors.

As a semiconductor material, silicon has several advantages over other materials.

1. Silicon is abundant in nature. Well established procedure and technology are available for its purification, crystal growth and processing.
2. Compared to germanium, silicon devices can be operated at higher temperature due to its wider band gap.
3. A stable oxide (SiO2) is available for silicon which can be used:

    a. as mask during fabrication process,

    b. for isolation,

    c. as passivation layer,

    d. as gate oxide in MOSFETs.

Because of these, the fabrication process is simpler for silicon devices. Therefore, most of the Integrated Circuits (ICs) and other electronic devices are made of silicon.

A semiconductor constituted by two or more different species of atoms is called compound semiconductor.

There are lot of semiconductors which are compounds of two or more elements. Following are some of the compound semiconductors.

III-V compounds: Compounds formed by elements from third and fifth groups are called III- V compounds.

II-VI compounds: Compounds formed by elements from second and sixth groups are called II-VI compounds.

A compound semiconductor consisting of two elements is called binary compound. Elements from fourth group also form binary compounds as listed in Table 1.1. In addition to these, compounds of three and four elements are also available. Compounds of three elements are called ternary compounds e.g., GaAsP, AlGaAs. Compounds of four elements are called quaternary compounds e.g., AlGaAsP, InGaAsP.

Table 1.1 List of binary compounds

IV group compounds	III-V compounds	II-VI compounds
SiC	AlP	ZnS
SiGe	AlAs	ZnSe
	AlSb	ZnTe
	GaP	CdS
	GaAs	CdSe
	GaSb	CdTe
	InP	
	InAs	
	InSb	

The properties of compound semiconductors (conductivity, band gap. etc) can be varied by varying the percentage composition of its costituent elements. Semiconductor materials most commonly used for fabricating some of the electronic devices are listed in Table 1.2.

1.2 INTRODUCTION TO QUANTUM MECHANICS

Many of the observed phenomena such as discrete spectral lines emitted by heated gas etc., could not be explained by classical mechanics. It also fails to describe microscopic phenomena related to semiconductors such as energy bands separated by band gaps etc. To analyze microscopic phenomena in semiconductor materials, application of quantum mechanics is essential. This section provides a brief introduction to quantum mechanics. Table 1.3 compares classical mechanics with quantum mechanics.

Table 1.2 Semiconductor Devices and Materials used for their Fabrication

Device	Material
Diodes, Transistors	Si, Ge
FETs, Monolithic IC's	Si
LEDs	GaAs, GaP, GaAsP
Solar cells	Si, GaAs
Photo detector	InSb, CdSe
Fluorescent screen (TV,-CRT, etc.)	ZnS
Gunn diode (microwave diode)	GaAs,InP
Semiconductor laser	GaAs, AlGaAs
SCR, TRIAC	Si

Table 1.3 Comparison Between Quantum Mechanics and Classical Mechanics

Classical mechanics	Quantum mechanics
1. Deals with macroscopic objects. These can be defined with infinite details.	Quantum mechanics abandons this hypothesis of infinite detailed experience.
2. Elements of observation and calculation are identical.	Elements of observation and calculation need not be identical. Quantum mechanics allows greater freedom of abstract notations which may not have immediate physical meaning.
3. No restriction in the accuracy of measurement or calculation.	Sharp prediction of observation is prevented by Heisenberg's Uncertainty principle.
4. State of physical system is described by dynamical variables, (dynamical variables are properties associated with a physical system such as position, momentum, etc.)	State is described by state function ψ.

1.2.1 Heisenberg's Uncertainty Principle

Heisenberg's uncertainty principle predicts the uncertainty in simultaneous measurement of any pair of variables with dimension ML^2/T such as position and momentum, energy and time etc., as

$$\Delta P \, \Delta x \geq h \tag{1.1}$$

$$\Delta E \, \Delta t \geq h \tag{1.2}$$

Where, $h = \frac{h}{2\pi}$ and h is Planck's constant

ΔP is uncertainty in momentum
Δx is uncertainty in position
ΔE is uncertainty in energy
Δt is uncertainty in time.

Example 1.1 A particle of mass 1g moves with a velocity of 150 m/s. The momentum of the particle is not known to an accuracy more than 10^{-5} percentage. What is the limiting value of accuracy in simultaneous measurement of position?

$$\text{Momentum} \quad P = mv = 10^{-3} \times 150 = 0.15 \text{ kgm/s}$$

$$\Delta P = 10^{-5}\ \%\ \text{of } P$$

$$= 0.15 \times \frac{10^{-5}}{100} = 1.5 \times 10^{-8} \text{ kgm/s}$$

$$\Delta P \,.\, \Delta x = \frac{h}{2\pi}$$

$$\therefore \Delta x = \frac{h}{2\pi.\Delta P}; \quad h = 6.62 \times 10^{-34} \text{ Js}$$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times 1.5 \times 10^{-8}} = 7.02 \times 10^{-27}\text{m}$$

i.e., the uncertainty about the position of the particle is 7.02×10^{-27}m.

1.2.2 Postulates of Quantum Mechanics

(1) There exists a state function $\psi(x,y,z,t)$, which contains all the measurable information about each particle of a physical system.

(2) Operators are used to bridge between states and measurement. Every dynamical variable has a corresponding linear operator. This operator is used on the state function to obtain measurable information about the system. The dynamical variables and corresponding operators are given in Table 1.4.

Table 1.4 Dynamical Variables and Quantum Mechanical Operators

Dynamical variable	**Operator**
Position x	x
Momentum P_x	$\frac{h}{j}.\frac{\partial}{\partial x}$
Total energy	$\frac{-h}{j}\frac{\partial}{\partial t}$
Potential energy	V(x)

(3) (a) The state function ψ(x,t) and its space derivative $\frac{\partial \psi}{\partial x}$ are continuous, finite and single valued for all values of x.

(b) The function ψ is normalised

i.e, $$\int_{-\infty}^{\infty} \psi\psi^* dx = 1 \quad \text{or} \quad \int_{-\infty}^{\infty} |\psi|^2 dx = 1 \tag{1.3}$$

where ψ* is the complex conjugate of ψ.
In three dimension,

$$\iiint \psi\psi^* dv = 1 \tag{1.4}$$

where *dv* is an elemental volume about the point where ψψ* is determined and ψψ* is the probability density. Equation (1.4) shows that the probability of finding the particle anywhere in the volume is unity.

(4) The average value of a dynamical variable (y) corresponding to state function ψ is given by

$$\langle y \rangle = \int_{-\infty}^{\infty} \psi^* y \psi dy \tag{1.5}$$

< y > is the average value or expectation value over many observations.

(5) The operator for any measurement, for which an operator is not postulated, is determined by expressing the measured quantities in terms of the basic variables. Then, substitute the operators for the basic variables.

1.2.3 Schrodinger Wave Equation

Schrodinger wave equation is the fundamental equation in quantum mechanics, (just like Newton's laws of motion in classical mechanics). The one dimensional Schrodinger wave equation is represented by

$$jh\frac{\partial \psi}{\partial t} = \frac{-h^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \tag{1.6}$$

Equation (1.6) may be separated into time dependent and time independent equations as follows

$$\frac{-jh}{\phi(t)}\frac{\partial \phi(t)}{\partial t} = E \tag{1.7}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\mathrm{h}^2}(E - V)\psi = 0 \tag{1.8}$$

where, ψ(x,t) = ϕ(t) ψ(x) and E is the kinetic energy of particle.

1.2.4 Schrodinger Equation Applied to a Free Particle

A particle moving in a region of constant potential energy is called a free particle. Such a particle is not subjected to any force. For such a particle V = 0. and Schrodinger wave equation (time independent) reduces to

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \tag{1.9}$$

where $$k = \frac{\sqrt{2mE}}{\text{h}} = \frac{P}{\text{h}} = \frac{2\pi}{\lambda} \tag{1.10}$$

and is called the propagation constant or wave vector.

Equation (1.9) has solution

$$\psi(x) = Ae^{\pm jkx}$$

so that $\psi\psi^* = AA^* = |A|^2$ is independent on x i.e., probability of finding the particle everywhere is same and wave function does not provide any information about precise location of the particle.

1.2.5 Schrodinger Equation Applied to a Particle in a Potential Well

Consider an electron in a one dimensional potential well of width a as shown in Fig. 1.1. The potential at the sides of the well is ∞ and inside the well is zero. The wave function of the particle may be obtained by solving the time independent Schrodinger wave equation.

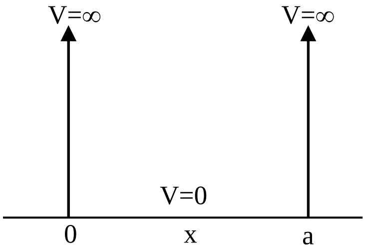


Fig. 1.1 An infinite potential well of width a

The Schrodinger equation inside the well may be written as

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \tag{1.11}$$

where, $$k^2 = \frac{2mE}{\text{h}^2} = \frac{8\pi^2 mE}{h^2} \tag{1.11a}$$

This equation has solution of the form

$$\psi_x = A \sin kx + B \cos kx \tag{1.12}$$

Since $\psi = 0$ at $x = 0$; $B = 0$

$$\therefore\ \psi_x = A \sin kx\ ; \tag{1.12a}$$

The electron cannot penetrate the infinite potential barrier.

i.e., at x = a, $\psi(a) = 0$, so that

$$A \sin ka = 0$$

$$\therefore \quad ka = n\pi$$

$$k^2 = \frac{n^2\pi^2}{a^2} \tag{1.13}$$

where, $\quad n = 1, 2, 3, \ldots$

$$E = \frac{h^2k^2}{8\pi^2 m} \quad \text{(by equation (l.lla))} \tag{1.14}$$

Substituting for k^2 from equation (1.13) in equation (1.14), energy of electron in nth state is

$$E_n = \frac{n^2h^2}{8ma^2} \tag{1.15}$$

From normalisation condition for the wave function (equation (1.3))

$$\int_0^a AA^* \sin^2 kx\,dx = 1 \tag{1.16}$$

ie.,

$$A^2 \int_0^a \frac{1 - \cos 2kx}{2}\,dx = 1$$

$$A^2 \left[\frac{x}{2} - \frac{\sin 2kx}{4k}\right]_0^a$$

But sin2ka = 0 Q $\quad ka = n\pi$

$$\therefore \quad A = \sqrt{\frac{2}{a}}; \quad \psi = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} \tag{1.17}$$

The normalised wave function in the lowest three energy states are

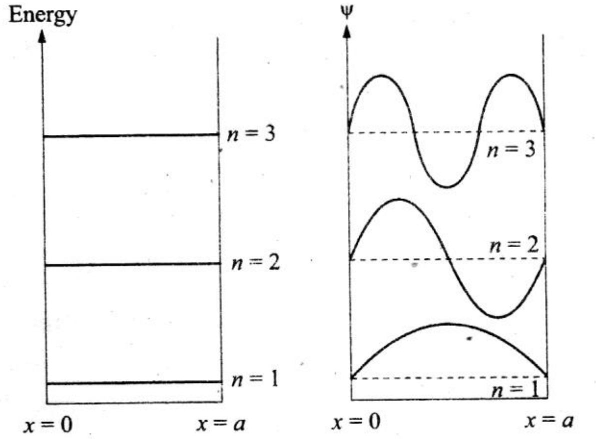
$\psi_1 = \sqrt{\frac{2}{a}} \sin\frac{\pi x}{a};$ half cycle variation over a $\left(a = \frac{\lambda}{2}\right)$ (1.17a)

$\psi_2 = \sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a};$ full wave variation over a (a = λ) (1.17b)

$\psi_3 = \sqrt{\frac{2}{a}} \sin\frac{3\pi x}{a};$ three half cycle variation over a $\left(a = \frac{3\lambda}{2}\right)$ (1.17c)

For nth state a = $\frac{n\lambda}{2}$ (1.17d)

The energy levels and corresponding wave functions are shown in Fig. 1.2.



(a) Energy levels (b) Wave function ψ

Fig. 1.2 First three energy levels and wave functions for electrons in an infinite potential well

a = $\frac{n\lambda}{2}$ (by equation (1.14))

k = $\frac{n\pi}{a}$ (by equation (1.13))

$$E = \frac{1}{2}mv^2 = \frac{P^2}{2m} \tag{1.18}$$

E = $\frac{h^2k^2}{8\pi^2 m}$ (by equation (1.14))

$$= \frac{\mathrm{h}^2k^2}{2m} \tag{1.19}$$

From equations (1.18) and (1.19),
$P = \mathrm{h}k$ where P is the electron momentum and k is the wave vector or wave number.

1.2.6. Schrodinger Equation Applied to a Particle in a Potential Wall

This considers the probability of locating an electron in a finite potential wall (potential wall) of height V_0 ($V_0 > E$, energy of the electron).

Consider an electron with total energy E moving in region 1 of potential energy zero as shown in Fig. 1.3.

At x = 0, the potential energy increases to V_0. Schrodinger equation in region 1

$$\frac{d^2\psi_1}{dx^2} + k_1^2\psi_1 = 0$$

$$K_1^2 = \frac{8\pi^2 mE}{h^2} \tag{1.20}$$

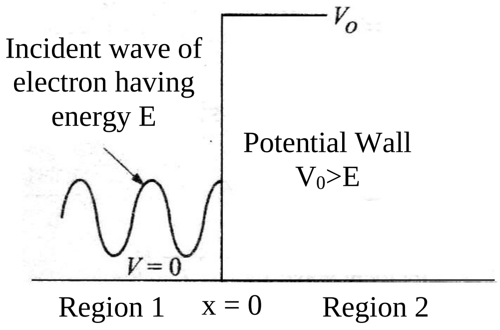


Fig. 1.3 Electron movement towards a potential wall

The solution to equation (1.20) is

$$\psi_1 = C \sin k_1x + D\cos k_1x \tag{1.21}$$

In region 2, the Schrodinger wave equation becomes

$$\frac{d^2\psi_1}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi_2 = 0 \tag{1.22}$$

$$\frac{d^2\psi_2}{dx^2} - k_2^2\psi_2 = 0; \quad k_2^2 = \frac{8\pi^2 m(V_0 - E)}{h^2} \tag{1.23}$$

The solution to this equation is

$$\psi_2 = Ae^{-k_2x} + Be^{k_2x} \tag{1.24}$$

By postulate 3(a) (ψ and $\frac{d\psi}{dx}$ are continuous)

$$\psi_1 = \psi_2 \quad \text{and} \tag{1.25}$$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \quad \text{at } x = 0 \tag{1.25b}$$

ψ must be finite as $x \to \infty$

$$\therefore \quad B = 0$$

Using equation (1.21)

$$C\sin k_1x + D\cos k_1x|_{x=0} = Ae^{-k_2x}|_{x=0}$$

i.e., D = A

Differentiating equations (1.21) and (1.24) and applying condition (1.25b)

$$k_1C\cos k_1x - k_1D \sin k_1x|_{x=0} = -k_2 Ae^{-k_2x}|_{x=0}$$

$$k_1C = -k_2A$$

$$C = \frac{-k_2A}{k_1}$$

$$\therefore \psi_1 = -k_2\frac{A}{k_1}\sin k_1x + A\cos k_1x \tag{1.26a}$$

$$\psi_2 = Ae^{-k_2x} \tag{1.26b}$$

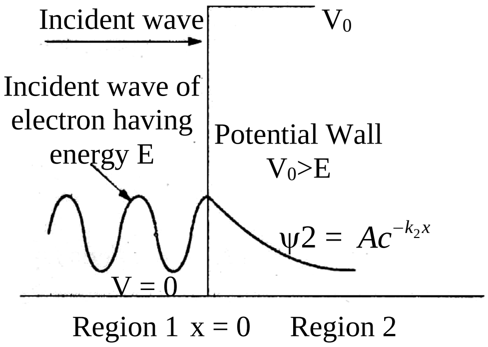


Fig. 1.4 Incident and penetrating wave into a potential barrier

The above solution shows that there is a finite probability that electron exists in region 2 where the potential energy barrier is higher than the energy of electrons as shown in Fig. 1.4.

Let an electron in region 1 be separated from region 3 by a finite potential energy barrier (V_o > E) of finite width as shown in Fig. 1.5. Then, it can be shown that there will be a finite probability that the electrons penetrate into region 3. This process is called tunneling. The probability of tunneling increases with decrease in width (d) of the barrier. This principle is made use of in the study of tunnel diodes, Zener break down, etc.

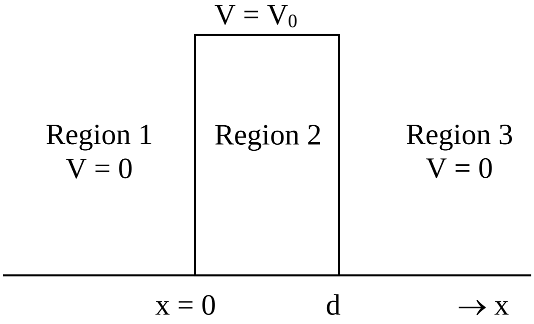


Fig. 1.5 Potential barrier of finite height and width

Example 1.2 For an electron in a one dimensional potential well of infinite height and width 2 A, calculate the expectation value of position (a) and momentum (P_x). Assume that potential inside the well is zero.

Solution

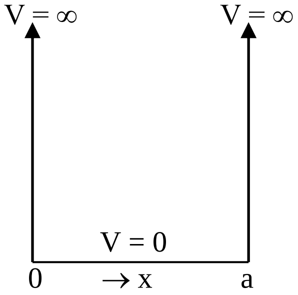


Fig. Ex. 1.2 Infinite potential well

Expectation value of x = < x > = $\int_0^a \psi^* x\psi dx$

From equation (1.17)

$$\psi = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}, \quad \therefore \quad \psi\psi^* = \psi^2$$

$$\therefore <x> = \int_0^a x\left(\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\right)0dx$$

$$= \frac{2}{a}\int_0^a x\sin^2\frac{n\pi x}{a}dx$$

$$= \frac{2}{a}\int_0^a x\left[\frac{1-\cos\left(\frac{2n\pi x}{a}\right)}{2}\right]dx$$

$$= \frac{1}{a}\int_0^a xdx\frac{1}{a}\int_0^a x\cos\frac{2n\pi x}{a}dx$$

$$= \frac{1}{a}\left[\frac{x^2}{2}\right]_0^a - \frac{1}{a}\left[ax.\frac{\sin\frac{2n\pi x}{a}}{2n\pi} - \int 1.a\sin\frac{\frac{2n\pi x}{a}}{2n\pi}dx\right]_0^a$$

$$= \frac{a}{2} - \frac{1}{a}\left[\frac{ax}{2n\pi}.\sin\left(\frac{2n\pi x}{a}\right) + \frac{a^2}{2n\pi}.\frac{\cos\frac{2n\pi x}{a}}{2n\pi}\right]_0^a$$

but sin 2nπ = 0 and cos 2nπ = 1

$$\therefore\ <x> = \frac{a}{2} = 1.0A^\circ$$

$$<P_x> = \int_0^a \psi^* \frac{h}{j}\frac{\partial}{\partial x}\psi dx$$

$$= \frac{2h}{ja}\int_0^a\left[\sin\frac{n\pi x}{a}\frac{\partial}{\partial x}\left(\sin\frac{n\pi x}{a}\right)\right]dx$$

$$= \frac{2h}{ja}\left[\sin\frac{n\pi x}{a}\times\sin\frac{n\pi x}{a} - \int\cos\frac{n\pi x}{a}\times\frac{n\pi}{a}\times\sin\frac{n\pi x}{a}dx\right]_0^a$$

$$= \frac{2h}{ja}\left[\sin^2\frac{n\pi x}{a} - \frac{n\pi}{a}\int\frac{1}{2}\frac{2n\pi x}{a}dx\right]_0^a$$

$$= \frac{2h}{ja}\left[\sin^2\frac{n\pi x}{a} - \frac{n\pi}{2a}\times\cos\frac{2n\pi x}{a}\times\frac{a}{2n\pi}\right]_0^a$$

$$= \frac{2h}{ja}\left[\sin^2\frac{n\pi x}{a} - \frac{1}{4}\cos\frac{2n\pi x}{a}\right]_0^a = 0$$

1.3 STATISTICAL MECHANICS

The objective of statistical mechanics is to treat the behavior of very large number of identical systems in a probabilistic fashion without going into the details of each and every individual component of the ensemble (collection of physical systems). The result obtained by this approach predicts the average behavior of the system based on the most probable values of the properties.

The most important characteristic of statistical mechanics is the distribution function. The three distribution functions that govern the distribution of particles among the various available energy levels are:

(1) Maxwell-Boltzmann distribution function
(2) Fermi-Dirac distribution function and
(3) Bose-Einstein distribution function.

1.3.1 Maxwell-Boltzmann Distribution Function

Maxwell-Boltzmann distribution is applicable to classical particles such as gas molecules. These particles are distinguishable[1] and any number of them can occupy a given energy state, i.e., Pauli's exclusion principle is not applicable.

For a system with a continuous distribution of energy, the probability f(E) that a state E is occupied is given by Maxwell-Boltzmann distribution as

$$f(E) = B(T)e^{-E/kT} \tag{1.27}$$

where B(T) is a constant and T is the absolute temperature.

1.3.2 Fermi-Dirac Distribution Function

This distribution is applicable to indistinguishable particles at thermal equilibrium, which obey Fault's exclusion principle. The Fermi-Dirac distribution function is given by

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}} \tag{1.28}$$

where, f(E) represents the probability of occupancy of available energy state E at absolute temperature T

E_F the Fermi energy level or Fermi level and

k the Boltzmann constant.

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[1] A set of identical particles are said to be distinguishable if they do not interact with each other except for occasional random collisions. Separation between particles will be large compared to de-Broglie wave length ($\lambda = h/mv$). There is no interaction between wave functions of such particles. A set of particles is said to be indistinguishable if separation between particles is comparable to or less than the de-Broglie wave length. Wave functions of such particles interact with each other.

$$f(E_F) = \frac{1}{1+e^{(E-E_F)/kT}} = \frac{1}{1+e^0} = \frac{1}{2} \quad \text{for } T > 0$$

i.e., the probability of occupancy of Fermi level is half, for any material under thermal equilibrium at any temperature other than 0 K (T > 0 K). Fermi level is a conceptual energy level, at which the probability of occupancy would be half, if it exists. Fermi level may be considered as the energy level corresponding to average energy of electron in a semiconductor.

When $E - E_F >> kT$

$$f(E) \cong e^{-(E-E_F)/kT} = e^{E_F/kT}e^{-E/kT}$$
$$= B(T)\,e^{-E/kT}$$

i.e., Fermi-Dirac distribution function decays to Maxwell-Boltzmann distribution.

In the case of a semiconductor, $E_C - E_F >> kT$ implies that the electron concentration is low. The probability of occupancy of quantum states in the conduction band is small in lightly doped semiconductor. Thus, interaction between wave functions of particles become negligible or particles become distinguishable. So, Pauli's exclusion principle is of no significance. This is called diluteness condition. In such case Fermi-Dirac distribution function may be replaced with Maxwell-Boltzmann distribution function. Particles that obey Fermi Dirac statistics are called fermions. Eg. electrons and holes in semiconductors. For more details of Perm Dirac distribution function, see section 1.8.

1.3.3 Bose-Einstein Distribution Function

Some particles like photons and phonons are indistinguishable, but do not obey Pauli's exclusion principle. The distribution function for indistinguishable particles that do not obey Pauli's exclusion principle is given by

$$f(E) = \frac{1}{e^{(E-E_B)/kT}-1} \tag{1.29}$$

Equation (1.29) is called Bose-Einstein distribution function. For particles like helium atoms parameter EB is obtained from the condition that the total number of particles in the system is constant. For particles like photons and phonons, which has no rest mass, these conditions are not valid because photons and phonons can be destroyed and recreated.

For such particles $E_B = 0$ and Bose-Einstein statistics takes the form

$$f(E) = \frac{1}{e^{E/kT}-1} \tag{1.30}$$

1.3.4 Photons and phonons

Pilot on is a quantum of electro-mechanical energy whereas phonon is a quantum of mechanical energy of lattice vibration.

The momentum associated with a photon of energy hν is $\frac{h\nu}{c}$ and that with a phonon is $\frac{h\nu}{u}$ where c is the velocity of light and u is the velocity of sound in solid medium. Since, u<<c, the momentum associated with a phonon is very large compared to that of a photon. The rest mass of photon and phonon are zero and these particles obey Bose-Einstein statistics.

1.4 CRYSTALLINE SOLIDS AND ENERGY BANDS

The valence electrons in an atom determine its chemical behavior and influence the type of bound formed between the atoms that result in the formation of molecules and solids. The strength with which the atoms are bound together determines the phase of the substance (gas. liquid, solid etc). When atoms of a substance are separated by large distance, they form a gas. When the average distance is comparable to the size of the atom. the substance behave as a liquid. When the atoms are brought, close together so that their outer electron orbits overlap, very strong interaction between atoms results which binds them together as a solid.

1.4.1 The Bonding of Atoms

A solid is formed by the chemical bonding of large number of atoms together. In this process, the total energy of the system is reduced. The energy required to separate 1 gram atom (or 1 mole) of a solid into individual atoms is known as cohesive energy. This energy is different from binding energy, which denotes the strength of a bond and is the negative of (lie energy required to break the bond. e.g.. binding energy of electron in ground state of hydrogen atom is 13.6 eV. This also implies that this much energy is lost during formation of bond.

When atoms are brought, together to form a molecule, two kinds of forces become important: attractive and repulsive. As the separation is decreased the attractive force comes into play first. But as the spacing is decreased, the other orbits of electrons overlap and repulsive force also arises. The chemical bond between atoms is formed at equilibrium spacing where the two forces balance each other and the total potential energy reaches its minimum. Depending on (lie nature of interaction between atoms, there are four different types of chemical bonding: ionic covalent, metallic and molecular.

Ionic bond: The ionic bond is formed predominantly between electronegative and electropositive elements e.g., NaCl. In ionic bonding the columbic forces (attractive between Na^+ and Cl^- ions) pull the lattice together until a balance is reached with the repulsive forces at equilibrium.

The ionic bond is fairly strong. Ionic substances are usually hard and have high melting and boiling points. Since the valence electrons are tightly bound to the respective ions, the movement of these electrons under an applied electric field is not possible and most ionic substances arc insulators at room temperature. At higher temperature the ions themselves become mobile and give rise to ionic conduction.

Covalent bond: The covalent bond is formed by sharing of electrons between the bonded atoms. This sharing results from the overlap of bonding orbitals. The covalent bond is a bond between atoms of the same polarity hence it is known as a homopolar bond.

Unlike ionic bond, the covalent bond is highly directional. The bond is very strong and the material formed has very high melting and boiling points. The covalently bonded substances are relatively poor conductors of electricity at normal temperatures.

Metallic bond: The metallic bond is formed between electropositive elements. The valence orbit of all metal atoms is either an s subshell or a p subshell with usually 1 or 2 electrons. The size of the outer shells in these atoms is rather large and the valence electrons are not as tightly bound to the nucleus as in non-metals. In metals, the outer electron of each atom is contributed to the crystal as a whole. The solid is made up of ions immersed in a sea of free electrons. The forces holding the lattice together arises from the interaction ^between the positive ion core and the sea of electrons.

In metals, the bonds remain unsaturated and a large number of atoms can be held together by the mutual sharing of electrons. The density of electrons is considerably lower than that allowed by Pauli's exclusion principle. The number of states available is more than the number of electrons. As a result, the electrons are able to move freely through the metal without experiencing any significant change in their energy. Therefore metals are good conductors of electricity. There are differences in the bonding in different metals.

Molecular bond: This bond is seen in inert gases and in some organic molecules. The binding force arises from the dipolar forces between the bonded species. The dipolar forces are quite weak and the substances exhibiting this type of bonding are characterized by low melting and boiling points, and are poor conductors of electricity.

1.4.2 Crystalline State

Solid materials can be divided into two categories-amorphous and crystalline. In amorphous materials, atoms are arranged in an irregular and random manner like the molecules in a liquid, e.g., glass. A crystalline solid has a regular periodic arrangement of atoms. If this regular arrangement of atoms extends over the whole sample, the material is said to be single crystalline. A polycrystalline material consists of groups or clusters of single crystals of various orientations that are joined together as shown in Fig. 1.6. The line separating the crystal section of two different orientations is known as grain boundary.

1.4.3 Crystal Structure

Extremely pure semiconductor in crystalline form are used for fabricating electronic devices. A crystal is a three dimensional structure composed of atoms arranged on a lattice. A lattice is a periodic arrangement of points in space. It is defined by three fundamental translation vectors a, b and c. The arrangement of lattice points looks the same from any point r as it looks from any other point r' given by

$$r' = r + n_1a + n_2b + n_3c = r + T$$

where n_1, n_2, n_3 are integers. The vector $T = n_1a + n_2b + n_3c$ is called the lattice translation vector.

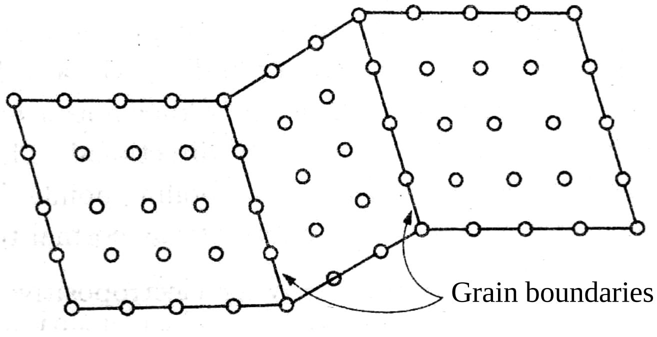


Fig. 1.6 Two-dimensional representation of arrangement of atoms in polycrystalline material

1.4.4 Terminology Related with Crystal Structure

Unit cell: A unit cell is the smallest structure that will generate the entire crystal from simple translation in three directions.

Primitive unit cell: If there is only one lattice point per unit cell, it is called a primitive unit cell. Fig. 1.7 shows unit cells and primitive unit cells.

Co-ordination number: The number of nearest neighbors (number of nearest lattice points to any lattice point) in a crystal arrangement is called co-ordination number.

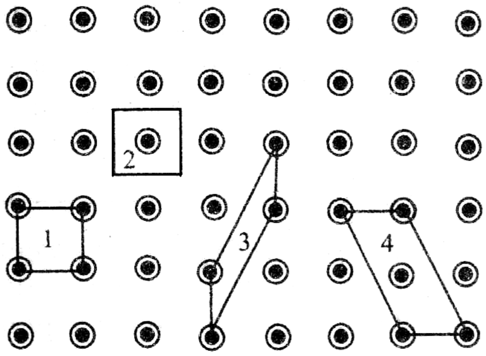


Fig. 1.7 Two-dimensional crystal lattices 1, 2, 3 and 4 represent unit cells. 1, 2 and 3 are primitive unit cells and 4 is non-primitive unit cell

Tetrahedral radius: The radius of atoms (in place of lattice points) constituting the crystal is called tetrahedral radius.

Cubic lattice: If the unit cell of crystal lattice has the shape of cube, the resulting lattice is called cubic lattice.

Lattice constant: In a cubic lattice the length of one side of the cube (a) is called lattice constant.

Packing efficiency: It is the ratio of the actual volume occupied by atoms in a lattice to the total volume, expressed as a percentage.

1.4.5 Important Cubic Crystal Structures

Simple cubic: For a simple cubic structure, lattice points are located only at the corners of the unit cell. Its co-ordination number is 6 and nearest neighbor distance is a. The structure of a simple cubic crystal is shown in Fig. 1.8.

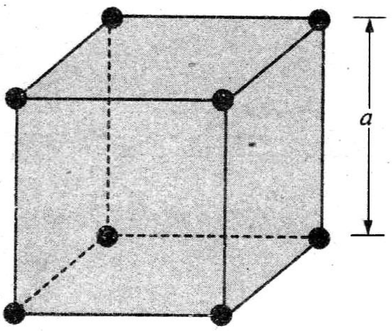


Fig. 1.8 Unit cell of simple cubic crystal

Body Centered Cubic (BCC): In addition to those in simple cubic structure, a lattice point is located at the centre of the cube as shown in Fig. 1.9. Its co-ordination number is 8 and nearest neighbor distance is $\frac{\sqrt{3}}{2}a$.

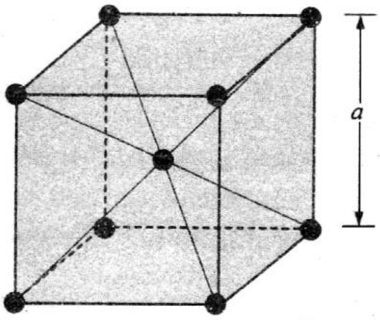


Fig. 1.9 Structure of body centered cubic crystal

Face Centered Cubic (FCC): In addition to those in simple cubic, lattice points are located at the centre of all the faces of the cube as shown in Fig. 1.10. Co-ordination number is 12 and nearest neighbor distance is $\frac{a}{\sqrt{2}}$.

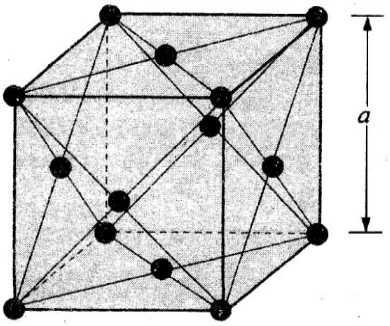


Fig. 1.10 Structure of face centered cubic crystal

Diamond lattice: Diamond lattice is formed by two inter penetrating FCCs, with the second FCC lattice displaced from the first by $\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)$ as shown in Fig. 1.11. The coordination number is 4 and nearest neighbor distance is a. Silicon, germanium and carbon (diamond) have this crystal structure.

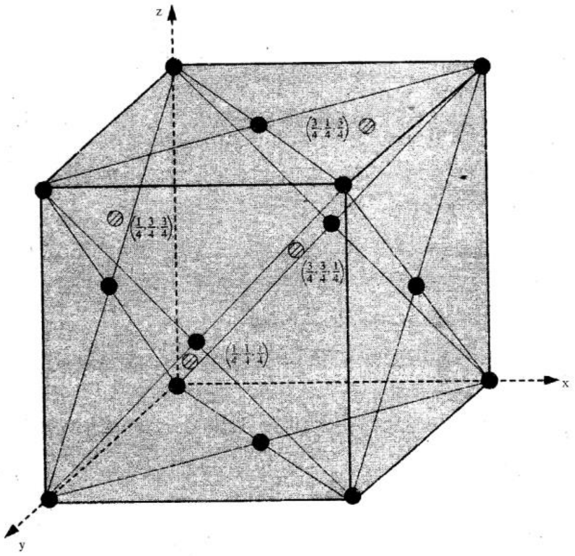


Fig. 1.11 Structure of diamond lattice; hatched-circles represent atoms of second FCC located inside first FCC

Zinc blend lattice: In diamond lattice, if the lattice points in the two FCCs are atoms of different elements, it becomes a zinc blend lattice, e.g., GaAs, in which one FCC is formed by Ga and the other FCC by As atoms. The co-ordination number is 4 and the nearest neighbor distance is $\frac{\sqrt{3}}{4}a$.

Example 1.3 Determine the packing efficiency of simple cubic, body centered cubic, face centered cubic and diamond lattices.

Solution

a. Simple cubic

$$\text{Tetrahedral radius r} = \frac{a}{2}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{\pi a^3}{6}$$

$$\text{Number of atoms per unit cell} = \frac{1}{8}\times 8 = 1$$

$$\text{Volume of unit cell} = a^3$$

$$\therefore \text{ Packing efficiency} = \frac{\textit{Volume of spheres} \text{ in } \textit{one unit cell}}{\textit{Volume of unit cell}}\times 100$$

$$= \frac{\pi\frac{a^3}{6}}{a^3}\times 100 = \frac{\pi}{6}\times 100 = 52\%$$

b. BCC

$$\text{Tetrahedral radius r} = \frac{\sqrt{3}}{4}a$$

Volume or sphere = $\frac{4\pi}{3}\frac{3\sqrt{3}}{4^3}a^3 = \frac{\pi\sqrt{3}a^2}{16}$

Number of atoms per unit cell = $\frac{1}{8}\times 8+1=2$ -x8+l=2

Packing efficiency = $\frac{2\times\pi\sqrt{3}\frac{a^3}{16}}{a^3}\times 100$

= $\frac{\pi\sqrt{3}}{8}\times 100 = 68\%$

c. FCC

Tetrahedral radius r = $\frac{a}{2\sqrt{2}}$

Volume or sphere = $\frac{\pi a^3}{12\sqrt{2}}$

Number of atoms per unit cell = $\frac{1}{8}\times 8+\frac{1}{2}\times 6=4$

Packing efficiency = $\frac{4\times\frac{\pi a^3}{12\sqrt{2}}}{a^3}\times 100$

= $\frac{\pi}{3\sqrt{28}}=72\%$

d. Diamond lattice

Tetrahedral radius r = $\frac{\sqrt{3}}{8}a$

Volume or sphere = $\frac{\pi\sqrt{3}}{128}a^3$

Number of atoms per unit cell = $\frac{1}{8}\times 8+\frac{1}{2}\times 6+4=8$

Packing efficiency = $\frac{8\times\pi\sqrt{3}\frac{a^3}{128}}{a^3}\times 100$

= $\frac{\pi\sqrt{3}}{16}=34\%$

1.4.6 Crystal Axes and Planes

Cubic crystal directions are described in terms of Miller notation. Consider, for example, any plane in space which satisfies the equation

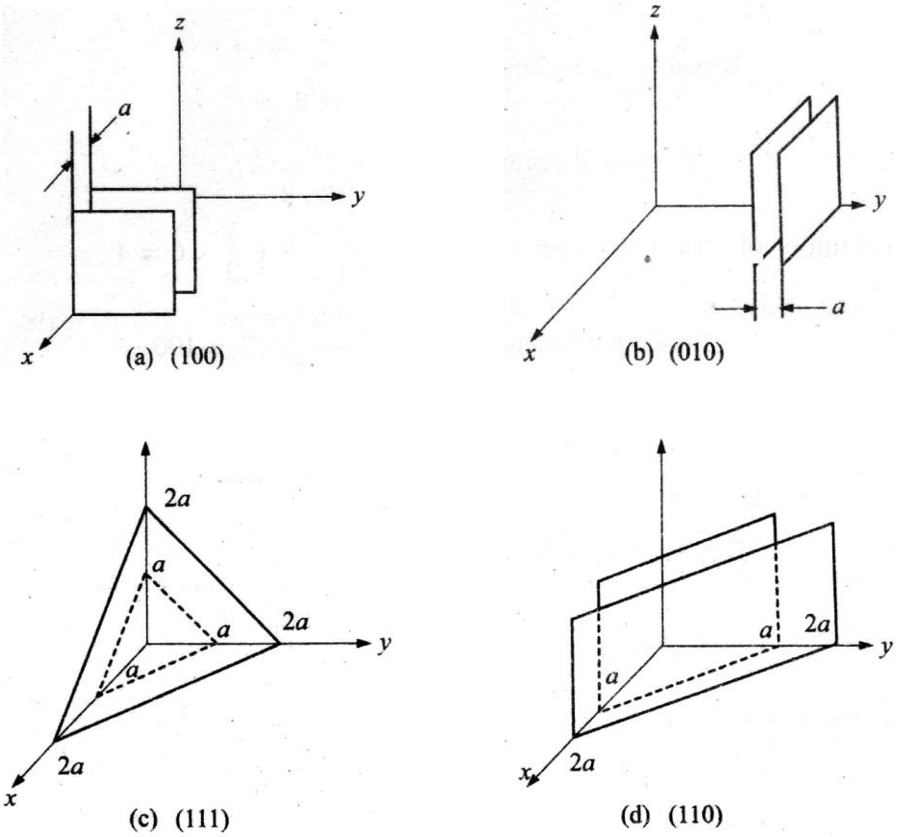
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (1.31)$$

Here, a, b and c are intercepts made by the plane at x, y and z axes respectively. Writing h, k and l as the reciprocals of these intercepts, the plane may be described by

$$hx + ky + lz = 1 \tag{1.32}$$

The Miller indices for this plane is written as (hkl). Integer values are chosen for these indices as multiples of the edges of the unit cell. If an intercept is negative a bar is used for that Miller index. For example, (Ill) shows that the intercept on the x-axis is -1.

The configuration of atoms in many of the Miller planes in a cubic crystal are identical. Thus, the planes (001), (010), (100), (001), (010) and (100) are essentially similar in nature and they are denoted as {001} planes. Some of the principle crystal planes are shown in Fig. 1.12.



(a) (100) (b) (010)

(c) (111) (d) (110)

Fig. 1.12 Principal crystal planes

Indices of lattice plane direction (i.e., the line normal to the lattice plane) are simply the vector components of the direction resolved along the co-ordinate axes. Thus the (111) plane has a direction written as [111], and so on. This is an extremely convenient feature of the Miller index system for cubic crystals. The set of directions [001], [010], [100], [001], [010], [100] is written as < 001 >. Fig 1.13 show different crystal directions.

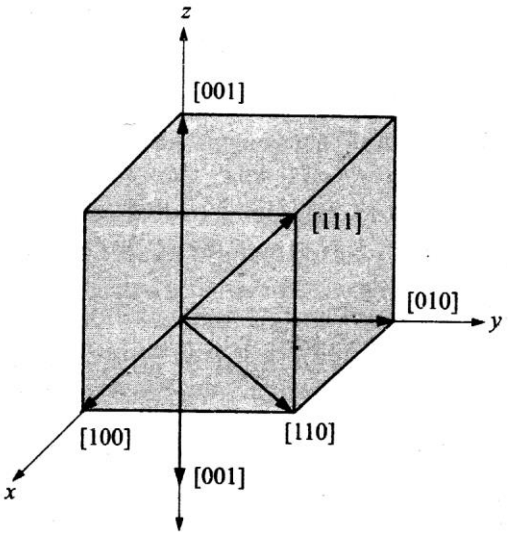


Fig. 1.13 Crystal directions

The separation between two adjacent parallel planes {hkl} is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \tag{1.33}$$

d = a for {100} planes
d = 0.707a for {110} planes
d = 0.577a for {111} planes

{111} planes are the closest spaced among low index planes.

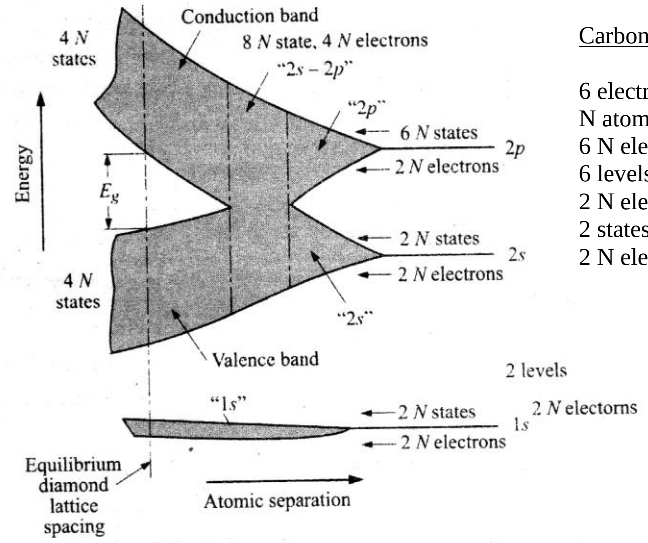
1.5 ENERGY BANDS IN SOLIDS

When atoms are brought together the application of Pauli's exclusion principle becomes important. When two atoms are completely isolated from each other so that there is no interaction of electron wave functions between them, they can have identical electronic structures. Electrons of isolated atoms occupy discrete energy levels as implied by Bohr postulates and Pauli's exclusion principle. As the spacing becomes smaller, electron wave functions begin to overlap. The Pauli's exclusion principle states that no two electrons in a given interacting system may have the same quantum state. Thus, there must be a splitting of the discrete energy levels of the isolated atoms into new levels belonging to the pair rather than to the individual atoms.

In a solid, many atoms are brought together, so that the split energy levels essentially form continuous bands of energies. Fig. 1.14 illustrates the imaginary formation of diamond crystal from isolated carbon atoms.

Each isolated carbon atom has an electronic structure $1s^2\ 2s^2\ 2p^2$ in the ground state. Each atom has available states of 2 in 1s, 2 in 2s and 6 in 2p. If we consider N atoms there will be 2 N, 2 N and 6 N states of 1s, 2s and 2p respectively.

As the inter-atomic spacing reduces, these energy levels split into bands beginning with the outer shell. As the '2s' and '2p' bands grow, they merge into a single band composed of mixture of energy levels of 2s and 2p. This is called hybridization of bands. This band of levels from 2s and 2p consists of 8 N states and 4 N electrons.



<u>Carbon atoms</u>

6 electrons/atom
N atoms
6 N electrons
6 levels
2 N electrons
2 states
2 N electrons

Fig. 1.14 Formation of energy band in diamond crystal. Imaginary diagram with electron energy as a function of inter-atomic spacing

As the distance between atoms decreases further, this band again splits into two different bands separated by a gap as shown in Fig. 1.14. At equilibrium inter-atomic spacing of diamond, these bands are separated by an energy gap. The upper band, called the conduction band, contains 4N states as does the lower band, called the valence band. The energy gap between these two bands does not contain any available energy states for the electrons to occupy. Thus, this band is called forbidden band.

At absolute zero temperature, the electrons will occupy the lowest energy levels available to them. Therefore 1s band will consists of 2 N electrons, and the remaining 4 N electrons will occupy the valence band. Thus the valence band is completely filled and the conduction band is empty at 0 K.

For the electrons to experience acceleration in an applied electric field, they must be able to move into other energy states. This means that there must be empty states available for the electrons to move. In diamond structure at 0 K, the valence band is completely filled so that no charge transport within the valence band is possible as there is no empty state for the electrons to move. There is no electron in the conduction band and so there is no charge transport in the conduction band. Thus diamond is a perfect insulator at 0 K.

If energy equal to the band gap is given to an electron in the valence band, it gets excited to the conduction band. This creates a vacancy in the valence band. This vacancy in the valence band is called a hole. The electron in the conduction band is called a free electron, because there are plenty of states for the electrons to move. Thus an Electron Hole Pair (EHP) is generated. If the energy used for this type of generation is thermal energy, the generation process is called thermal generation.

At a given temperature the number of electrons and holes generated depends on the band gap. The lower the band gap, the more the EHPs generated at a given temperature. The electrons can easily move in the conduction band resulting in a current flow, if a field is applied.

Similarly, the holes or vacancies in valence band can also provide charge transport in the valence band.

Energy band diagram is a characteristic property of solids. It depends on the coupling of atoms in the solid. Fig. 1.15 represents a typical energy band diagram where E_C represents the minimum energy in the conduction band and E_V represents the maximum energy of electron in the valence band.

$E_C - E_V = E_g$ is the band gap of the semiconductor.

$E_V + \frac{E_g}{2}$ represents the middle of the band gap. E_i is called intrinsic level, which is the Fermi level in an intrinsic semiconductor and is located at the middle of the band gap.

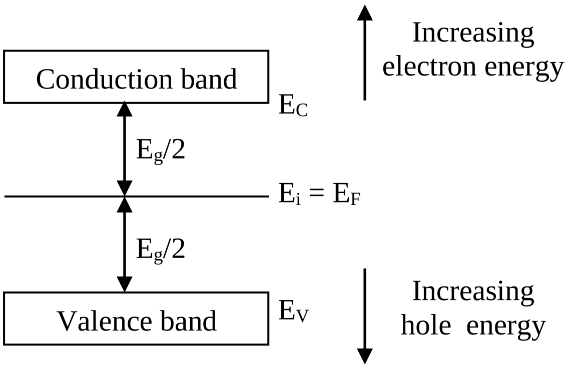


Fig. 1.15 Energy band diagram of intrinsic semiconductor

In the energy band diagram, usually x-axis represents distance (along the direction of carrier movement) and y-axis represents electron energy, i.e., electron energy increases upward and hole energy increases downward.

The upper end of conduction band and lower end of valence band are usually not shown in the energy band diagram, the conduction band minimum (E_C) and valence band maximum (E_V) are sufficient. The energy band diagram is useful as a tool for analysing devices.

1.5.1 Energy Band Diagram of Insulator, Semiconductor and Metal

An insulator has a large band gap, so that the thermally generated carriers are negligible at room temperatures as shown in Fig. 1.16. The band gap of insulators are usually greater than 4 eV.

The energy band structure of semiconductor is similar to that of diamond except for the difference in band gap (Fig. 1.16). The conductivity of semiconductor is also zero at 0 K. Its conductivity can be increased by thermal or optical generation of EHPs. At room temperature, semiconductors have higher conductivity than insulators, due to its smaller band gap. The band gap of a semiconductor is generally less than 4 eV.

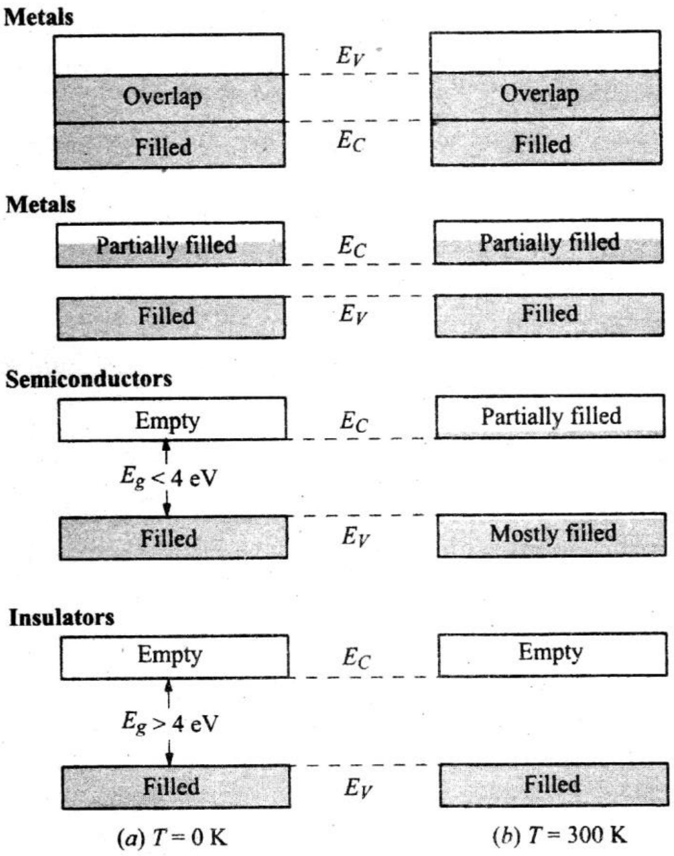


Fig. 1.16 Energy band diagram of metal, semiconductor and insulator

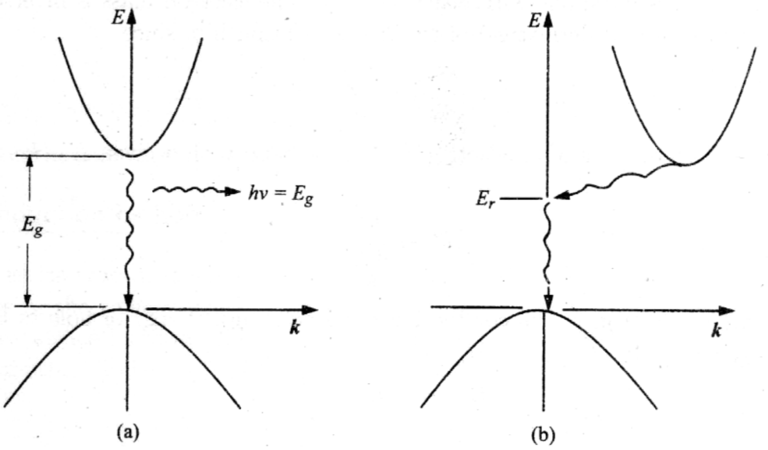
In metals the bands either overlap or are only partially filled. Thus electrons and empty states are mixed in the bands, so that electrons can move freely under the influence of electric of field.

Table 1.5 Band gaps of some insulators and semiconductors

Material	InSb	InAs	Ge	Si	InP	GaAs	GaP	GaN	ZnS	Si_3N_4	Diamond	SiO_2
Band gap	0.18	0.36	0.67	1.11	1.35	1.43	2.26	3.4	3.6	5.0	5.5	9.0

1.5.2. Direct and Indirect Band Gap Semiconductors

The variation of electron energy (E) as a function of wave vector (K) along the principal direction of crystal is shown in figure 1.17(a) and (b). If the K values are equal for an electron with minimum energy in the conduction band (electron at the bottom of the conduction band) and an electron with maximum energy in the valence band (electron at the top of the valence band) as in Fig. 1.17(a), the semiconductor is called direct band gap semiconductor. For direct band gap semiconductor, electrons in the conduction hand minimum and valence band maximum have the same momentum. Therefore an electron in the conduction band can directly recombine with a hole in the valence band, releasing the difference in energy ($E \approx Eg = h\nu$) as a photon. (Electrons in the conduction band usually occupy available states at bottom of conduction band, where the electron energy is minimum and holes occupy available states at top of the valence band where hole energy is minimum.) Such materials are used for making LEDs. e.g., GaAs.



(a) (b)

Fig. 1.17 E- k plots (a) direct bandgap semiconductor (b) indirect bandgap semiconductor

In Fig. 1.17(b) the electron in conduction band minimum and valence band maximum have different values of k. Thus, the momentum values (P = hk) are different for electrons at conduction band minimum and valence band maximum. Therefore, an electron cannot recombine directly with a hole in the valence band by releasing a photon as the momentum cannot be conserved. The transition must follow conservation of energy and momentum.

The momentum of photon is negligibly small, and momentum of the system cannot be conserved by release of photon. In this case, electron first transfers to an intermediate level E_r called recombination centre by release of phonon and from there to the valence band. Semiconductors with different k values for conduction band minimum and valence band maximum, shown in Fig. 1.17(b) are called indirect band gap semiconductors, e.g., Ge, Si, AlAs. In these semiconductors recombination process results in release of energy as heat.

1.5.3 Effective Mass

The electrons in a crystal are not completely free, but interact with the periodic potential of the lattice. Therefore, the "wave particle" motion is not same as that in free space. Thus while applying the usual equations of electrodynamics to charge carriers in solids we must use altered values of mass.

The electron momentum P is given by

$$P = mv = hk$$

$$E = \frac{1}{2}mv^2$$

$$= \frac{P^2}{2m} = \frac{h^2k^2}{2m}$$

Electron energy is parabolic with wave vector k. The electron mass is inversely related to the curvature (second derivative) of the E - k relationship

$$\frac{d^2E}{dk^2} = \frac{h^2}{m}$$

Thus, the effective mass of an electron (m_n^*) in a band with a given E - k relationship is given by

$$m_n^* = \frac{h^2}{\frac{d^2E}{dk^2}} \qquad (1.34)$$

The energy of electron in the conduction band (E_n) and energy of hole in the valence band (E_h) are given by

$$E_n = E_C + \frac{P^2}{2m_n^*} \qquad (1.35)$$

$$E_h = E_V + \frac{P^2}{2m_p^*} \qquad (1.35a)$$

m_p^* is the effective mass of hole.

Kinetic energy of electron at $E = E_C$ and hole at $E = E_V$ are zero.

The energy band diagram of GaAs is shown in Fig. 1.18. It has three conduction band minima L,τ and χ. The curvature $\left(\frac{d^2E}{dk^2}\right)$ is maximum for r minimum. Therefore the effective mass of electron in the r band is much less than that in other two bands.

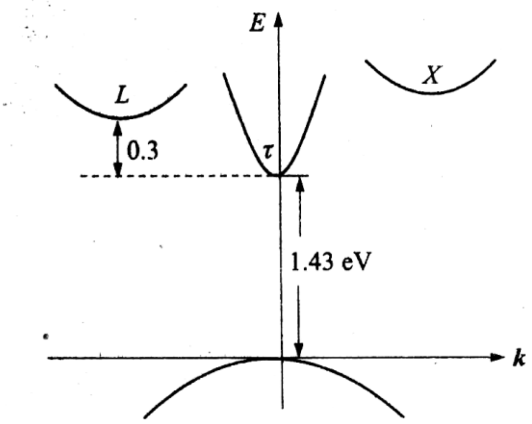


Fig. 1.18 Energy band diagram of GaAs

An electron at conduction band minimum has positive effective mass (positive curvature) and at valence band maximum has negative effective mass (negative curvature). Valence band electrons with negative charge and negative mass move in an electric field in the same direction as holes with positive charge and positive mass. Thus charge transport in valence band can be fully accounted by hole motion.

The effective mass of electrons and holes in various semiconductors are listed in Table 1.6.

Table 1.6

	Ge	Si	GaAs
m_n^*	0.55m$_o$	1.1m$_o$	0.067m$_o$
m_p^*	0.37m$_o$	0.56m$_o$	0.48m$_o$

Note: mo is the free space mass of electron.

1.5.4 Conduction by Holes

Consider a semiconductor specimen of volume V with N semiconductor atoms. There will be 4N states in the valence band and 4N states in the conduction band. At 0 K all the states in the valence band are filled with electrons. For every electron (j) with velocity, v_j, there is another electron (j') with opposite velocity ($-v_j$) as shown in Fig. 1.19.

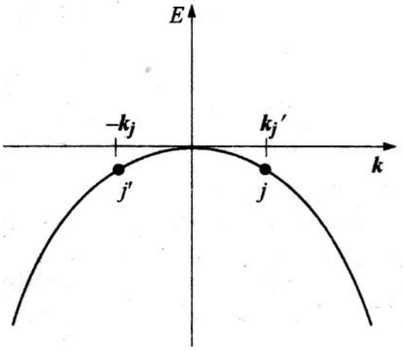


Fig. 1.19 Valence band of semiconductor with all states filled (electrons j and j' are shown)

There fore, the current density due to holes is

$$J = \frac{-q}{V}\sum_{k=1}^{4N}\upsilon_k = 0\,, \quad \text{for filled valence band} \tag{1.36}$$

where V is the volume of semiconductor specimen and v is the velocity of electron.
If jth electron is removed from valence band 4N

$$J = \frac{-q}{V}\left(\sum_{k=1}^{4N}\upsilon_k - \upsilon_j\right) = \frac{-q}{V}(0-\upsilon_j) \tag{1.37}$$

i.e., removing an electron with velocity v_j from the valence band produces a current density in the direction of velocity. This may be considered as hole current density, which is in the direction of movement of hole.

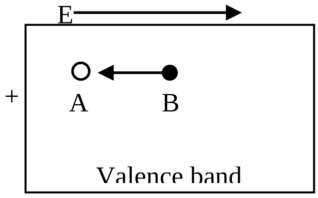


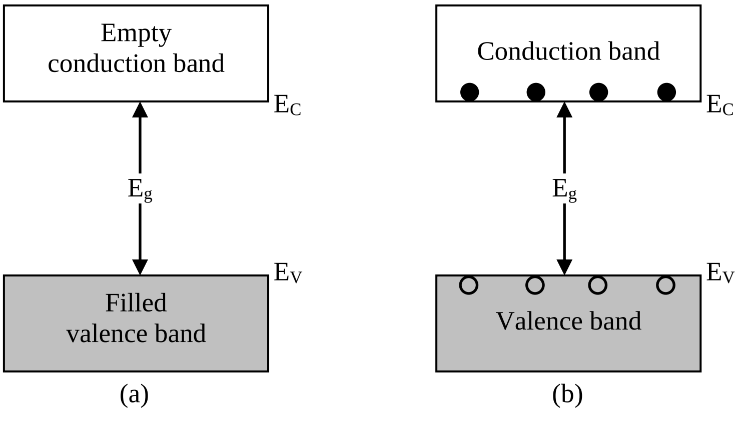
Fig. 1.20 Hole Movement in Valence Band

Consider a vacant state in the valence band or hole at A. When an electric field is applied in the positive x-direction as shown in Fig. 1.20, an electron at B moves into the vacant state at A. Now, the position of hole is transferred from A to B. i.e., hole moves in a direction opposite to the direction of movement of electron. This process continues and hole continues to move in the direction of electric field resulting in a current flow in the direction of hole movement. This current is called hole current since it is due to the presence of holes in the valence band and no free electrons are involved in this conduction process.

1.6 INTRINSIC SEMICONDUCTOR

A semiconductor crystal with no impurities or defects is called an intrinsic semiconductor. At 0 K the conduction band is empty and valence band is completely filled as in Fig. 1.21(a). Thus, there are no mobile charge carriers at 0 K. Hence, the conductivity of an intrinsic semiconductor at absolute zero temperature is zero. But as temperature increases, electrons from the valence band are excited to the conduction band and the conductivity increases with temperature. The conductivity of intrinsic semiconductors decrease with increase in band gap, as more energy is required to excite electron from valence band to conduction band when the band gap is more.

If an electron in the valence band gets energy equal to band gap (E_g) it gets excited from valence band to conduction band. As a result a vacancy is created in the valence band (vacancy in the valence band is nothing but a hole) and an electron in the conduction band. i.e., an Electron-Hole Pair (EHP) is generated. Generation due to thermal excitation of electrons from valence band to conduction band is called intrinsic generation.



(a) (b)

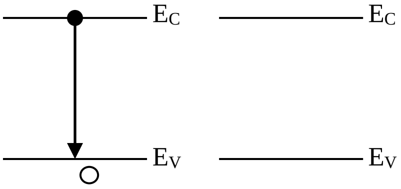
Fig. 1.21 Energy band diagram of intrinsic semiconductor

Since electrons and holes are generated in pair, the concentration of electrons in the conduction band and holes in the valence band are equal as shown in Fig. 1.21(b). The concentration of electrons or holes in the intrinsic semiconductor under thermal equilibrium (n_o, p_o) is referred to as intrinsic carrier concentration (n_i). For an intrinsic semiconductor,

$$n_o = p_o = n_i$$

Intrinsic carrier concentration is a function of temperature. As temperature increases thermal generation rate increases, resulting in the increase of intrinsic carrier concentration.

Recombination is the opposite process of generation. When an electron in the conduction band falls to a vacancy in the valence band, by releasing energy, an electron in the conduction band and a hole in the valence band vanish as shown by Fig. 1.22. This process is called recombination of EHP.



(a) Before (b) After

Fig. 1.22 Recombination of electron hole pair

Under thermal equilibrium, the rate of generation and rate of recombination are equal in any semiconductor so that carrier concentrations remain steady. The thermal generation rate at temperature T is given by $g(T) = \alpha_r n_i^2$. Recombination rate (r) is proportional to the product of electron and hole concentrations and is expressed as

$$r = \alpha_r np$$

where α_r - is a proportionality constant depending on the recombination mechanism
n - total electron concentration
p - total hole concentration

At thermal equilibrium, $n = n_o$ and $p = p_o$

$$\therefore \quad r = \alpha_r n_o p_o$$

where n_o - electron concentration under thermal equilibrium
p_o - hole concentration under thermal equilibrium

For intrinsic semiconductor $n_o = p_o = n_i$

$$\therefore \quad r_i = \alpha_r . \, n_i^2 = g(T)$$

where g(T) represents the thermal generation rate.

1.6.1 Valence Bond Model

Each atom of semiconductor (Si) shares its four valence electrons with four other atoms, so that all atoms get a stable structure with 8 electrons in valence shell as shown in Fig. 1.23 All the electrons are tightly bound to the nucleus at 0 K. No free electron or hole exists at 0 K. If energy equal to binding energy is given, electrons come out of the bond, breaking the bonds. (This energy is equivalent to the energy band gap). Now, these electrons become part of the crystal as a whole and are called free electrons. The broken bond is called a hole. As temperature increases more and more electrons break the bonds resulting in generation of more EHPs.

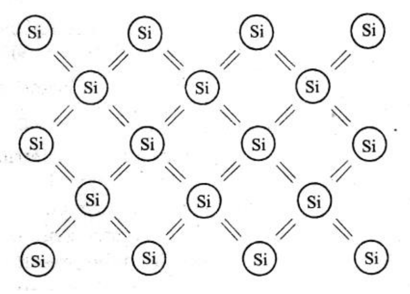


Fig. 1.23 Valence bond model of intrinsic semiconductor

1.7 EXTRINSIC SEMICONDUCTOR

In a semiconductor, charge carriers may be purposely introduced by adding impurities to it. This process is called doping. Depending on the type of impurities, the doped semi conductor may be n-type or p-type. In a doped semiconductor, the electron concentration (n_o) and hole concentration (p_o) are different ($n_o \neq p_o \neq n_i$). Such a semiconductor is called extrinsic semiconductor. When impurities are added to an otherwise pure semiconductor, additional energy levels are introduced in the band structure of the semiconductor. Usually, these energy levels lie within the band gap.

1.7.1 n-type Semiconductors

A semiconductor doped with a pentavalent impurity is called n-type semiconductor. Pentavalent impurity introduces energy level in the band gap, close to the conduction band edge as shown in Fig. 1.24. At 0 K these levels are completely occupied by electrons (5th electron of the pentavalent impurity at absolute zero temperature occupy this level). These levels donate free electrons to the semiconductor on increasing the temperature slightly. So, these levels are called donor levels. The energy difference between conduction band and donor level (E_C - E_D) is called ionization energy. (The ionization energy is defined as the energy required to ionize a donor or an acceptor atom).

The ionization energy is very small (of the order of meV), so that very small energy sufficient to excite electrons from donor level to conduction band. Therefore, at room temperature all the electrons in the donor level are excited to the conduction band. Each pentavalent impurity atom donates a free electron to the semiconductor. So, pentavalent impurities are also known as donor impurities.

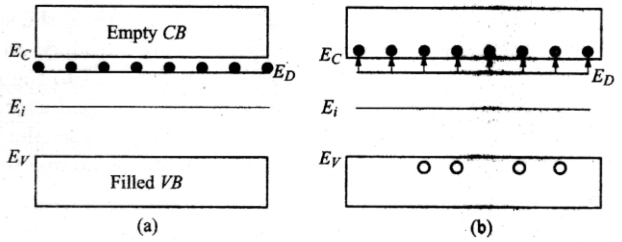


Fig. 1.24 Energy band diagram of n-type semiconductor (a) at 0 K (b) at room temperature

Due to doping, electron concentration increases from intrinsic carrier concentration (n_i) to a value approximately equal to the donor impurity concentration (doping concentration). Increase in electron concentration results in an increased recombination rate, [$r \alpha np$], thus reducing electron and hole concentrations. Under thermal equilibrium, generation and recombination rates balances each other, resulting in a reduced concentration of holes compared to the intrinsic value. Thus in an n-type semiconductor $n_o > n_i$ and $p_o < n_i$. Hence, electrons are referred to as majority carriers and holes as minority carriers in an n-type semiconductor.

The thermal generation rate in a semiconductor material at a given temperature is a constant, independent of doping. Therefore, for n-type semiconductor

$$\text{thermal generation rate } g(T) = \alpha_r n_i^2$$

$$\text{recombination rate } r = \alpha_r np$$

$$= \alpha_r n_o p_o \text{ under thermal equilibrium.}$$

The recombination rate balances with generation rate under thermal equilibrium.

$$\therefore \quad r = g(T)$$

$$\text{i.e.,} \quad \alpha_r n_o p_o = \alpha_r n_i^2$$

$$\text{or} \quad n_o p_o = n_i^2$$

Table 1.7 Ionization Energies of Impurities in Silicon

Element	Type of level	Ionization energy (eV)
Boron	Acceptor	0.045
Aluminium.	Acceptor	0.057
Gallium	Acceptor	0.065
Phosphorous	Donor	0.044
Arsenic	Donor	0.049
Antimony	Donor	0.039

Valence Bond Model

When a pentavalent impurity is added to the silicon crystal, one of the silicon atoms is replaced, by an impurity atom as shown in Fig. 1.25. This introduces an excess of one electron after completing the bond. At 0 K this electron is bound to the impurity atom itself and is not free to move. So, the conductivity of n-type semiconductor is zero at 0 K. If thermal energy equal to binding energy is supplied to this electron, the impurity atom gets ionized and electron becomes free (not bound to any of the atoms) to participate in the conduction process.

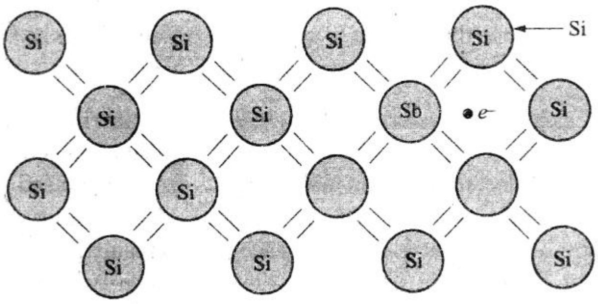
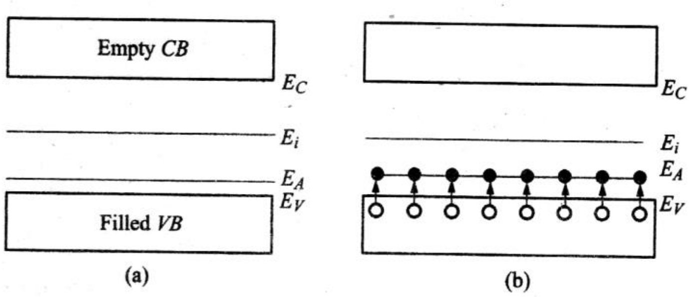


Fig. 1.25 Valence Bond Model of n-Type Semiconductor

1.7.2 p-type Semiconductor

A semiconductor doped with trivalent impurities like Boron, Gallium etc., forms a p-type semiconductor. These impurities introduces energy levels in the band gap of silicon, germanium etc., near the valence band edge as shown in Fig. 1.26. These energy levels are vacant at 0 K. Thus, these levels can accept electrons from the valence band and are called acceptor levels (E_A).

The energy difference between acceptor level and valence band edge (E_A - Ev) is very small (~ meV). So, as the temperature increases slightly (~100-150 K), electrons excited from valence band to acceptor level creating holes in the valence band. The hole concentration increases to the acceptor doping concentration (N_A). As the hole concentration increases, recombination rate increases. This results in the reduction of electron and hole concentrations. Under thermal equilibrium, generation rate ($\alpha_r n_i^2$) and recombination rate ($\alpha_r n_o p_o$) balances each other, so that $p_o > n_i$ and $n_o < n_i$. Thus, holes are majority carriers and electrons are minority carriers in a p-type semiconductor.



(a) (b)

Fig. 1.26 Energy band diagram of p-type semiconductor (a) at 0 K (b) at room temperature

Generation process by thermal excitation of electrons from donor level to conduction band or valence band to acceptor level is called extrinsic generation. In extrinsic generation only majority carriers are generated.

There are impurities, which can act as donors as well as acceptors. For example, silicon in GaAs usually acts as a donor impurity when a silicon atom replaces a Ga atom. But, if there is an excess of As vacancies during growth process, Si impurities may sit on As sites. In that case, silicon acts as an acceptor impurity. This type of impurity, which can serve as acceptor as well as donor is called amphoteric impurity.

1.8 CARRIER CONCENTRATION

Free electrons and holes are the charge carriers in a semiconductor. To obtain expression for carrier concentration we must investigate the distribution of carriers over the available energy states. For this we can use Fermi-Dirac distribution function, as electrons in semiconductors obey Fermi-Dirac statistics.

The distribution of electrons over a range of allowed energy levels at thermal equilibrium is given by

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

where, k is Boltzmann's constant = 8.62×10^{-5} eV/K = 1.38×10^{-23}J/K

T is absolute temperature

E_F is Fermi level

f(E) is Fermi-Dirac distribution function.

f(E) represents the probability that an available energy state at E is occupied by an electron at absolute temperature T under thermal equilibrium. (Thermal equilibrium refers to a semiconductor specimen not acted upon by any external force. Under thermal equilibrium the only generation process is thermal generation. Thermal generation and recombination rates balance each other under equilibrium.)

Probability of occupancy of Fermi level in any material under thermal equilibrium is $\frac{1}{2}$, for r > 0. Thus, if an energy level is available at the Fermi level, its probability of occupancy will be half. The Fermi-Dirac distribution function f(E) has a rectangular distribution at absolute zero temperature as shown in Fig. 1.27.

At T = 0K and $E < E_F$; $f(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$

At T = 0K and $E > E_F$; $f(E) = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = 0$

This implies that at 0 K all available energy states up to E_F are filled with electrons and all the available states above E_F are empty. At T > 0K some probability exists for states above E_F to be filled and states below E_F to be vacant. This is illustrated in Fig. 1.27. Fermi Dirac distribution function has a fixed shape with respect to the Fermi level at a given temperature.

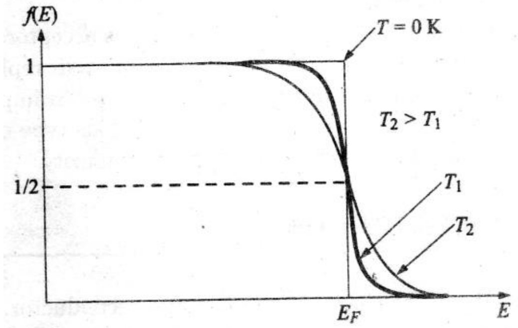


Fig. 1.27 Fermi-Dirac distribution function f(E) vs E for different temperatures

While applying Fermi-Dirac distribution function to semiconductors it should be noted that f(E) is the probability of occupancy of an available energy state at E. Thus, if there is no available state at E (e.g., in the band gap of a semiconductor) there is no possibility of finding an electron at E.

Example 1.4 Show that the probability of a state ΔE above E_F is filled equals probability that a state ΔE below E_F is empty. (i.e., f(E) vs. E diagram is antisymmetric about $E = E_F$).

Solution

Probability that a state ΔE above E_F is filled

$$= f(E_F + \Delta E) = \frac{1}{1+e^{(E_F+\Delta E-E_{fF})/kT}}$$

$$= \frac{1}{1+e^{\Delta E/kT}} \qquad \text{(Ex.1.4a)}$$

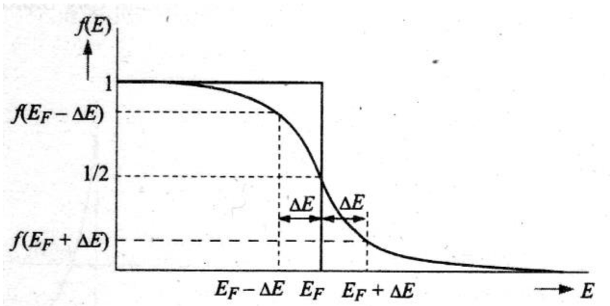


Fig. Ex. 1.4 Fermi-Dirac Distribution Function

Probability that a state ΔE below E_F is filled

$$= f(E_F - \Delta E)$$

Probability that a state ΔE below E_F is vacant

$$= 1 - f(E_F - \Delta E)$$

$$= 1 - \frac{1}{1+e^{(E_F-\Delta E-E_F)/kT}}$$

$$= 1 - \frac{1}{1+e^{-\Delta E/kT}} = \frac{e^{-\Delta E/kT}}{1+e^{-\Delta E/kT}}$$

Dividing numerator and denominator with $e^{-\Delta E/kT}$

$$1 - f(E_F - \Delta E) = \frac{1}{1+e^{\Delta E/kT}} \qquad \text{(Ex.l.4b)}$$

From equations (Ex.l.4a) and (Ex.l.4b), the probability of a state ΔE above E_F is filled equals the probability that a state ΔE below E_F is empty. Or the Fermi dirac distribution function is antisymmetric with respect to the Fermi level.

Example 1.5. Determine, a. the probability of occupancy of following energy levels at 0 K, 300 K and 600 K;

(i) 0.26 eV above E_F

(ii) 0.52 eV above E_F

b. the probability of vacancy of the following levels at 0 K, 300 K and 600 K

(i) 0.26 eV below E_F

(ii) 0.52 eV below E_F

Solution

a. Probability of Occupancy

at 0 K

(i) $f(E_F + 0.26) = 0$

(ii) $f(E_F + 0.52) = 0$

at 300 K, kT = 0.026 eV

(i) $f(E_F + 0.26) = \dfrac{1}{1+e^{\frac{0.26}{0.026}}} = 4.54 \times 10^{-5}$

(ii) $f(E_F + 0.52) = \dfrac{1}{1+e^{\frac{0.52}{0.026}}} = 2.061 \times 10^{-9}$

at 600 K, kT = 0.052 eV

(i) $f(E_F + 0.26) = \dfrac{1}{1+e^{\frac{0.26}{0.052}}} = 6.693 \times 10^{-3}$

(ii) $f(E_F + 0.52) = \dfrac{1}{1+e^{\frac{0.52}{0.052}}} = 4.54 \times 10^{-5}$

1 + e 0.052

b. Probability of Vacancy

at 0 K

(i) $1 - f(E_F - 0.26) = 0$

(ii) $1 - f(E_F - 0.52) = 0$

at 300 K

(i) $1 - f(E_F - 0.26) = \dfrac{1}{1+e^{\frac{-0.26}{0.026}}} = 4.54 \times 10^{-5}$

(ii) $1 - f(E_F - 0.52) = \dfrac{1}{1+e^{\frac{-0.52}{0.026}}} = 2.061 \times 10^{-9}$

at 600 K

(i) $1 - f(E_F - 0.26) = \dfrac{1}{1+e^{\frac{-0.52}{0.026}}} = 6.693 \times 10^{-3}$

(ii) $1 - f(E_F - 0.52) = \dfrac{1}{1+e^{\frac{-0.52}{0.052}}} = 4.54 \times 10^{-5}$

Remark: The symmetry of f(E) vs E plot with reference to E_F can be observed in the above results.

1.8.1 Fermi-Dirac Distribution Function Applied to Semiconductors

Fig. 1.28 shows the energy band diagram of an intrinsic semiconductor along with f(E) with common energy axis. For intrinsic semiconductors, probability of occupancy in conduction band equals the probability of vacancy in valence band. i.e., $1 - f(E_V) = f(E_C)$. Because of the symmetry of f(E) vs E diagram, to satisfy the above condition E_F must be at the middle of the band gap. i.e. the Fermi level in an intrinsic semiconductor is at the middle of the band gap and is called the intrinsic level.

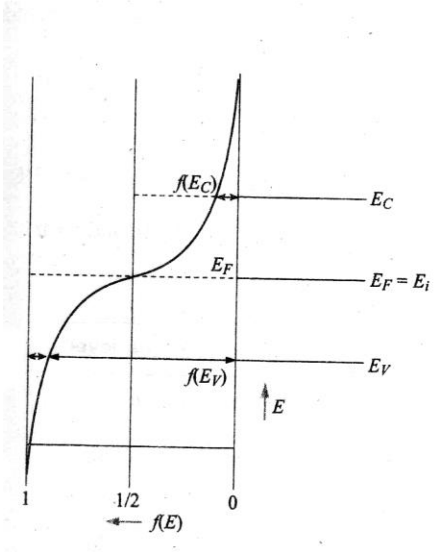


Fig. 1.28 Relative plot of Fenni-Dirac distribution function and energy band diagram of intrinsic semiconductor

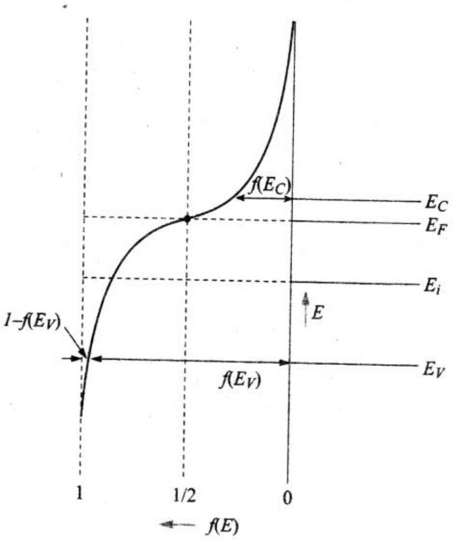


Fig. 1.29 Energy band diagram along with f(E) for n-type semiconductor

For an n-type semiconductor probability of occupancy in the conduction band is much greater than the probability of vacancy in the valence band. i.e., $f(E_C) >> 1 - f(E_V)$. To satisfy this condition E_F must be above the middle of the band gap as shown in Fig. 1.29. As doping increases E_F moves towards E_C.

Similarly for a p-type semiconductor, probability of vacancy in valence band is much greater than probability of occupancy in the conduction band at room temperature. The shape of f(E) vs E diagram remains, unchanged for a given temperature. It changes only with change in temperature. Therefore, Fermi level must be closer to E_V compared to E_C as shown in Fig. 1.30.

In short, position of Fermi level is an indication of carrier concentrations in a semiconductor.

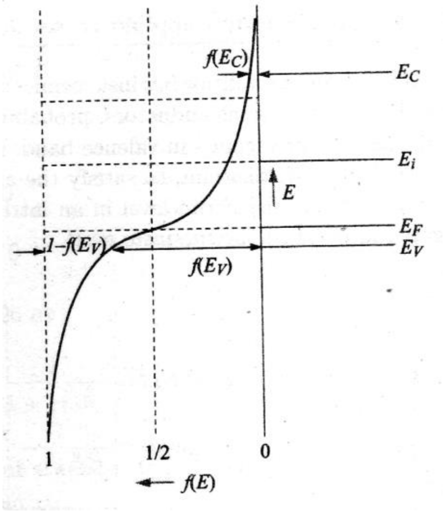


Fig. 1.30 Energy band diagram along with f(E) for p-type semiconductor

Table 1.8

Type of semiconductor	**Position of Fermi level**
1. Intrinsic semiconductor	At the middle of the band gap (E_i)
2. n-type semiconductor	Above E_i E_F moves up towards E_C with increase in electron concentration.
3. p-type semiconductor	Below E_i E_F moves down towards with increase in hole concentration.

1.8.2. Electron and Hole Concentration at Equilibrium

The Fermi-Dirac distribution function can be used to evaluate electron and hole concentrations in a semiconductor. The concentration of electrons in the conduction band is given by

$$n_o = \int_{E_C}^{\infty} f(E)\, N(E) dE \qquad (1.38)$$

where N(E)dE represents the density of states in the energy range dE. The number of electrons per unit volume in the energy range dE is the product of density of states N(E)dE and probability of occupancy f(E). Thus, the total electron concentration in the conduction band (n_o) is the integral of product of density of states and probability of occupancy over the entire conduction band as given by equation (1.38).

N(E) can be evaluated using quantum mechanics and Pauli's exclusion principle. It can be shown that N(E) α $E^{\frac{1}{2}}$ i.e., density of states in conduction band increases with increase in electron energy. But f(E) decreases exponentially with increase in energy and becomes extremely small at large energies, i.e., f(E)N(E) decrease rapidly above E_C and very few electrons occupy far above E_C. Therefore, most of the electrons occupy the lowest available energy levels in the conduction band. 1 - f(E_V) is the probability of vacancy (probability of ccupancy of hole) at E_V . 1 - f(E) decrease rapidly below E_V i.e., holes occupy top most available states in the valence band.

$$p_o = \int_{E_V}^{\infty} [1 - f(E)]\, N(E) dE \tag{1.39}$$

Fig.1.31 makes a graphical estimate of carrier concentrations in intrinsic semiconductor.

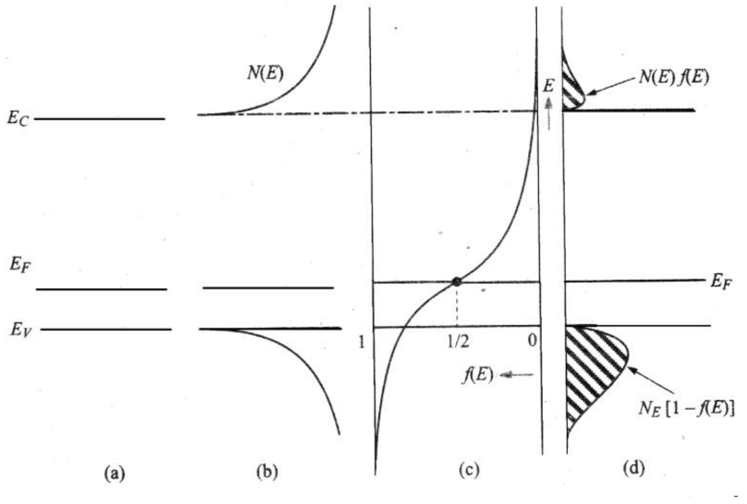


Fig. 1.31 Graphical evaluation of carrier concentrations (intrinsic semiconductor) (a) Energy band diagram (b) Variation of density of states with energy (c) Fermi-Dirac distribution function (d) Carrier concentrations

Area under f(E) N(E) in the conduction band is a measure of electron concentration. Similarly, area under N(E)[l - f(E)] in the valence band is a measure of hole concentration a the valence band. For intrinsic semiconductor, these quantities are equal as is evident from Fig. 1.31.

For n-type semiconductor, N(E) f(E) in the conduction band has a larger area than N_V[1-f(E)] in the valence band i.e., $n_0 >> p_0$ (Fig. 1.33). Similarly, for a p-type semiconductor N(E)f(E) in the conduction band is much less than N(E)[1 - f(E)] in the valence band. s$_0$ $p_0 >> n_0$. (Fig. 1.32)

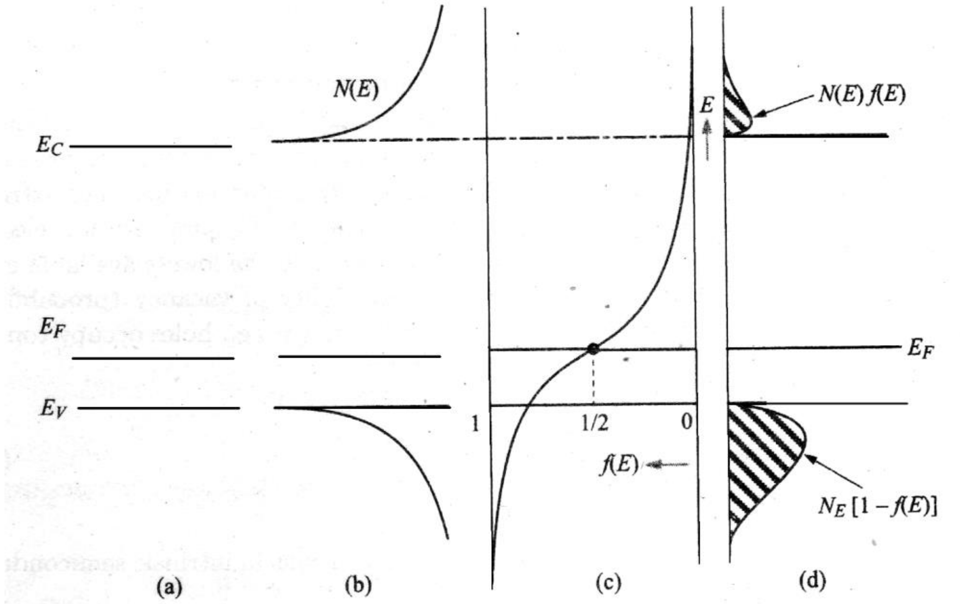


Fig. 1.32 Graphical evaluation of carrier concentrations in a p-type semiconductor

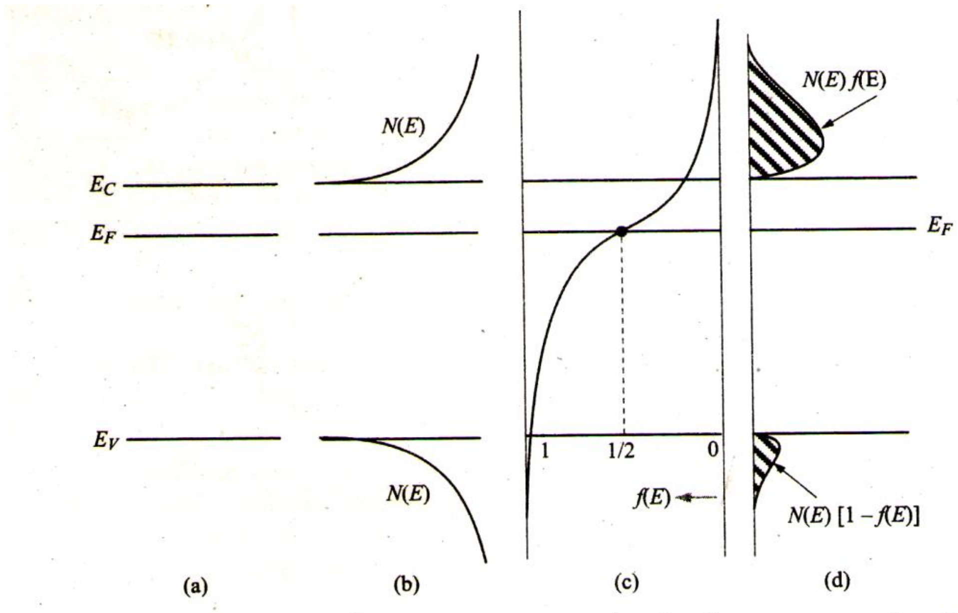


Fig. 1.33 Graphical evaluation of carrier concentration in a n-type semiconductor

The electron concentration in the conduction band can be obtained by representing the distributed electron states in the conduction band by an effective density of states (N_C) located at the conduction band edge E_C. The effective density of states in the conduction band (N_C) is given by

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}} \tag{1.40}$$

In terms of effective density of states (N_C), the electron concentration in the conduction band is given as

$$n_0 = N_C\, f(E_C) \tag{1.41}$$

i.e., the electron concentration in the conduction band equals the product of effective density of states at E_C. and the probability of occupancy at E_C.

$$f(E_C) = \frac{1}{1+e^{(E_C-E_F)/kT}} \tag{1.42}$$

if $E_C - E_F >> kT$, i.e., if electron concentration is not very high, the above equation reduces to

$$f(E_C) = \frac{1}{e^{(E_C-E_F)/kT}} = e^{-(E_C-E_F)/kT} \tag{1.43}$$

i.e., for lightly doped semiconductor, Fermi-Dirac distribution function decays to Maxwell-Boltzmann distribution function

$$\therefore \quad n_0 = N_C f(E_C) = N_C e^{-(E_C-E_F)/kT} \tag{1.44}$$

Similarly, the concentration of holes in the valence band is given by

$$p_0 = N_V[1 - f(E_V)] \tag{1.45}$$

$$\text{where} \qquad N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}} \tag{1.46}$$

is the effective density of states in the valence band.

1 - $f(E_V)$ is the probability of vacancy in the valence band.

$$1 - f(E_V) = 1 - \frac{1}{1+e^{(E_V-E_F)/kT}}$$

$$= \frac{e^{(E_V-E_F)/kT}}{1+e^{(E_V-E_F)/kT}}$$

if $E_F - E_V >> kT$, i.e., if hole concentration is not very high

$$1 - f(E_V) = e^{(E_V-E_F)/kT} = e^{-(E_F-E_V)/kT} \tag{1.47}$$

Substituting equation (1.47) in equation (1.45)

$$p_0 = N_V\, e^{-(E_F-E_V)/kT} \tag{1.48}$$

For intrinsic semiconductor, equations (1.44) and (1.48) can be modified to

$$n_0 = n_i = N_C\, e^{-(E_C - E_i)/kT} \quad (\therefore\ E_F = E_i) \qquad (1.49)$$

$$p_0 = n_i = N_V\, e^{-(E_i - E_V)/kT} \qquad (1.50)$$

where E_i represent the Fermi level in intrinsic semiconductor.

$\therefore$ by equations (1.49) and (1.50) for intrinsic semiconductor, at temperature T,

$$n_0 p_0 = n_i^2 = N_C N_V\, e^{-(E_C - E_V)/kT}$$

$$= N_C N_V\, e^{-E_g/kT} \qquad (1.51)$$

From equations (1.44) and (1.48), for a semiconductor with any temperature T,

$$n_0 p_0 = N_C N_V\, e^{-(E_C - E_V)/kT}$$

$$= N_C N_V\, e^{-E_g/kT} \qquad (1.52)$$

From equations (1.51) and (1.52), $n_0 p_0 = n_i^2$ for any semiconductor under thermal equilibrium. This relation is called mass action law.

Mass action law states that at a given temperature, the product of equilibrium electron and hole concentrations in a semiconductor is a constant for any doping and is equal to the square of the intrinsic carrier concentration at that temperature.

Substituting value of N_C in terms of n_i, from equations (1.50) in (1.45), we get

$$n_0 = n_i\, e^{E_F - E_i/kT} \qquad (1.53)$$

and substituting value of N_V in terms of n, from equation (1.50) in equation (1.48), we get

$$p_0 = n_i\, e^{E_i - E_F/kT} \qquad (1.54)$$

Equations (1.53) and (1.54) relate Fermi level position and carrier concentrations in a semiconductor. These equations are valid for semiconductors with low doping ($E_C - E_F >> kT$ and $E_F - E_V >> kT$). Under low doping, Fermi-Dirac statistics may be replaced by Maxwell-Boltzmann statistics. Physically, this means that under low doping number of mobile charge carriers in the bands are so small that only a small fraction of available states are occupied. This is known as diluteness condition, because the electrons (solute) form a dilute solution in the available energy states (solvent). A dilute electron gas is said to be non-degenerate, and the resulting semiconductor is called non-degenerate semiconductor.

1.8.3 Temperature Dependence of Intrinsic Carrier Concentration

By equation (1.51) intrinsic carrier concentration (n_i) is

$$n_i(T) = \sqrt{N_C N_V}\, e^{-E_g/2kT} \qquad (1.55)$$

Substituting values for N_C and N_V,

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{\frac{3}{2}} \left(m_n^* m_p^*\right)^{\frac{3}{4}} e^{-E_g/2kT} \qquad (1.56)$$

$$= K_1 T^{\frac{3}{2}} e^{-E_g/2kT} \;| \qquad (1.57)$$

In silicon and germanium, the band gap decreases monotonically with increases in temperature and can be expressed as

$$E_g(T) = E_{g_0} - b_1 T \qquad (1.58)$$

where, E_{g_0} is the interpolated value of band gap at 0 K

b_1 is the rate of decrease of band gap with increase in temperature.

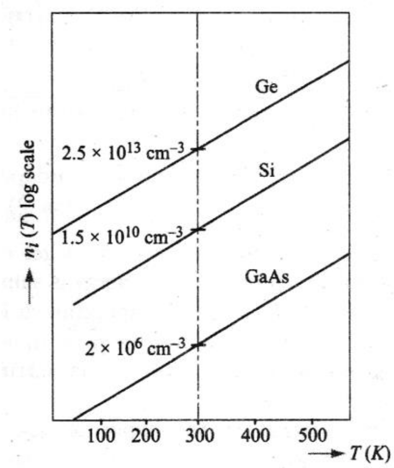
Using equation (1.58), $n_i(T)$ becomes

$$n_i(T) = K_2 T^{\frac{3}{2}} e^{-E_{g0}/2kT} \qquad (1.59)$$

where $$K_2 = K_1 e^{b_1/2k} \qquad (1.60)$$

i.e., $n_i \,\alpha\, T^{\frac{3}{2}}$ and $n_i \,\alpha\, e^{-E_{g0}/2kT}$

The exponential dependence is stronger than the other one and the intrinsic carrier concentration increases exponentially with increase in temperature.



1.34 Approximate plot of variation of intrinsic carrier concentration with temperature for GaAs, Si and Ge

An approximate plot of intrinsic carrier concentration, as a function of temperature, is given in Fig. 1.34 for GaAs, Si and Ge. It can be shown that the intrinsic carrier concentration is minimum for GaAs and maximum for Ge at a given temperature. This is because of the wider band gap of GaAs and lower band gap of Ge.

Following empirical relationships are valid for temperature above 50 K.

$$n_i(T) = 1.76 \times 10^{16}\, T^{\frac{3}{2}} e^{\frac{-4550}{T}}\, cm^{-3} \quad \text{for Ge} \qquad (1.61)$$

$$n_i(T) = 3.88 \times 10^{16}\, T^{\frac{3}{2}} e^{\frac{-7000}{T}}\, cm^{-3} \quad \text{for Si} \qquad (1.62)$$

[Note: Conductivity of intrinsic semiconductor also varies with temperature in a similar manner.]

1.8.4 Temperature Dependence of Majority Carrier Concentration in Extrinsic Semiconductor

Consider an n-type semiconductor with doping concentration $N_D = 10^{15}$ cm^{-3}. At 0 K, the carrier concentrations are zero as there is no band to band generation or extrinsic generation.

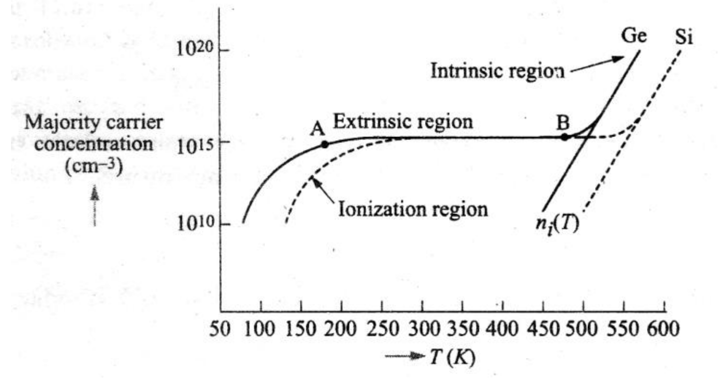


Fig. 1.35 Variation of majority carrier concentration in a doped semiconductor (n-type semiconductor with $N_D = 10^{15}$ cm^{-3})

As temperature increases, electrons in the donor level get excited to the conduction band (extrinsic generation). For this process small energy is sufficient. As a result the free electron concentration increases with increase in temperature at lower range of temperature. The electron concentration in this region is given by $n_0 = n_i + N_D^+ \cong N_D$; i.e., electron concentration equals ionized donor concentration, as the intrinsic carrier concentration (n_i) is negligible at these temperatures.

At temperature of around 150 K all the impurities are ionized, so that the electron concentration in the conduction band become equal to the donor concentration ($n_0(T) = n_i(T) + N_D^+ \cong N_D$; point A in Fig. 1.35) as intrinsic carrier concentration at this temperature is negligibly small ($n_i(T) << N_D$).

As temperature increases further, the electron concentration remains almost unchanged at N_D, until the band to band generation (n_i,(T)) become comparable to the impurity ionisation (N_D).

At a particular temperature the band to band generation [n_i,(T)] become comparable to the impurity ionisation (point B in Fig. 1.35).

$$\therefore \qquad n_0(T) = n_i(T) + N_D$$

After this temperature, majority carrier concentration increases with increase in temperature. In this region, as band to band generation exceeds impurity ionisation, minority carrier concentration has a value comparable to majority carrier concentration ($n_o = n_i + N_D \cong n_i$) and $p_o \cong n_i$. The temperature dependence of n_o is shown in Fig. 1.35. It consists of three regions:

1. Ionization region: In this region the majority carrier concentration increases with increase in temperature due to ionisation of impurities.
2. Extrinsic region: In this region majority carrier concentration remains constant and minority carrier concentration is negligible. The majority carrier concentration and the

conductivity of the semiconductor is almost' independent of temperature in this legion. A device can be operated satisfactorily, only in this range of temperature. This shows that the doping process makes the conductivity of a semiconductor almost independent of temperature, over a wide range of temperature. The wider the band gap (or lower n_i) of a semiconductor, the higher will be the extrinsic limit of temperature. For extrinsic behaviour, majority carrier concentration must be atleast 5 n_i.

3. Intrinsic region: At higher values of temperature the intrinsic carrier concentration become comparable to or greater than the doping concentration. Therefore, in this region, majority as well as minority carrier concentration increases with increase in temperature and the semiconductor behaves like an intrinsic semiconductor.

Example 1.6 A semiconductor device requires n-type material. It is to be operated at 455 K. Silicon doped with 5×10^{15} As atoms/cm^3 and Ge doped with 5×10^{15} Sb atoms/cm^3 are available. Which one of these materials can be used for fabricating the device?

Solution

For extrinsic behaviour $n_0 > 5n_i$

At 455 K, n_i for Si $\quad n_i = 3.88 \times 10^{16} T^{\frac{3}{2}} . e^{-7000/T}$

$$= 7.84 \times 10^{13} \text{ cm}^{-3}$$

$$5n_i = 3.92 \times 10^{14} \text{ cm}^{-3}$$

$$n_o > 5n_i$$

Therefore, silicon doped with 5×10^{15} Arsenic atoms/cm^3 can be used for fabricating the device.

At 455 K, n_i for Ge $\quad n_i = 1.76 \times 10^{16}\ T^{\frac{3}{2}} . e^{-4500/T}$

$$= 7.755 \times 10^{15} \text{ cm}^{-3}$$

$$5n_i = 3.878 \times 10^{16} \text{ cm}^{-3}$$

$$n_0 \ngtr 5n_i$$

Therefore, Germanium doped with 5×10^{15} Antimony atoms/cm3 cannot be used as n-type material at 455 K.

1.8.5 Equilibrium Electron Hole Concentrations and Charge Neutrality

Consider a homogeneous non-degenerate semiconductor with uniform doping of N_D donors/cm^3 and N_A acceptors/cm^3. Under thermal equilibrium the semiconductor as a whole remains neutral, i.e., total positive charge = total negative charge.

$$p_o + N_D^+ = n_0 + N_A^- \qquad (1.63a)$$

or

$$n_o - p_o = N_D^+ - N_A^- \qquad (1.63b)$$

where N_D^+ represents ionized donor concentration.

N_A^- represents ionized acceptor concentration.

Equation (1.63) is called charge neutrality equation. Above 150 K almost all the impurities get ionized in a non-degenerate semiconductor, so that

$$N_D^+ = N_D \text{ and } N_A^- = N_A$$

Therefore, equation (1.63b) becomes

$$n_o - p_o = N_D - N_A \qquad (1.64)$$

We also have the equation,

$$n_o p_o = n_1^2 \tag{1.65}$$

Equations (1.64) and (1.65) can be solved to get carrier concentrations (n_o and p_o) in a semiconductor under thermal equilibrium.

If a semiconductor is doped with both donors and acceptors, the net doping is equal to the difference between the two. i.e., if $N_A > N_D$, the resulting semiconductor is n-type and if $N_A > N_D$, the resulting semiconductor is p-type.

If net doping is much greater than n_i, the majority carrier concentration equals net doping. If net doping is comparable to n_i, solve equations (1.64) and (1.65) for majority carrier concentrations. In both cases, the minority carrier concentration is evaluated from the expression $n_o p_o = n_i^2$ Examples 1.7 to 1.9 illustrate the procedure of determining carrier concentrations.

Example 1.7 A silicon sample is doped with 5×10^{16} As atoms/cm^3 and 2×10^{16} B atoms/cm^3. Determine a. electron and hole concentrations at room temperature b. the position of Fermi level. Assume n. for silicon at room temperature as 1.5×10^{10} cm^{-3}.

(a) $N_D = 5 \times 10^{16}\text{cm}^{-3}$, $N_A = 2 \times 10^{16}$ cm^{-3}

$$\begin{aligned} \text{Net doping} &= N_D - N_A \\ &= 5 \times 10^{16} - 2 \times 10^{16} \\ &= 3 \times 10^{16}\ \text{cm}^{-3} \end{aligned}$$

$N_D > N_A$ ∴ Semiconductor is n-type

Net doping $N_D - N_A >> n_i$.

Therefore, majority carrier concentration ≅ net doping

i.e., $n_o = N_D - N_A = 3 \times 10^{16}$ cm^{-3}

$$p_o = \frac{n_i^2}{n_0} = \frac{2.25\times10^{20}}{3\times10^{16}}$$

$$= 7.5 \times 10^3\ \text{cm}^{-3}$$

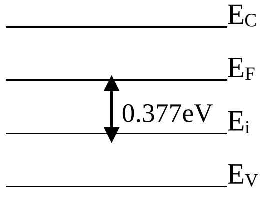


Fig. Ex.1.7

(b) $n_o = n_i\, e^{E_F - E_i/kT}$

$$\frac{n_0}{n_i} = e^{(E_F - E_i)/kT}$$

$$\frac{(E_F - E_i)}{kT} = \ln\frac{n_0}{n_i}$$

Taking natural logarithm,

$$E_F - E_i = kT\ln\frac{n_0}{n_i}$$

$$kT = 26 \text{ meV at room temperature}$$

$$\therefore E_F - E_i = 0.026\times\ln\frac{3\times10^{16}}{1.5\times10^{10}}$$

$$= 0.377 \text{ eV.}$$

The Fermi level position is shown in Fig. Ex.1.7.

Example 1.8 A Ge sample is doped with 10^{17} Boron atoms per cm^3. Determine the carrier concentrations and Fermi level position at room temperature. n_i for Ge = 2.5×10^{13} cm^{-3} at room temperature.

Solution

$$N_A = 10^{17} \text{ cm}^{-3}$$

$$n_i = 2.5 \times 10^{13} \text{ cm}^{-3},\ N_A >> n_i$$

Therefore, majority carrier concentration = doping concentration.

i.e., $p_0 = N_A = 10^{17}$ cm^{-3} and

$$n_o = \frac{n_i^2}{p_0}$$

$$= \frac{(2.5\times10^{13})^2}{10^{17}} = 6.25 \times 10^9 \text{ cm}^{-3}$$

$$p_0 = n_i\, e^{E_F - E_i / kT}$$

$$E_i - E_F = kT\ln\frac{p_0}{n_i}$$

$$= 0.026\times\ln\frac{10^{17}}{2.5\times10^{13}}$$

$$= 0.2156 \text{ eV.}$$

Example 1.9 A Ge sample is doped with 5 x 10^{13} Arsenic atoms/cm^3. Determine, the carrier concentration and Fermi level position at 300 K. (n_i for Ge at 300 K = 2.5×10^{13} cm^{-3}).

Solution

$$N_D = 5 \times 10^{13} \text{ cm}^{-3}$$

$$N_D \cong n_i$$

In this case, doping concentration is comparable to intrinsic carrier concentration. Therefore, it is required to solve the charge neutrality equation to find out majority carrier concentration.

$$N_D + p_0 = n_0$$

$$N_D + \frac{n_i^2}{n_0} = n_0$$

$$n_0^2 - n_0 N_D - n_i^2 = 0$$

$$n_0 = \frac{N_D \pm \sqrt{N_D^2 + 4n_i^2}}{2}$$

$$= \frac{5\times10^{13} \pm \sqrt{(5\times10^{13})^2 + 4\times(2.5\times10^{13})^2}}{2}$$

$$= \frac{5\times10^{13} \pm 7.07\times10^{13}}{2}$$

$$= 6.035 \times 10^{13} \text{ cm}^{-3}$$

(Negative answer is invalid)

$$p_0 = \frac{n_i^2}{n_0}$$

$$= \frac{(2.5\times10^{13})^2}{6.035\times10^{13}}$$

$$= 1.035\times 10^{13} \text{ cm}^{-3}$$

To find Fermi level position

$$n_0 = n_i e^{E_F - E_i/kT}$$

$$E_F - E_i = kT \ln\frac{n_0}{n_i}$$

$$= 0.026 \ln\frac{6.035\times10^{13}}{2.5\times10^{13}}$$

$$= 0.023 \text{ eV.}$$

1.9 CARRIER TRANSPORT IN SEMICONDUCTORS-MOBILITY AND CONDUCTIVITY

Carrier transport in semiconductors is mainly by two different mechanisms; drift and diffusion. Drift current results from the movement of electrons (holes) under an electric field similar to the current flow in a metal. Diffusive motion is due to gradients in carrier concentrations.

Drift carriers in an electric field: The charge carriers in a solid are in random thermal motion under thermal equilibrium. (The mean thermal velocity of electron is given by $v_{th} = \sqrt{\frac{3kT}{m_n^*}}$. For silicon at 300 K, $v_{th} = 2.3 \times 10^7$ cm/s).

At room temperature the thermal motion of an electron may be visualised as random scattering from lattice atoms, impurities, other electrons, defects etc. Since the scattering is random, there is no net motion along any direction for a group of n electrons/cm^3 in a given time. This is not true for an individual electron as shown in Fig. 1.36.

If an electric field is applied in the x-direction (E_x), each electron experiences a force $-qE_x$ from the field. Thus, the group of electrons will have a net average velocity in the x-direction. This mean velocity is called drift velocity. If P is the total momentum of the group of n electrons/cm^3 the accelerating force due to electric field is

$$\left.\frac{dP_x}{dt}\right|_{field} = -nq\mathrm{E}_x \tag{1.66}$$

Even though this equation shows a continuous acceleration in the x-direction, this force due to electric field is balanced by the decelerating force due to the collision process, under steady state condition.

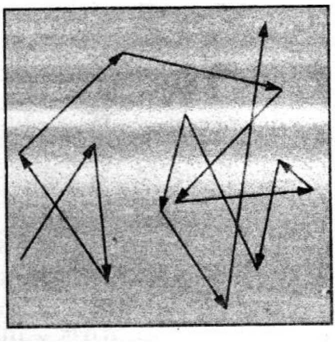


Fig. 1.36 Thermal motion of electron in solid

To evaluate the change in momentum due to collisions, we must investigate the collision probabilities, which will be constant if the collision is purely random.

Let us consider a group of N_0 electrons at time t = 0. Let N(t) be the number of electrons unscattered (have not undergone collision) by time t. The rate of decrease N(t) at any time t is proportional to N(t).

$$\frac{-dN(t)}{dt} \propto N(t)$$

or $$\frac{-dN(t)}{dt} = \frac{N(t)}{\bar{t}} \qquad (1.67)$$

Solution to this equation is

$$N(t) = N_0 e^{-t/\bar{t}} \qquad (1.68)$$

$\bar{t}$ is the mean time between collisions or mean free time.

The probability of collision for an electron in any time interval dt is dt $/\bar{t}$. Therefore, the differential change in momentum due to collisions is

$$-dP_x = P_x . \frac{dt}{\bar{t}}$$

or $$\left.\frac{dP_x}{dt}\right|_{collision} = \frac{-P_x}{\bar{t}} \qquad (1.69)$$

At steady-state, sum of accelerating and decelerating effects must be zero. i.e., from equations (1.66) and (1.69),

$$\frac{-P_x}{\bar{t}} - nqE_x = 0 \qquad (1.70a)$$

$$<P_x> = \frac{P_x}{n} = -q\bar{t}E_x \qquad (1.70b)$$

This indicates that the electrons have constant average velocity in the negative x- direction given by

$$<v_x> = \frac{<P_x>}{m_n^*} = \frac{-q\bar{t}\,E_x}{m_n^*} \qquad (1.71)$$

This represents the net drift velocity of an average electron in response to the electric field. Usually, this velocity is much less than mean thermal velocity (v_{th}).

The current density due to electron drift is the number of electrons crossing unit area per unit time (n$<v_x>$) multiplied by the charge of an electron (q), which can be verified dimensionally as

$$\begin{aligned} J_n \text{ drift} &= -q(C)\ n\ (cm^{-3}) v_x\ (cm\ s^{-1}) \\ &= -qn(v_x) C/s\ cm^2 = -qn(v_x) A/cm^2 \\ J_n \text{ drift} &= -qn <v_x> \\ &= \frac{nq^2\bar{t}}{m_n^*} E_x, \text{ substituting for } <v_x> \text{ from 1.71} \end{aligned} \qquad (1.72)$$

By Ohm's law,

$$J_n \text{ drift} = \sigma_n E_x$$

$$\therefore \sigma_n = \frac{nq^2\bar{t}}{m_n^*} \text{ by equation (1.72)} \qquad (1.73)$$

The conductivity (σ_n), can be written as

$$\sigma_n = qn\mu_n, \text{ where } \mu_n = \frac{q\bar{t}}{m_n^*} \qquad (1.74)$$

μ_n is called electron mobility, which represents the ease with which an electron drift in a material.

Mobility can also be defined as the average drift velocity per unit electric field.

$$\mu_n = \frac{-<v_x>}{E_x}\ cm^2/Vs \qquad (1.75a)$$

The current density in terms of mobility is

$$J_{n\ drift} = qn\mu_n E_x$$

Similarly $$\mu_p = \frac{+<v_x>}{E_x} \qquad (1.75b)$$

Total drift current is

$$\begin{aligned} J_{drift} &= J_{n\ drift} + J_{p\ drift} \\ &= q(n\mu_n + p\mu_p)\, E_x = \sigma E_x \end{aligned} \qquad (1.76)$$

where $$\sigma_n + \sigma_p = qn\mu_n + qp\mu_p \qquad (1.77)$$

Equation (1.74) shows that lesser the effective mass higher the mobility, i.e., lighter particles are more mobile than heavier particles. It can also be shown that as the curvature of the band increases mobility increases.

Generally the mobility of electrons is found to be higher than that of holes in all semiconductors. This is due to the fact that the position of holes is close to the nucleus than that of electrons. (Valence band where holes occupy is close to the nucleus than conduction band where electrons occupy). The nuclear interaction is more on holes.

Derivation of Conductivity of Semiconductor

By Ohms law, the electron drift current density may be expressed as

$$J_n = \sigma_n E \tag{1.78}$$

where

σ_n is the conductivity due to electrons and

E is the electric field.

By definition of current density,

$$J_n = -nqv_n \tag{1.79}$$

where

n is the electron concentration

q is the charge of electron = 1.6×10^{-19}C

v_n is the drift velocity of electron.

The mobility of electrons is defined as

$$\mu_n = \frac{-v_n}{E} \tag{1.80}$$

Substitute equation (1.80) in (1.79), we get

$$J_n = qn\mu_n E \tag{1.81}$$

From equations (1.78) and (1.81) we get

$$\sigma_n = qn\mu_n E$$

Similarly

$$\sigma_p = qp\mu_p$$

$$\therefore \quad \sigma = \sigma_n + \sigma_p$$

$$= q[n\mu_n + p\mu_p]$$

1.9.1 Compensation

The process of adding opposite type of impurity to an already doped semiconductor is called compensation. In a semiconductor if equal amount of opposite type of impurities are added the resulting material become intrinsic. Such a material is called completely compensated semiconductor. By adding opposite type of impurities an n-type semiconductor can be converted to p-type semiconductor and vice versa. The carrier mobilities and conductivity of a compensated semiconductor are different from those of an uncompensated semiconductor with equal carrier concentrations.

Example 1.10 The Fermi level position in a silicon sample at 300 K is 0.29 eV below E_C. Determine the carrier concentrations and conductivity of the specimen. Given that $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_n = 1350$ cm^2/Vs, $\mu_p = 480$ cm^2/Vs.

Solution

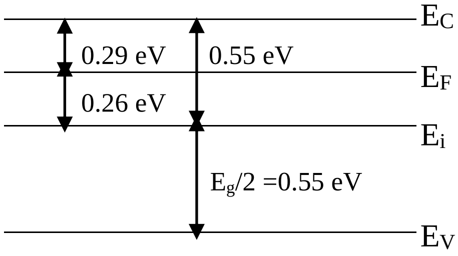


Fig. Ex.1.10

For silicon E_g = 1.1 eV

$$\therefore\ E_F - E_i = \frac{E_g}{2} - (E_C - E_F)$$
$$= 0.55 - 0.29$$
$$= 0.26 \text{ eV}$$

The specimen is n-type as the Fermi level is above E_i.

$$n_0 = n_i e^{(E_F - E_i)/kT}$$
$$= 1.5 \times 10^{10}\, e^{\frac{0.26}{0.026}}$$
$$= 3.30 \times 10^{14} \text{ cm}^{-3}$$
$$= p_0 = \frac{n_i^2}{n_0}$$
$$= \frac{(1.5\times10^{10})^2}{3.30\times10^{14}} = 6.8 \times 10^5 \text{ cm}^{-3}$$

Conductivity

$$\sigma = q(n_0\,\mu_n + p_o\mu_p)$$
$$\sigma = 1.6 \times 10^{-19}\,(3.3 \times 10^{14} \times 1350 + 6.8 \times 10^5 \times 480)$$
$$= 0.071\ (\Omega \text{ cm})^{-1}.$$

Example 1.11 Show that the conductivity of a silicon sample under thermal equilibrium is minimum when it is slightly p-type. Derive expression for minimum conductivity.

Solution

$$\sigma = q(n_0\,\mu_n + p_o\mu_p) \qquad \text{Ex.1.11}$$
$$= q\left(n_0\mu_n + \frac{n_i^2}{n_0}\mu_p\right)$$

For minimum conductivity,

$$\frac{d\sigma}{dn_0} = 0; \quad \frac{d^2\sigma}{dn_0^2} > 0$$

$$\frac{d\sigma}{dn_0} = q\left(\mu_n - \frac{n_i^2\mu_p}{n_0^2}\right) = 0$$

$$n_0^2\mu_n = n_i^2\mu_p$$

$$n_0 = n_i\sqrt{\frac{\mu_p}{\mu_n}} < n_i (Q \;\; \mu_n > \mu_p) \qquad \text{Ex.1.11(2)}$$

Also $\frac{d^2\sigma}{dn_0^2} = \frac{2qn_i^2\mu_p}{n_0^3} > 0$

Similarly, for minimum conductivity

$$\frac{d\sigma_0}{dp_0} = 0; \quad p_0 = n_i\sqrt{\frac{\mu_n}{\mu_p}} > n_i (Q \;\; \mu_n > \mu_p) \qquad \text{Ex.1.11(3)}$$

When conductivity is minimum, $n_0 < n_i$ and $p_0 > n_i$ which implies that silicon sample is p-type when conductivity is minimum. Substituting n_0 and p_0 from equation (Ex. 1.11(2)) and (Ex. 1.11(3)) into equation (Ex. 1.11(1)) gives the expression for minimum conductivity (σ_{min}) as

$$\sigma_{min} = qn_i\sqrt{\frac{\mu_p}{\mu_n}}\mu_n + qn_i\sqrt{\frac{\mu_n}{\mu_p}}\mu_p$$

$$= 2qn_i\sqrt{\mu_n\mu_p}$$

Example 1.12 Estimate the ratio of minimum conductivity to intrinsic conductivity of silicon at 300K.

Solution

μ_n = 1350 cm^2/Vs, μ_p = 480 cm^2 / Vs

σ_{min} = $2qn_i\sqrt{\mu_n\mu_p}$

= $2 \times 1.6 \times 10^{-19} \times 1.5 \times 10^{10} \times \sqrt{1350 \times 480}$

= 3.86×10^{-6} (Ω cm)$^{-1}$

σ_i = $qn_i(\mu_n + \mu_p)$

= $1.6 \times 10^{-19} \times 1.5 \times 10^{10}$ (1350 + 480)

= 4.39×10^{-6} (Ω cm)$^{-1}$

$$\frac{\sigma_{min}}{\sigma_i} = \frac{3.86\times10^{-6}}{4.39\times10^{-6}} = 0.88\,.$$

1.9.2 Thermal relaxation time

When an electric field is applied to a semiconductor, the charge carriers will acquire a net mean velocity whose direction depends on the type of carriers and direction of electric field. If the electric field is removed it will take some time for the electrons to acquire thermal equilibrium condition under which the mean velocity is zero. The time required to attain equilibrium after the removal of electric field is called thermal relaxation time ($\bar{t}$). The velocity decays exponentially with time on removal of electric field as follows.

$$v(t) = v_0 e^{-t/\bar{t}} \qquad (1.82)$$

Thermal relaxation time is also equal to the time taken by the charge carriers to attain steady-state after applying electric field.

Example 1.13 Determine, the mean free time for electron and hole in intrinsic silicon.

$m_0 = 9.11 \times 10^{-28}$ g, $q = 1.6 \times 10^{-19}$ C

$\mu_n = 1350 \text{cm}^2/\text{Vs}$, $\mu_p = 480 \text{ cm}^2/\text{Vs}$

$m_n^* = 0.26\, m_0$, $m_p^* = 0.38\, m_0$

$$\bar{t}_n = \frac{m_n^* \mu_n}{q}$$

$$= \frac{0.26 \times 9.11 \times 10^{-28} \times 1350}{1.6 \times 10^{-19}} = 1.99 \mu s$$

$$\bar{t}_p = \frac{m_n^* \mu_p}{q}$$

$$= \frac{0.38 \times 9.11 \times 10^{-28} \times 480}{1.6 \times 10^{-19}} = 1.038 \mu s$$

1.9.3 Scattering Mechanisms

The different scattering mechanisms limit the mobility of charge carriers in a semiconductor. The important scattering mechanisms are

1. Lattice scattering
2. Ionized impurity scattering
3. Neutral impurity scattering and
4. Carrier carrier scattering

Lattice scattering: This is the scattering mechanism due to vibration of lattice. As temperature increases the lattice vibrates at a faster rate (with increased energy). Therefore, scattering of carriers with lattice vibration increases with increase in temperature. Mobility decreases with rise in temperature such that $\mu_L \propto T^{\frac{-3}{2}}$.

Ionized impurity scattering: Ionized impurity scattering is the scattering of charge carriers with ionized impurities. Usually ionized impurities are dopants in a semiconductor. This is a dominant mode of scattering in doped semiconductors. This type of scattering increases with increase in doping and decrease in temperature. Mobility due to this type of scattering (μ_I) is proportional to $T^{\frac{3}{2}}$.

Neutral impurity scattering: The size of a neutral impurity atom is different from that of a host lattice atom and this difference cause strain in the lattice. This strain produces a bump in the potential near the neutral impurity atom, and the electric field produces scatters a carrier close enough to this impurity. This is negligible because the number of neutral impurities is very small.

Carrier carrier scattering: In semiconductors holes and electrons may deflect each other during their movement through the crystal. This scattering is negligible in extrinsic semiconductor because the minority carriers are in short supply. Collision between same type of carriers is unimportant because there is only an exchange of momentum for carrier moving in the same direction.

1.9.4 Variation of Mobility with Temperature

Mobility of charge carriers is decided by the scattering or collision mechanisms. As the scattering increases, mobility decreases. Lattice scattering increases with increase in temperature as the thermal agitation of lattice increases. Therefore, mobility due to lattice scattering decreases with increase in temperature $\left(\mu_L \alpha T^{\frac{-3}{2}}\right)$.

At lower temperature, the thermal motion of carriers is slower. Slowly moving carrier is scattered more strongly by interaction with a charged ion, compared to carriers with greater momentum. Therefore, mobility due to ionized impurity scattering decreases with decrease in temperature $\left(\mu_L \alpha T^{\frac{3}{2}}\right)$.

The variation of mobility with temperature is shown in Fig. 1.37. The effective mobility due to different scattering mechanisms is given by

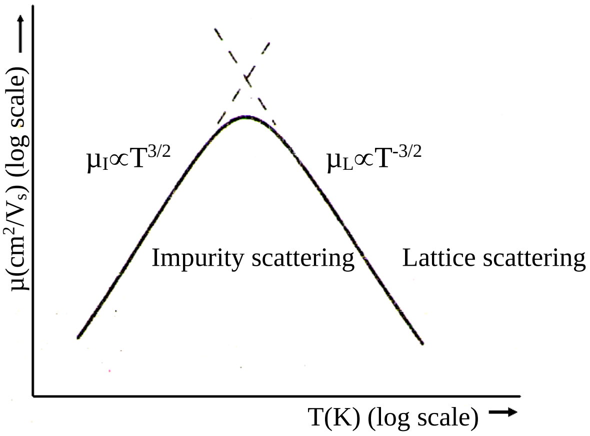


Fig. 1.37 Variation of mobility with temperature

$$\frac{1}{\mu}=\frac{1}{\mu_I}+\frac{1}{\mu_L}+\ldots \tag{1.83}$$

where μ is the effective mobility.

μ_I is mobility due to ionized impurity scattering.

μ_L is mobility due to lattice scattering.

Therefore, mechanism with lowest mobility will dominate at a given temperature.

1.9.5 Variation of Mobility with Doping

As the concentration of impurities increases (increase in doping), due to increased ionized impurity scattering, mobility decreases. An approximate plot of mobility as a function of doping on log scale is given in Fig. 1.38.

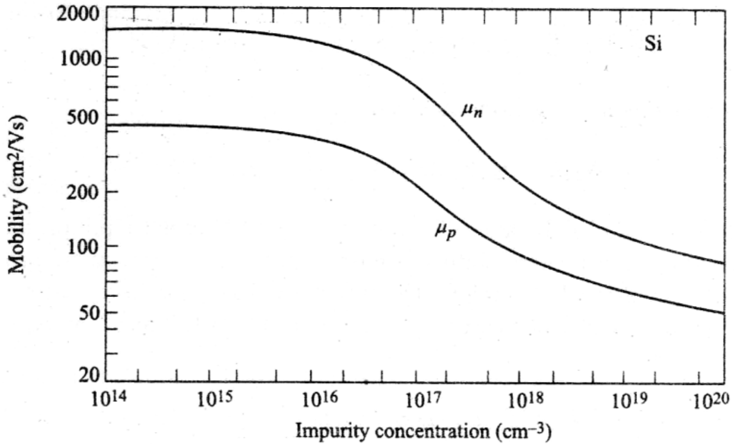


Fig. 1.38 Variation of Mobility with Doping

Doping dependant mobilities in Si at 300 K is given by emperical formulae

$$\mu_n = 88 + \frac{1252}{1+0.698\times10^{-17}N} cm^2/Vs$$

$$\mu_p = 54.3 + \frac{407}{1+0.374\times10^{-17}N} cm^2/Vs$$

where N represents doping concentration in cm^{-3}.

1.10 CONSTANCY Of FCRMI LEVEL AT EQUILIBRIUM

Assume that a contact be made between two materials as shown in Fig. 1.39.

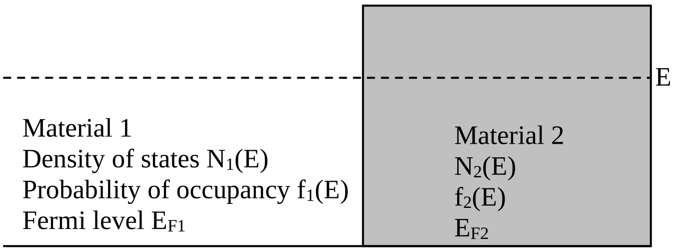


Fig. 1.39 Two materials in contact at equilibrium

Consider a small energy interval ΔE around an absolute value of energy E that is identical in the two materials. Assume that both materials have partially filled states around ΔE. Therefore, electrons are able to move from material 1 to 2 without expenditure of any energy.

Number of electrons moving from material 1 to material 2 in energy range ΔE is propotional to total number of filled states in material 1 and total number of vacant states in material 2 in ΔE. i.e., electrons moving from material 1 to 2

$$\propto [N_1(E).\ f_1(E)\ \Delta E]\ [N_2(E)[l - f_2(E)]\ \Delta E] \qquad (1.84a)$$

Similarly, electrons moving from material 2 to 1

$$\propto [N_2(E).\ f_2(E)\ \Delta E]\ [N_1(E)[l - f_1(E)]\ \Delta E] \qquad (1.84b)$$

By principle of detailed balance, for any system at thermal equilibrium, the rate of physical process in a given energy interval must be balanced by its inverse process occurring in the same energy interval. Therefore, the transfer of electrons from material 1 to 2 must be balanced by the transfer of electrons from material 2 to 1. Thus from equations (l.84a) and (1.84b), we get

$$[N_1(E)\ f_1(E).\ \Delta E]\ [N_2(E)[l - f_2(E)]\ \Delta E]$$
$$= [N_2(E)\ f_2(E)\ \Delta E]\ [N_1(E)[l - f_1(E)]\ \Delta E]$$

i.e., $N_1(E)\ f_1(E)\ N_2(E)[l - f_2(E)]$
$= N_2(E)\ f_2(E)\ N_1(E)[l - f_1(E)]$
$f_1(E)\ [l - f_2(E)] = f_2(E)\ [l - f_1(E)]$

i.e., $f_1(E) = f_2(E)$

i.e., $$\frac{1}{1+e^{(E-E_{F_1})/kT}} = \frac{1}{1+e^{(E-E_{F_2})/kT}}$$

which is true only if $E_{F_1} = E_{F_2}$. i.e., the gradient in Fermi level (c-^) is zero at thermal equilibrium.

1.11 HALL EFFECT

When electric and magnetic fields are applied simultaneously in perpendicular directions to a semiconductor specimen, an electric field develops in a direction mutually perpendicular to the applied magnetic and electric fields. This phenomenon is known as Hall effect.

Consider a p-type semiconductor bar of length L, thickness t and width w as shown in Fig. 1.40. Let B_z be the applied magnetic field along the z direction and I_x be the current along the x-direction due to the applied electric field. The net force experienced by a hole due to the combined electric and magnetic fields is given by

$$\mathbf{F} = q\ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (1.85a)$$

where $$v \times B = \begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \qquad (1.85b)$$

It may be noticed that

$B_x = B_y = 0$ and $v_y = v_z = 0$

The net force experienced by a hole along the y direction is the sum of the forces due to electric field and magnetic field along the y direction.

Force due electric field along the y-direction = qE_y

Force due magnetic field along the y-direction = q × y component of u × B

$$= q[-v_x B_z - B_z v_z]$$
$$= q(-v_x B_z) \quad , \text{Since } v_z = 0$$

Therefore the net force experienced by the hole along the y direction due to the combined electric and magnetic fields becomes

$$F_y = q(E_y - v_x B_z) \tag{1.86}$$

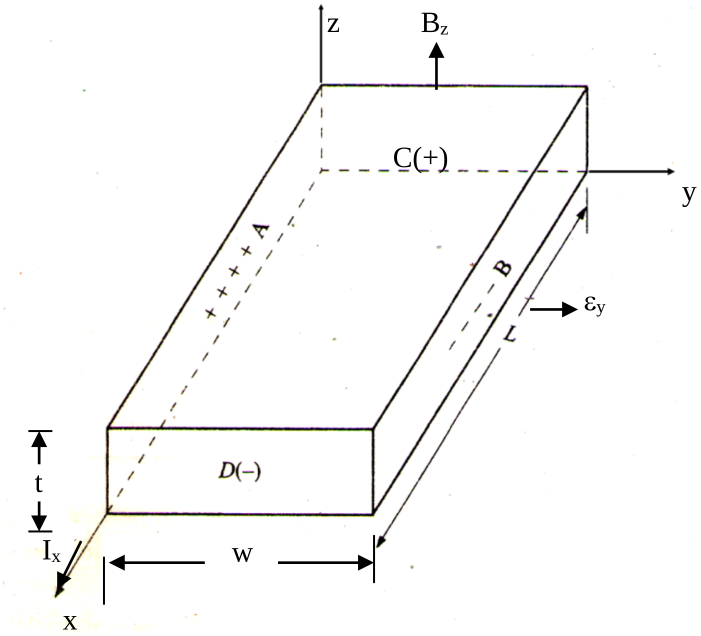


Fig. 1.40 Hall Effect

Equation 1.86 shows that unless an electric field is established along the y-direct the hole will experience a net force and acceleration in the y-direction due to v_xB_z product.

To maintain steady-state condition, E_y must just balance the product v_xB_z ($E_y = v_xB_z$), so that the net force $F_y = 0$. Physically, this electric field is set up when the holes are shifted slightly in the negative y-direction (towards side A in the figure) due to magnetic field. As the force due to this field balances the force due to magnetic field, the lateral force on the holes becomes zero, as they drift along the bar. The establishment of electric field E_y is known as Hall Effect and the resulting voltage $V_{AB} = E_y w$ is called Hall voltage.

$$v_x = \frac{J_x}{qp_0} \quad (Q \;\; J_{p(x)} = qp_0v_x)$$

$$\therefore \; E_y = \frac{J_xB_z}{qp_0} = R_H J_x B_z \tag{1.87}$$

i.e., the Hall field is proportional to the product of current density and magnetic flux density. The proportionality constant $\left(R_H = \frac{1}{qp_0}\right)$ is called Hall coefficient.

From Eqn 1.87

$$p_0 = \frac{J_x B_z}{q\varepsilon_y} = \frac{\left(\frac{I_x}{wt}\right)B_z}{q\left(\frac{V_{AB}}{w}\right)} = \frac{I_x B_z}{qtV_{AB}} \tag{1.88}$$

Since, all the quantities in equation (1.88) are measurable the majority carrier concentration can be measured using Hall Effect.

If a measurement of sample resistance R is made, the sample resistivity ρ can be calculated.

$$\rho = \frac{RA}{L}$$

$$= \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt} \tag{1.89}$$

For a p-type semiconductor, $\sigma = qpo\mu\rho$

Therefore mobility, $$\mu_p = \frac{\sigma}{qp_0} = \frac{R_H}{\rho} = \frac{1}{qp_o\rho} \tag{1.90}$$

$$= \frac{V_{AB}}{V_{CD}}\frac{L}{B_z w} \quad \text{(by eqns 1.88 and 1.89)}$$

Hall Effect can be used for:

1. measurement of majority carrier concentration,
2. measurement of majority carrier mobility and,
3. to determine the type of unknown semiconductor sample (n or p).

Example 1.14 With reference to Fig. 1.40, the following data are given: $B_z = 6$ kG, $I_x = 1$mA, w = 0.2 mm, t = 100 μm, L = 4 mm, $V_{AB} = -2$ mV, $V_{CD} = 500$ mV. Find the type of semiconductor, majority carrier concentration and its mobility.

Solution

V_{AB} negative, therefore the semiconductor is n-type.

$$n_0 = \frac{I_x B_z}{qtV_{AB}}$$

$$B_z = 6 \text{ kG} = 6 \times 10^{-5} \text{ Wb/cm}^2$$

$$\therefore \quad n_0 = \frac{1\times10^{-3}\times6\times10^{-5}}{1.6\times10^{-19}\times100\times10^{-4}\times2\times10^{-3}}$$

$$= 1.875 \times 10^{16} \text{ cm}^{-3}$$

$$\rho = \frac{V_{CD}/I_x}{L/wt} = \frac{V_{CD}wt}{I_x L}$$

$$= \frac{500\times10^{-3}\times0.02\times100\times10^{-4}}{1\times10^{-3}\times0.40}$$

$$= 0.25\Omega \text{ cm}$$

$$\mu_n = \frac{\sigma}{qn_0} = \frac{1}{qno\rho}$$

$$= \frac{1}{1.6\times10^{-19}\times1.875\times10^{16}\times0.25}$$

$= 1333\ \text{cm}^2/\text{Vs}.$

1.12 HIGH FIELD EFFECTS

The relation $v_d = \mu_n E$ is valid only for low values of electric fields. As the electric field is increased the drift velocity reaches an upper limit. This limit occurs near the mean thermal velocity (~ 10^7 cm/s at 300 K for Si). After that the energy from the field is transfered to the lattice rather than increasing the carrier velocity. The result of the scattering limited velocity is fairly constant at high fields as shown in fig 1.41. Due to velocity saturation, mobility decrease with increase in electric field at high electric fields $\left(\mu = \frac{v_s}{E}\right)$. This behaviour is typical of Si, Ge etc.

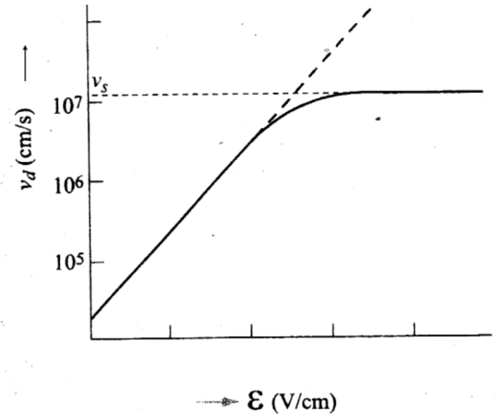


Fig. 1.41 Electron drift velocity vs electric filed for silicon

But for GaAs there is a region of negative resistance due to the transfer of electrons from a higher curvature band to a lower curvature band. As the electric field increases electrons from τ band where mobility is high are transferred to L band where the electron mobility is low (see Fig. 1.18). During the transfer, drift velocity decreases due to reduced mobility. Once all the electrons are transferred to lower mobility band, drift velocity reaches its minimum. After that the velocity again increases with electric field and saturates at high electric field to mean thermal velocity as shown in Fig. 1.42.

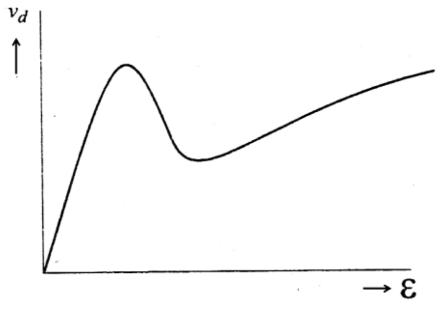


Fig. 1.42 Variation of electron drift velocity with electric field for GaAs

Example 1.15 What is the average time taken by an electron to drift 1 μm in pure silicon at an electric field of a. 100 V/cm b. 10^5 V/cm? $\mu_n = 1350\ cm^2/Vs$, $\mu_p = 480\ cm^2/Vs$ at room temperature.

Solution

a. $v_d = \mu_n E$

$= 1350 \times 100 = 1.35 \times 10^5$ cm/s

$$t = \frac{L}{v_d} = \frac{1\times10^{-4}}{1.35\times10^{5}} = 0.74ns$$

b. $v_d = \mu_n E$

$= 1350 \times 10^5 = 1.35 \times 10^8$ cm/s > v_s (saturation velocity)

$$\therefore\ t = \frac{L}{v_s} = \frac{1\times10^{-4}}{10^{7}} = 10^{-11}s.$$

1.13 EXCESS CARRIERS IN SEMICONDUCTORS

Charge carriers in semiconductors which are in excess of their thermal equilibrium value are called excess carriers. Excess carriers can be generated by optical excitation, electron bombardment, injection across a p-n junction etc.

1.13.1 Photo Generation

This is the process of generating charge carriers (excess) by shining light on the semiconductor. If a photon of energy greater than the band gap of semiconductor falls on it, the energy of photon is absorbed by an electron in the valence band, and is excited to the conduction band. This results in the generation of an EHP. i.e., optically generated electron and hole concentrations are equal. These carrier concentrations are in excess of the thermal equilibrium value.

i.e., $n = n_o + \delta_n$

$p = p_o + \delta_p$

$\delta_n = \delta_p$

where, n is the total electron concentration

p is the total hole concentration

n_o is the equilibrium electron concentration

p_o is the equilibrium hole concentration

δ_n is the excess electron concentration

δ_p is the excess hole concentration

If light of intensity Io falls on one face of a semiconductor sample of thickness t as shown in Fig. 1.43, the amount of light transmitted through it is given by

$$I(t) = I_o\, e^{-\alpha t} \tag{1.91}$$

where α is the absorption coefficient.

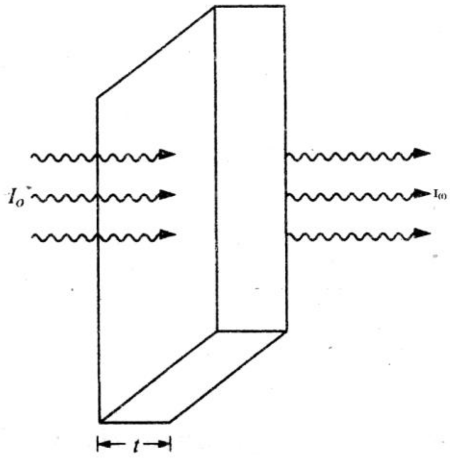


Fig. 1.43 Absorption and transmission of light through a semiconductor specimen of thickness t

The absorption coefficient of a material depends on the incident wave length or energy $\left(E = \frac{hc}{\lambda}\right)$. The absorption coefficient as a function of incident photon energy is shown in Fig. 1.44.

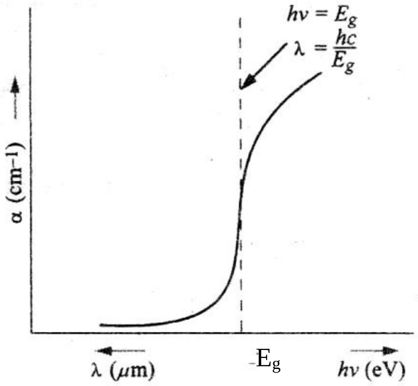


Fig. 1.44 Absorption coefficient as a function of energy of photon

If the incident photon has energy greater than the band gap energy, the velocity of the excited electron will be higher than the mean thermal velocity. The excess energy of electron is released to the lattice as heat during scattering events until its velocity becomes the thermal velocity at that temperature. Photogeneration and recombination process are illustrated in Fig. 1.45.

Photo conductivity: The conductivity due to the photogenerated carriers is called photo conductivity.

$$\sigma_{op} = qn_{op}\mu_n + qp_{op}\mu_p$$
$$= q\delta n\mu_n + q\delta p\mu_p \qquad (1.92)$$

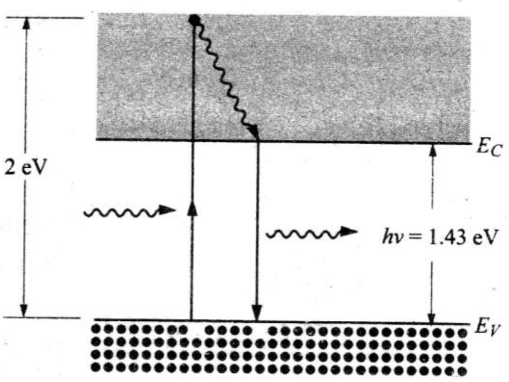


Fig. 1.45 Photo excitation and radiative recombination

Example 1.16 A 100 mW laser beam with wave length $\lambda = 6328\ \overset{o}{A}$ is focussed onto a GaAs sample 100 μm thick. The absorption coefficient at this wave length is 3×10^4 cm^{-1}.Find the number of photons emitted per second by radiative recombination in the GaAs specimen, assuming perfect quantum efficiency; what power is delivered to the sample as heat?

Solution

Transmitted power, $P_t = I_o e^{-\alpha t}$

$P_o = 100 \text{ mW} = 100 \times 10^{-3}$ W

$\alpha = 3 \times 10^4 \text{ cm}^{-1} = 3 \times 10^6 \text{ m}^{-1}$

$t = 100\ \mu\text{m} = 100 \times 10^{-6}$ m

$P_t = 100 \times 10^{-3} \times e(-3\times10^6\times100\times10^{-6})$

$\cong 0$

Absorbed power = Incident power - transmitted power

= 100 mW

Band gap of GaAs = 1.43eV

Energy of incident photon E $= \dfrac{hc}{\lambda}$

$= \dfrac{1.24}{\lambda(in\,\mu m)}$

$E = \dfrac{1.24}{0.6328} = 1.96$ eV

Energy of emitted photon is equal to the band gap of GaAs. The difference between energies of absorbed and emitted photons will be released to the lattice as heat.

Therefore, the amount of power converted to heat

$= \dfrac{1.96 - 1.43}{1.96} \times 100 \times 10^{-3}$ W

= 27 mW

Power of emitted photon = 100 - 27

= 73 mW

$$\text{Number of emitted photons} = \frac{\text{power of emitted photons}}{\text{energy of emitted photons}}$$

$$= \frac{73\times10^{-3}}{1.43\times1.6\times10^{-19}}$$

$$= 3.19 \times 10^{17}.$$

1.13.2 Low-level and High Level Injections

When the injected excess carrier concentration is small compared to the equilibrium majority carrier concentration, the injection of excess carriers is called low level injection. [i.e., δ_n, δ_p << p_o for n-type semiconductors and δ_n, δ_p << p_o for p-type semiconductors]. In low level injection, the majority carrier concentration remains almost unchanged.

When the injected excess carrier concentration is comparable to or greater than the equilibrium majority carrier concentration, the injection is called high level injection. In high level injection, the majority carrier concentration also changes significantly. One tenth of equilibrium majority carrier concentration is usually considered as the boundary between low level and high level injection.

1.13.3 Excess Carrier Lifetime (Transient Decay of Excess EHP) - Direct Recombination

In direct recombination, electron from the conduction band directly falls into a vacant state (hole) in the valence band and the difference in energy is released as a photon. The probability of recombining a hole and electron is constant at any time. Thus, the decay of excess carriers will have an exponential form.

The rate of decay at any time t is proportional to the product of number of electrons and number of holes remaining at time t. Therefore, the net rate of change of conduction band electron concentration $\left(\frac{dn(t)}{dt}\right)$ equals thermal generation rate $\left(\alpha_r n_i^2\right)$ minus the recombination rate ($\alpha_r$np).

$$\frac{-dn(t)}{dt} = \alpha_r n(t)\,p(t) - \alpha_r n_i^2 \tag{1.93}$$

Assume that excess EHP is created at t = 0 by a flash of light and let the initial excess electron and hole concentrations be Δn and Δp respectively. Also, Δn =Δp. As electrons and holes recombine in pairs, the instantaneous values of carrier concentrations δn(t) and δp(t) are also equal.

$$\therefore\ n(t) = n_o + \delta n(t)$$

$$p(t) = p_o + \delta p(t)$$

and equation (1.93) becomes

$$\frac{-d}{dt}\delta n(t) = \alpha_r\,[n_0 + \delta n(t)][p_0 + \delta p(t)] - \alpha_r n_i^2 \left(\because \frac{dn_0}{dt} = 0\right)$$

$$= \alpha_r \left[n_0 p_0 + \delta n(t)(p_0 + n_0) + \delta^2 n(t)\right] - \alpha_r n_i^2 \because \delta p(t) = \delta n(t)$$

$$= \alpha_r \left[(n_0 + p_0)\delta n(t) + \delta^2 n(t)\right]\left(\therefore\ \ \alpha_r n_0 p_0 = \alpha_r n_i^2\right) \tag{1.94}$$

For low-level injection δ^2n(t) is negligible compared to other terms.

For p-type semiconductor,

$$n_0 + p_0 \cong p_0$$

$$\therefore \frac{-d}{dt}\delta n(t) = \alpha_r p_0 \delta n(t)$$

Solution to this equation is

$$\delta n(t) = \Delta n\ e^{-\alpha_r p_0 t} = \Delta n e^{-t/T_n} \qquad (1.95a)$$

where $\tau_n = (\alpha_r p_0)^{-1}$ is called recombination lifetime. τ_n is also called minority carrier lifetime. Δn is the excess carrier concentration at t = 0.

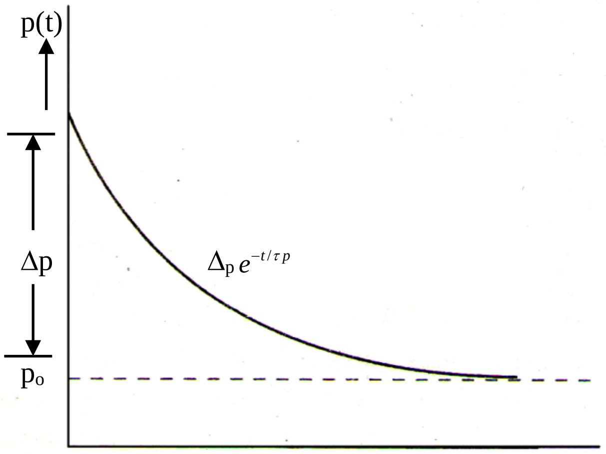


Fig. 1.46 Exponential decay of excess holes in a p-type semiconductor

Similarly, the decay of excess holes in n-type semiconductor is given by

$$\delta p(t) = \Delta p\, e^{-\alpha_r p_0 t} = \Delta p e^{-t/\tau_p} \qquad (1.95b)$$

where $\tau_p = (\alpha_r n_0)^{-1}$ and Δp is δp at t = 0.

In the case of direct recombination, the excess majority carriers decay at the same rate as the minority carriers.

1.13.4 Steady-State Carrier Generation

Under thermal equilibrium, the generation rate is balanced by recombination rate, so that equilibrium values of n_o and p_o are maintained

$$g_{(T)} = \alpha_r n_i^2 = \alpha_r n_0 p_o$$

Consider a steady-state optical generation rate g_{op}, added to thermal equilibrium. Now, the carrier concentration will increase to new steady-state values. Balance between generation and recombination is maintained under steady-state conditions.

i.e., $$g_{(T)} + g_{op} = \alpha_r np$$
$$= \alpha_r (n_0 + \delta n)(p_o + \delta p)$$

For direct recombination with no trapping, $\delta n = \delta p$

$$\therefore \quad g_{(T)} + g_{op} = \alpha_r n_0 p_o + \alpha_r [(n_0 + p_o)\delta n + \delta^2 n]$$

$\alpha_r n_0 p_o = g_{(T)}$ and $\delta^2 n$ term can be neglected for low-level injection.

$$\therefore g_{op} = \alpha_r (n_o + p_0)\delta n = \frac{\delta n}{\tau_n} \tag{1.96a}$$

where $\tau_n = \dfrac{1}{\alpha_r (n_0 + p_0)}$

$$\therefore \delta n = g_{op}\tau_n \tag{1.96b}$$

$$\delta p = g_{op}\tau_p$$

i.e., Excess carrier concentration = (excess carrier generation rate) x (minority carrier lifetime)

Example 1.17 A direct band gap semiconductor with $n_i = 10^{10}$ cm^{-3} is doped with 10^{15} cm^{-3} donors. Its low-level carrier lifetime $\tau_n = \tau_p = 10^{-7}$ s.

a. If a sample of this material is uniformly exposed to a steady optical generation rate of $g_{op} = 2 \times 10^{22}$ EHP/cm^{3}s; Calculate the excess carrier concentration $\Delta n = \Delta p$. Note: This excitation rate is not low-level, but you may assume that α_r is the same.

b. If the carrier lifetime (τ) is defined as the excess carrier concentration divided by the recombination rate, what is τ at this excitation level?

Solution

$N_D = 10^{15}\text{cm}^{-3}$; $n_0 = 10^{15}$ cm^{-3}

$$p_0 = \frac{n_i^2}{n_0} = \frac{10^{20}}{10^{15}} = 10^5 \text{ cm}^{-3}$$

$$\tau = \frac{1}{\alpha_r (n_0 + p_0)}$$

$\therefore$ $\alpha_r = 10^{-8}$ cm^{-3} s^{-1}

a. $g_{op} = 2 \times 10^{22}$ EHP/cm^{3}s

$\delta n = g_{op}\, \tau_n$

$= 2 \times 10^{22} \times 10^{-7} = 2 \times 10^{15}$

$\delta p = \delta n = 2 \times 10^{15}$ cm^{-3}

b. $n = n_o + \delta n = 10^{15} + 2 \times 10^{15} = 3 \times 10^{15}$

$p = p_o + \delta p = 10^5 + 2 \times 10^{15} = 2 \times 10^{15}$

$$\tau = \frac{\delta n}{\alpha_r np} = \frac{2\times10^{15}}{10^{-8}\times3\times10^{15}\times2\times10^{15}}$$

$= 3.33 \times 10^{-8}$ s.

1.13.5 Quasi Fermi Levels

The mass action law $n_o p_o = n_i^2$ is not applicable in the presence of excess carriers. Under thermal equilibrium, the carrier concentrations [n_o and p_o] are represented in terms of equilibrium fermi level, E_F. When excess carriers are present, the carrier concentrations cannot be represented by the equilibrium Fermi level. Seperate fermi levels are required to express the electron and hole concentrations and they are known as Quasi fermi levels. Electron quasi fermi level is expressed by F_n and hole quasi fermi level by F_p. In terms of quasi fermi levels, the carrier concentrations in the presence of excess carriers may be expressed as

$$n = n_i e^{(F_n - E_i)/kT} \tag{1.97a}$$

$$p = n_i e^{(E_i - F_p)/kT} \quad \text{and} \tag{1.97b}$$

$$np = n_i^2 e^{(F_n - F_p)/kT} \tag{1.97c}$$

The quasi-Fermi levels illustrate the deviation from equilibrium caused by excess carriers.

When excess carriers are present, the deviations of F_n and F_p from E_F indicate how far the electron and hole population are from the equilibrium values, n_o and p_o. Example 1.18 illustrates the effect of excess carriers on Fermi level position.

Example 1.18 A silicon sample with arsenic doping of 10^{16} atoms/cm^3 is steadily illuminated such that $g_{op} = 10^{21}$ EHP cm^{-3} s^{-1}. If $T_n = T_p = 10^{-6}$ s, calculate separation between quasi-Fermi levels and show the positions of equilibrium and quasi-Fermi levels at 300 K.

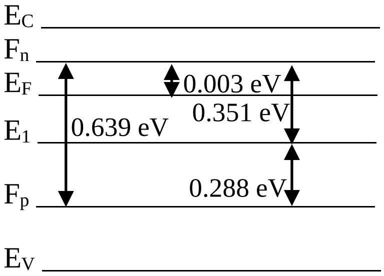


Fig. Ex.1.18

Take n_i at 300 K as 1.5×10^{10} cm^{-3}

$$N_D = 10^{16} \text{ cm}^{-3} >> n_i$$

$$\therefore\ n_o = N_D = 10^{16} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$\delta n = \delta p = g_{op}\,\tau_p$$

$$= 10^{21} \times 10^{-6} = 10^{15} \text{ cm}^{-3}$$

$$n = n_0 + \delta n$$

$$= 10^{16} + 10^{15} == 1.1 \times 10^{16} \text{ cm}^{-3}$$

$$p = p_o + \delta p = 2.25 \times 10^4 + 10^{15} = 10^{15} \text{ cm}^{-3}$$

$$n = n_i e^{(F_n - E_i)/kT}$$

$$F_n - E_i = kT \ln \frac{n}{n_i}$$

$$= 0.026 \ln \frac{1.1\times10^{16}}{1.5\times10^{10}} = 0.351 eV$$

$$p = n_i e^{\frac{(E_i - F_p)}{kT}}$$

$$E_i - F_p = kT \ln \frac{p}{n_i} = 0.026 \ln \frac{10^{15}}{1.5\times10^{10}} = 0.288 \text{ eV}$$

At equilibrium,

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT \ln \frac{n_0}{n_i}$$

$$= 0.026 \ln \frac{10^{16}}{1.5 \times 10^{10}} = 0.348 eV$$

$$F_n - F_p = F_n - E_i + E_i - F_p$$

$$= 0.351 + 0.288 = 0.639 \text{ eV.}$$

1.13.6 Recombination Mechanisms

i. Direct band to band recombination

In direct band to band recombination an electron at conduction band minimum directly falls into a hole at the valence band maximum, releasing the difference energy as a photon as illustrated in Fig. 1.47. As a result an electron hole pair vanishes. This type of recombination mechanism is seen in direct band gap semiconductors such as GaAs, InP, etc. Generation and recombination are continuous process in a semiconductor. Lifetime is the average time for which a charge carrier survives without recombination after it is generated.

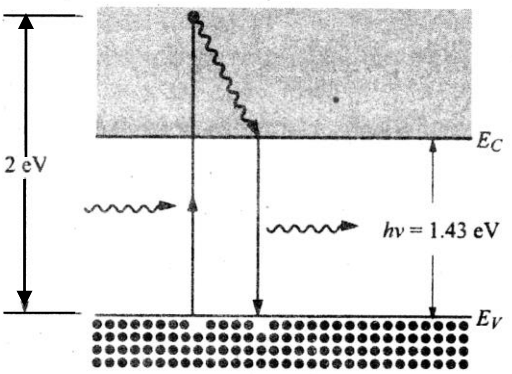


Fig. 1.47 Optical generation and direct recombination

ii. Indirect Recombination Via Deep Energy Levels in the Band Gap

Impurity atoms other than donors and acceptors and crystal defects in a semiconductor introduce localised energy levels deep in the band gap, away from the band edges. These levels act as stepping stones for electrons between conduction band and valence band, 'making a substantial enhancement in the recombination process. Depending on the location in the band gap, this level can act as a trap or recombination center for electron or hole.

An electron trap has a high probability of capturing a conduction band electron and setting it free after sometime. A hole trap has high probability of capturing a hole and subsequently releasing it to the valence band.

At a recombination centre, the probability of electron and hole captures are nearly equal, thus an electron capture is followed by a hole capture, and this results in the elimination of an EHP. Fig. 1.48 illustrates the four steps that occur in the recombination of an EHP through a deep level.

a) Capture of electron from conduction band by centre located at E_f.
b) Emission of an electron from the centre to the conduction band.

c) Capture of a hole from the valence band. Here, the centre emits electron to the valence band which is equivalent to the capture of a hole.

d) Centre captures an electron from the valence band, which is equivalent to the emission of a hole.

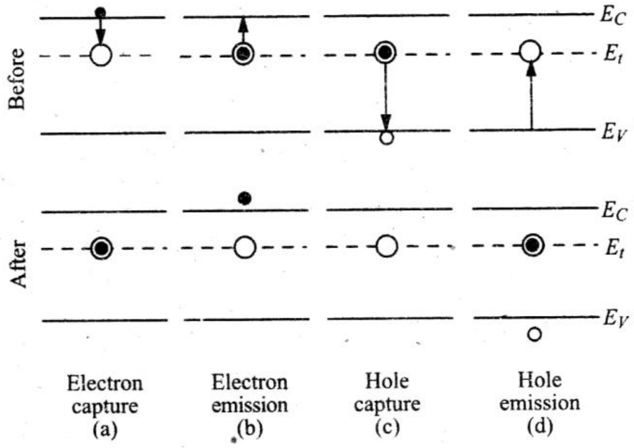


Fig. 1.48 Steps involved in indirect recombination via traps

For the level to act as a recombination centre, the electron capture process (a) must be followed by the hole capture process (c) and both these processes should have nearly equal probability. If electron capture (a) is followed by electron emission (b) this centre acts as electron trap. If hole capture (c) is followed by a hole emission (d) the centre acts as a hole trap.

iii. Auger Recombination

Auger recombination is a three particle process in which either two electrons and one hole or two holes and one electron are involved. This type of recombination is possible for direct band to band recombination and also for indirect recombination involving traps.

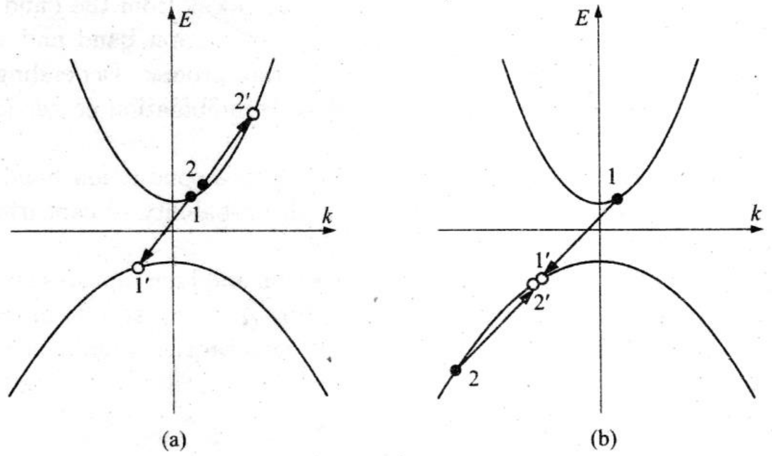


Fig. 1.49 Auger recombination (a) the two electrons and one hole (eeh) process (b) the one electron and two holes (ehh) process

The auger recombination process for a direct band to band recombination is illustrated in Fig 1.49. The two electrons and one hole process (eeh) is shown in Fig. 1.49(a). Hence electron 1 in the conduction band makes a transition to empty state 1' in the valence band. The energy of EHP is transferred to the nearby electron 2 and it is excited to 2'. The excited electron imparts its kinetic energy to the lattice and comes back to conduction band minimum.

Auger recombination involving two holes and one electron is shown in Fig. 1.49(b)). Here, the recombining electron 1 on transition to 2 imparts its energy to a hole at 2' which transfers to a state 2 deep in the valence band.

iv. Trap Aided Auger Recombination

In Fig. 1.50(a) the trapped electron makes a transition to the valence band by giving its energy to an electron in the conduction band. This process is significant only in a heavily doped n-type semiconductor with a high concentration of traps below E_F.

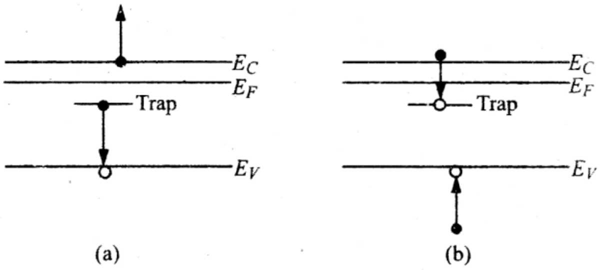


Fig. 1.50 Trap aided Auger recombination

Recombination involving two holes and one electron is shown in Fig. 1.50(b). Here, the electron makes a transition to an unoccupied trap level in the band gap and gives its energy to a hole in the valence band. This is significant in heavily doped p-type semiconductor with a large density of traps located above Fermi level.

1.13.7 Origin of Recombination Centres

The major origin of recombination centres are impurities, imperfection due to radiation damage and unsaturated bonds at the surface.

Impurities: Impurities which introduce levels near the middle of the band gap act, as efficient recombination centres. Eg: Au in Si, Cu in Ge. Gold introduces an acceptor level 0.54 eV below E_C in silicon. Gold doping increases recombination rate and therefore reduces lifetime. Thus, lifetime can be controlled by gold doping.

Radiation damage: High energy particles such as neutrons, protons, electrons and γ-rays can displace atoms in a semiconductor lattice from their normal positions creating vacancies and interstitials. The lattice defects created by them can behave like deep impurities. The lifetime (of minority carriers) decrease with radiation doze.

Electron irradiation is a process with greater control and uniformity to control lifetime. but it has the following disadvantage. Radiation induced recombination centres anneal out after a period of time, even if the device is not subjected to high temperature.

Unsaturated bonds at the surface: At the surface of a semiconductor, the lattice is abruptly terminated resulting in unsaturated bonds for the surface atoms. This irregularity introduces large density of localised energy levels in the forbidden gap and these levels are called surface states. Surface states with levels at the middle of the band gap act as recombination centres. The effect of surface recombination is illustrated in Fig. 1.51.

Density of surface states on clean surface ≅ 10^{15} cm^{-2}. Density of surface states on oxide coated surface ≅ 10^{12} cm^{-2}. This is because, due to presence of oxide layer, some of the bonds get saturated. Carrier recombination at the surface is characterized by surface recombination velocity (s).

$$s = \frac{\text{Excess carrier flux reaching the surface}}{\text{Concentration of excess carrier at the surface}}$$

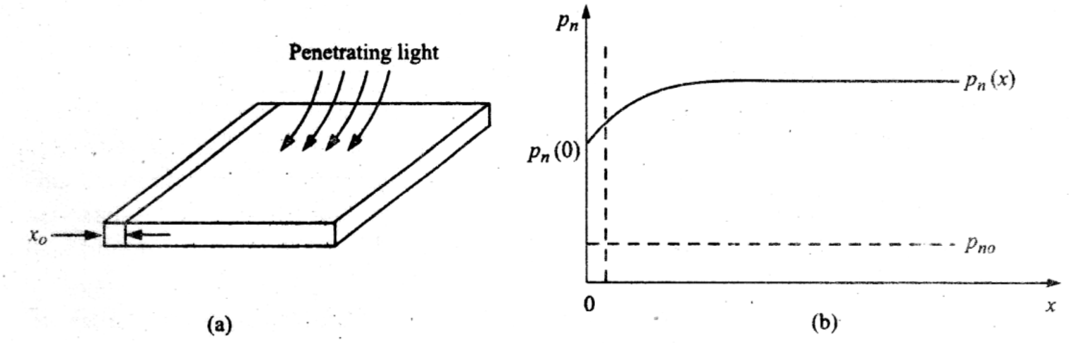


Fig. 1.51 Excess carrier recombination near the surface of a semiconductor with high density of recombination centres (a) illuminated semiconductor (b) excess minority carrier distribution

Due to increased recombination at the surface, $p_n(0) < p_n(x)$ beyond the surface.

$p_n(0)$ is the concentration at x = 0 (surface)

$p_n(x)$ is the concentration at depth x from the surface

This cause a net flux of excess carriers given by

$$-D_p \cdot \left.\frac{\partial p_n}{\partial x}\right|_{x=0} = s(p_n(0) - p_{no}) \qquad (1.98)$$

p_{no} is equilibrium hole concentration in n-type material.

s ≅ 105 cm/s for sand blasted surface

s ≅ 10 - 100 cm/s for clean etched surface

1.13.8 Diffusion of Charge Carriers

When excess carriers are generated non-uniformly in a semiconductor, the carrier concentrations vary with position in the sample. Thus, charge carriers will move from regions of high concentration to low concentration. This type of motion is called diffusion and the current due to diffusive motion of charge carriers is called diffusion current.

Fig. 1.52 shows the spreading of a pulse of electrons by diffusion. Diffusion is the natural result of the random motion of individual particles. A pulse of excess electrons injected at x = 0 at t = 0 will spread out in time as shown in Fig. 1.52. Initially excess electrons are concentrated at x = 0, but as time elapses these excess electrons spread out and finally the carrier concentration becomes uniform throughout and n(x) becomes constant.

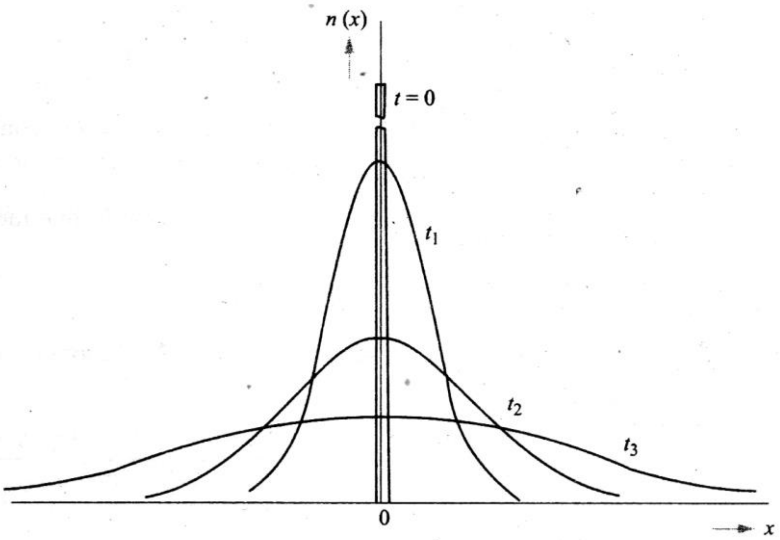


Fig. 1.52 Spreading of a pulse of electron by diffusion

Diffusion current (Derivation)

Consider an arbitrary one dimensional distribution of electrons n(x) as shown in Fig. 1.53. Divide x into segments of width $\bar{l}$ (mean free path which is a very small length), with n(x) evaluated at the centre of each segment. The electrons in any segment have equal chances of movement towards left or right since the movement is purely random.

In one mean free time, half of the electrons in segment 1 will cross x_o and move into segment 2 and half the electrons in segment 2 will cross x_o and move into segment 1.

Let electron concentration in segment 1 be n_1 and electron concentration in segment 2 be n_2, then

$$\text{Total number of electrons in segment 1} = n_1 \bar{l} A$$

$$\text{Total number of electrons in segment 2} = n_2 \bar{l} A$$

where A is the area of cross section of the sample.

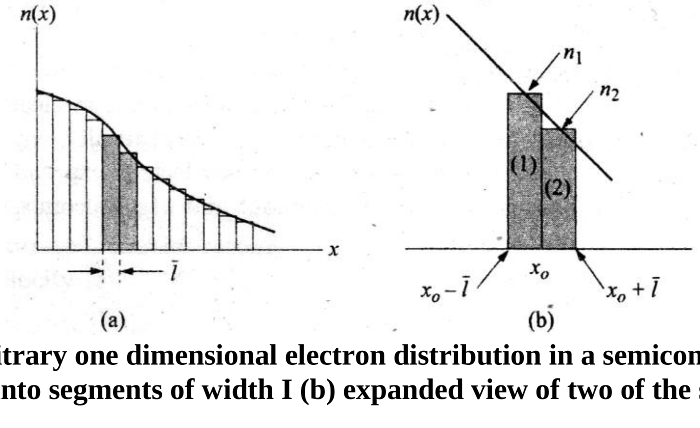


Fig. 1.53 An arbitrary one dimensional electron distribution in a semiconductor (a) n(x) is divided into segments of width I (b) expanded view of two of the segments

Therefore, net number of electrons crossing x_o from left to right in one mean free time

$$= \frac{1}{2}(n_1 \bar{l} A - n_2 \bar{l} A) \tag{1.99}$$

The rate of electron flow (electrons/unit time) in positive x-direction per unit area (electron flux density) is given by

$$\phi_n(x) = \frac{\text{Net number of electrons moving in x direction}}{\text{time } \times \text{Area of cross section}}$$

$$= \frac{\frac{1}{2}\left(n_1 \bar{l} A - n_2 \bar{l} A\right)}{\bar{t} A}$$

$$= \frac{\bar{l}}{2\bar{t}}(n_1 - n_2)$$

Since $\bar{l}$ is a small differential length, $n_1 - n_2$ can be written as

$$n_1 - n_2 = \frac{n(x) - n(x + \Delta x)}{\Delta x} \times \bar{l} = \frac{-dn(x)}{dx}\bar{l} \tag{1.100}$$

where x is taken as the middle of segment 1 and $\Delta x = \bar{l}$

$$\therefore \phi_n(x) = \frac{\bar{l}^2}{2\bar{t}}\left(\frac{-dn(x)}{dx}\right)$$

$$= \frac{-\bar{l}^2}{2\bar{t}}\frac{dn(x)}{dx}$$

$$= -D_n \frac{dn(x)}{dx} \tag{1.101a}$$

where D_n is the electron diffusion constant in cm^2/s.

$$D_n = \frac{\bar{l}^2}{2\bar{t}}$$

Similarly $\phi_p(x) = D_p \frac{dp(x)}{dx}$ (1.101b)

where D_p is hole diffusion constant.

The particle flux density multiplied by the charge of the particle gives current density.

$$J_{n\ diff} = (-q)\,\phi_n$$

$$= q D_n \frac{dn}{dx} \tag{1.102a}$$

$$J_{p\ diff} = q\,\phi_p$$

$$= -q D_p \frac{dp}{dx} \tag{1.102b}$$

The total current in a semiconductor is the sum of drift and diffusion currents due to electrons and holes. The current density due to electrons is

$$J_n(x) = q\mu_n n(x)\,E(x) + q\,D_n \frac{dn(x)}{dx} \tag{1.103a}$$

The current density due to holes is

$$J_p(x) = q\mu_p p(x)\,E(x) - q\,D_p \frac{dp(x)}{dx} \tag{1.103b}$$

and the total current is

$$J = J_n(x) + J_p(x)$$

The direction of carrier flow and current flow in a given electric field and carrier gradient is shown in Fig. 1.54.

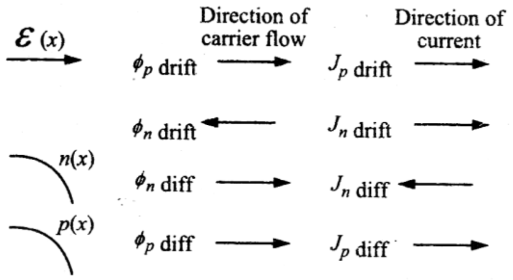


Fig. 1.54 Direction of drift and diffusion of charge carriers and the resulting currents

It can be observed that

1. For a given direction of field, electrons and holes move in opposite directions and the resulting drift current is along the same direction.
2. For the same gradient (sign) electrons and holes move in the same direction and the resulting currents are in opposite directions.
3. Minority carriers can provide large diffusion currents, because diffusion current is proportional to the gradient in carrier concentration and not to the number of carriers.

1.13.9 Energy Band Diagram of a Semiconductor with Applied Electric Field

The energy band diagrams of a uniformly doped n-type semiconductor under equilibrium condition and with an electric field applied along the positive x-direction are shown in Fig. 1.55.

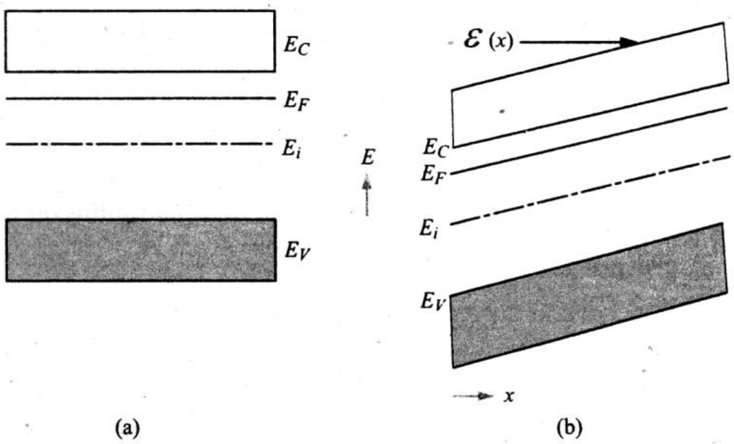


Fig 1.55 Energy band diagrams of a semiconductor (a) at equilibrium (b) in an electric field E_x

The applied electric field is proportional to the gradient in electron energy as given by equation (1.105). Therefore, energy bands bend upward in the direction of the electric field. Energy bands of a semiconductor will be flat when electric field in it is zero.

1.13.10 Einstein Relations

Einstein relation states that the ratio of diffusion coefficient to mobility of electrons and holes are equal under thermal equilibrium and is equal to the volt equivalent of temperature (kT/q).

i.e., $$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q} \tag{1.104}$$

Electrostatic potential and potential energy of a particle of charge q are related as follows

$$\text{Electrostatic potential } V(x) = \frac{E_{(x)}}{q} = \frac{\text{Potential energy}}{\text{Charge}}$$

$$= \frac{E_{(x)}}{-q} \text{ E(s) (for electrons)}$$

Also, electric field

$$E_{(x)} = \frac{-dV(x)}{dx}$$

$$= \frac{-d}{dx}\left(\frac{E_i}{-q}\right) \quad \left[\because \frac{dE_{(x)}}{dx} = \frac{dE_C}{dx} = \frac{dE_i}{dx} = \frac{dE_V}{dx}\right]$$

(i.e, All energies in the EB diagram bend equally in a given electric field)

$$E_{(x)} = \frac{1}{q}\frac{dE_i}{dx} \tag{1.105}$$

Under thermal equilibrium $J_n = J_p = 0$

i.e., $$J_p = qp_o(x)\,\mu_p E(x) - q\,D_p \frac{dp_0(x)}{dx} = 0$$

$$q\mu_p .\, p_o(x).\, E(x) = q\,D_p . \frac{dp_0(x)}{dx}$$

$$\therefore E(x) = \frac{D_p}{\mu p} . \frac{1}{p(x)} . \frac{dp_0(x)}{dx} \tag{1.106}$$

$$p_o(x) = n_i e^{E_i(x) - E_F(x)/kT} \text{ by equation (1.54)}$$

$$\frac{dp_0(x)}{dx} = n_i e^{E_i(x) - E_F(x)/kT} \times \frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$$

$$= \frac{p_0(x)}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right) \tag{1.107}$$

Substituting (1.107) in (1.106)

$$E_{(x)} = \frac{D_p}{\mu_p} \times \frac{1}{p_0(x)} . \frac{p_0(x)}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$$

Under thermal equilibrium,

$$\frac{dE_F}{dx} = 0$$

$$\frac{dE_i}{dx} = q\text{E}(x)\text{x x x x}\text{(by equation (1.105))}$$

$$\therefore E(x) = \frac{D_p}{\mu p} \cdot \frac{1}{kT} \cdot \qquad qE_x$$

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

i.e., $$\frac{D_n}{\mu_n} = \frac{kT}{q}$$

Similarly, $$\frac{D_n}{\mu_n} = \frac{kT}{q}$$

or $$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q}$$

1.13.11 Continuity Equation (Effect of Diffusion and Recombination)

Continuity equation is an important equation for the analysis of semiconductors. It can be used to determine the distribution of charge carriers in a semiconductor. It considers the time and space variation of charge carriers simultaneously. A simplified continuity equation in differential form is derived below.

Consider a semiconductor bar with area of cross-section A. Let $J_p(x)$ be the current density entering an elemental volume ΔxA and $J_p(x + \Delta x)$ be the current density leaving the elemental volume as shown in Fig. 1.56.

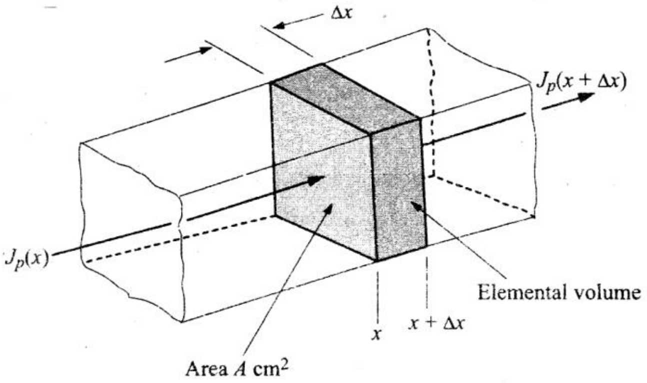


Fig. 1.56 Current density entering and leaving elemental volume ΔxA

The rate of increase of hole concentration in the elemental volume $\left(\frac{\partial p}{\partial t}\right)$ is given by

$\frac{\partial p}{\partial t}$ = rate of hole build up = net flux/unit volume of hole in elemental volume

- net recombination rate in it

= (hole flux/unit volume entering elemental volume ΔxA

- hole flux/unit volume leaving ΔxA - recombination rate)

$$J_p = q\phi_p$$

$$\phi_p = \frac{J_p}{q} = \text{flux/unit area}$$

$$\therefore \text{ flux/unit volume} = \frac{J_p}{q\Delta x}$$

i.e.,

$$\frac{\partial p}{\partial t} = \frac{J_p(x)}{q\Delta x} - \frac{J_p(x+\Delta x)}{q\Delta x} - \frac{\delta_p}{\tau_p}$$

$$= \frac{-1}{q}\frac{[J_p(x+\Delta x) - J_p(x)]}{\Delta x} - \frac{\delta_p}{\tau_p}$$

$$= \frac{-1}{q}\frac{\delta J_p}{\delta x} - \frac{\delta_p}{T_p} \tag{1.108a}$$

The above equation is called continuity equation for holes.
Similarly, the continuity equation for electrons is given as

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\delta J_n}{\delta x} - \frac{\delta_n}{T_n} \tag{1.108b}$$

For a uniformly doped specimen the current is only due to diffusion,

i.e., $J_p = -qD_p . \frac{dp}{dx} = -qD_p \frac{d}{dx}\delta p$

(because, $p = p_o + \delta p$ and $\frac{dp_0}{dx} = 0$ for uniformly doped semiconductor)

Similarly, $J_n = q\,D_n \,.\, \frac{dn}{dx} = -qD_n \frac{d}{dx}\delta n$

Therefore, Equations (1.108a) and (1.108b) reduces to

$$\frac{\partial p}{\partial t} = D_p \frac{\delta^2 \delta p}{\delta x^2} - \frac{\delta p}{T_p}$$

$$\frac{\partial n}{\partial t} = D_n \frac{\delta^2 \delta n}{\delta x^2} - \frac{\delta n}{T_n} \tag{1.109b}$$

Equations (1.109a) and (1.109b) are called diffusion equations.

1.13.12 Steady-State Carrier Injection and Diffusion Length

Under steady-state conditions the time derivatives are zero and the diffusion equations reduces to

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n T_n} = \frac{\delta n}{L_n^2} \tag{l.110a}$$

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{D_p T_p} = \frac{\delta p}{L_p^2} \tag{l.110b}$$

where $L_n = \sqrt{D_n \tau_n}$ is called electron diffusion length and $L_p = \sqrt{D_p \tau_p}$ is the hole diffusion length.

These equations are called steady-state diffusion equations. The solution of the steady- state diffusion equation for holes is of the form

$$\delta p(x) = C_1\, e^{x/Lp} + C_2\, e^{-x/Lp} \tag{1.111}$$

Consider a uniformly doped semi-infinite n-type semiconductor bar in which excess carriers are introduced at x = 0. Let $\delta p(x=0) = \Delta p$. The excess carrier concentration decays to zero at the end of the semiconductor bar ($x = \infty$).

At $x = 0$, $\delta p = \Delta p$; $C_1 + C_2 = \Delta p$
At $x \rightarrow \infty$, $\delta p = 0$; $\therefore C_1 = 0$
$\therefore\ C_2 = \Delta p$

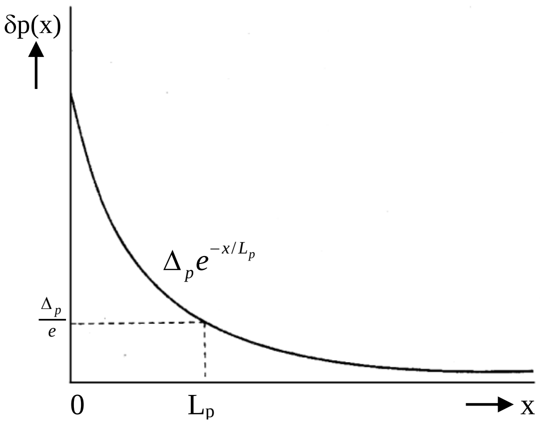


Fig. 1.57 Decay of excess holes and diffusion length

Thus the solution is $$\delta p(x) = \Delta p\, e^{-x/Lp} \tag{1.112}$$

The distribution of excess carrier given by eq 1.12 is shown in Fig. 1.17.

The injected excess carrier concentration decay exponentially to 1/e times its initial value at a diffusion length from the point of injection. It can be shown that L_p is the average distance a hole diffuse before it recombines.

Example 1.19 Prove that L_p is the average distance a hole diffuse before it recombines.

The probability that a hole injected at x = 0 survives to x without recombination is the ratio of the excess carrier concentrations at x and at 0, and is equal to $\frac{\Delta p e^{-x/L_p}}{\Delta p} = e^{-x/L_p}$.

The probability that a hole recombines in a subsequent interval dx is

$$\frac{\delta p(x)-\delta p(x+dx)}{\delta p(x)} = \frac{\delta p(x)-\delta p(x+dx)}{dx}\frac{dx}{\delta p(x)}$$

$$= \frac{-d}{dx}\delta p(x).\frac{dx}{\delta px} = \frac{-d}{dx}\Delta p e^{\frac{-x}{L_p}}.\frac{dx}{\delta px}$$

$$= -\Delta p e^{\frac{-x}{L_p}}\left(\frac{1}{-L_p}\right).\frac{dx}{\delta px} = \frac{1}{L_p}\delta p(x).\frac{dx}{\delta px}$$

$$= \frac{1}{L_p}dx$$

The probability P(x) that a hole injected at x = 0 recombines in a given distance dx, is the product of the probability that it survives to x without recombination and probability of recombination in subsequent interval dx.

i.e., $$p(x) = e^{\frac{-x}{L_p}}.\frac{dx}{L_p}$$

The average distance a hole diffuse before recombination is

$$< x > = \int_0^\infty x\frac{e^{-x/L_p}}{L_p}$$

$$= \frac{1}{L_p}\left[x.e^{-x/L_p}\left(-L_p\right) - \int e^{-x/L_p}\left(-L_p\right).1dx\right]_0^\infty$$

$$= \left[-xe^{-x/L_p} + e^{-x/L_p}\left(-L_p\right)\right]_0^\infty = L_p$$

1.13.13 Built in field

Built in field is the field present in a semiconductor under thermal equilibrium. Electric fieldbe present in a semiconductor at equilibrium, if a carrier concentration gradient exists in the semiconductor.

Example 1.20 In a silicon sample, the doping profile is in such a way that $n_o = Gx$ where G is a constant. Determine the built-in field in the semiconductor if no $n_0 >> n_i$.

Under thermal equilibrium,

$$J_p = qn_0\,\mu_n E_{(x)} + qD_n\frac{dn_0}{dx} = 0$$

$$E_{(x)} = \frac{-D_n}{\mu_n}\times\frac{1}{n_0}\frac{dn_0(x)}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{q}$$ by Einstein relation

$$\frac{dn_0(x)}{dx} = G$$

$$\therefore\ E_{(x)} = \frac{-kT}{q}\times\frac{1}{Gx}\times G$$

$$= \frac{-kT}{qx}$$

1.13.14 Minority Carrier Injection-Drift And Diffusion Currents

Consider a long semiconductor bar doped uniformly with N_D donor atoms/cm^3 so that $n_o \cong N_D$. Let A be its cross-sectional area. It is illuminated at x = 0 as shown in Fig. 1.58 so that there is a steady generation of EHP and the resulting excess carrier concentration be $\Delta p = \Delta n$ (at x = 0). Since the bar is of n-type, the hole drift current is negligible.

$$\therefore \quad J_p = -q D_p . \frac{dp}{dx}$$

$$\frac{\partial J_p}{\partial x} = -qD_p . \frac{\partial^2 p}{\partial x^2}$$

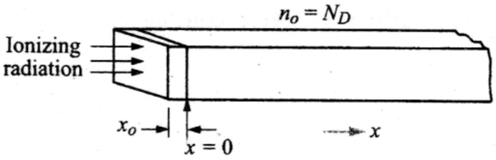


Fig. 1.58 Steady-state injection of minority carriers into a semiconductor

Applying this in the continuity equation,

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\frac{\partial Jp}{\partial x} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial p}{\partial t} = 0 \text{ (steady-state)}$$

$$\therefore \frac{d^2 p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2}$$

Solution to this equation is

$$\delta p(x) = \Delta p e^{-x/Lp}$$

i.e., the injected hole concentration decays exponentially.

Minority carrier diffusion current,

$$I_p(x) = -q A D_p \frac{d}{dx}(\Delta p e^{-x/Lp})$$

$$= q A \frac{D_p}{L_p} \Delta p e^{-x/Lp}$$

$$= q A \frac{D_p}{L_p} [p_n(0) - p_{n0}] e^{-x/Lp}$$

i.e., the minority carrier diffusion current also decays exponentially with x.

Assuming that electrical neutrality is maintained in the semiconductor, i.e., $\delta p = \delta n$, (under low-level injection) throughout the bar

$$\frac{dp}{dx} = \frac{dn}{dx}$$

Electron diffusion current,

$$I_{n\,diff} = q\,A\,D_n \frac{dn}{dx}$$

$$= q\,A\,D_n \frac{dp}{dx}$$

But $-q\,A\,D_p \dfrac{dp}{dx} = I_p$ (hole diffusion current)

$$\therefore\ I_{n\,diff} = \frac{-D_n}{D_p} I_p$$

Hence, electron and hole diffusion currents vary almost similarly with distance

$$\frac{D_n}{D_p} \cong 2 \text{ for Ge and 3 for Si}$$

Drif Currents

The total current must be zero everywhere in the semiconductor.

i.e., $I_p + (I_{n\,drift} - I_{n\,diff}) = 0$ (see Fig. 1.59)

$$I_p + \left(I_{n\,drift} - \frac{D_n}{D_p} I_p \right) = 0$$

$$\therefore I_{n\,drift} = \left(\frac{D_n}{D_p} - 1 \right) I_p$$

i.e., majority carrier drift current also decrease exponentially with distance.

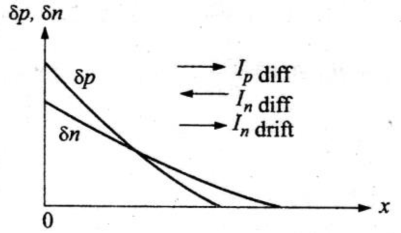


Fig. 1.59 Excess electron and hole distribution for the specimen shown in Fig. 1.58.

The drift current is due to the electric field present as a result of the difference in distribution of the excess electrons and holes as shown in Fig. 1.59.

$$E_{(x)} = \frac{J_{n\,drift}}{\sigma} = \frac{J_{n\,drift}}{qn\mu_n} = \frac{I_{n\,drift}}{Aqn\mu_n}$$

$$E_{(x)} = \frac{1}{Aqn\mu_n} \left(\frac{D_n}{D_p} - 1 \right) I_p \tag{1.113}$$

The derivation was based on the assumption that $J_{p\,drift} = 0$

But $I_{p\,drift} = Aqp\mu_p E_{(x)}$ (1.114)

Using (1.113) and (1.114)

$$I_{p\,drift} = \frac{p}{n} \frac{\mu_p}{\mu_n} \left(\frac{D_n}{D_p} - 1 \right) I_p$$

$$p << n \ \therefore I_{p\,drift} << I_{n\,drift}$$

1.13.15 Gradient in Quasi-Fermi Levels

At equilibrium, gradient in Fermi level is zero. But a combination of drift and diffusion with a net current implies a gradient in quasi Fermi levels. The total electron current is given by

$$J_n(x) = qn(x)\mu_n E_x + q D_n.\ \frac{dn(x)}{dx} \tag{1.115}$$

But

$$\frac{dn(x)}{dx} = \frac{d}{dx} n_i e^{(F_n - E_i)/kT}$$

$$= \frac{n(x)}{kT}\left[\frac{dF_n}{dx} - \frac{dE_i}{dx}\right] \tag{1.116}$$

Substituting equations (1.116) in (1.115) and applying Einstein relation,

$$J_n(x) = qn(x)\mu_n E_x + \mu_n n(x) \left[\frac{dF_n}{dx} - \frac{dE_i}{dx}\right]$$

$$= qn(x)\mu_n E_x + \mu_n n(x) \frac{dF_n}{dx} - \mu_n n(x) \frac{dE_i}{dx}$$

$$= qn(x)\mu_n E_x + \mu_n n(x) \frac{dF_n}{dx} - \mu_n n(x) qE_x \quad \text{,by equation (1.105)}$$

$$= \mu_n n(x) \frac{dF_n}{dx}$$

$$= q\mu_n n(x).\ \frac{d}{dx}\left(\frac{F_n}{q}\right)$$

$$= \sigma_n(x).\ \frac{d}{dx}\left(\frac{F_n}{q}\right) \tag{1.117}$$

Equation (1.117) shows that total electron current is proportional to the gradient in electron quasi-Fermi level.

Similarly,

$$J_p(x) = q\mu_p\, p(x) \frac{d}{dx}\left(\frac{F_p}{q}\right)$$

$$= \sigma_p(x) \frac{d}{dx}\left(\frac{F_p}{q}\right) \tag{1.118}$$

Table 1.9 Important properties of intrinsic germanium, silicon and gallium arsenide at 300 K

Properties	**Ge**	**Si**	**GaAs**
Energy band gap E_g (eV)	0.67	1.11	1.43
Electron mobility μ_n(cm^2/Vs)	3900	1350	8500
Hole mobility μ_p (cm^2/Vs)	1900	480	400
Resistivity ρ(Ω cm)	43	2.5×10^5	4×10^8
Lattice constant a(Å)	5.66	5.43	5.65
Relative permittivity $\int_r$	16	11.8	13.2

Density (g/cm^3)	5.32	2.33	5.31
Intrinsic carrier concentration n_i (cm^{-3})	2.5×10^{13}	1.5×10^{10}	2×10^{6}

Solved Problems

Problem 1.1

The lattice constant of diamond crystal is 3.56 A°. Calculate the number of atoms/cm^2 in the diamond crystal in (100) and (111) planes.

Solution

(100) plane is a face of the cube as in Fig. Sp. 1.16(b).

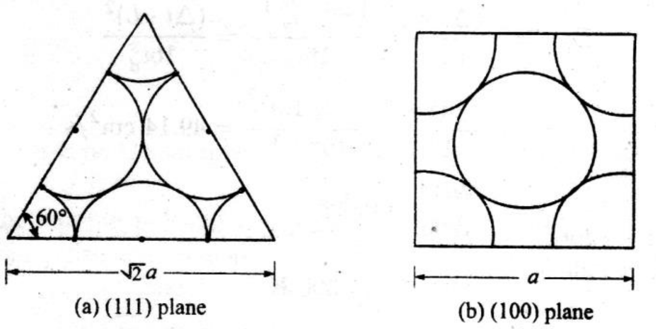
$$\text{Number of atoms on a face of the cube} = \frac{1}{4} \times 4 + 1 = 2$$

$$\text{Area of face} = a^2$$

$$\text{No. of atoms/cm}^2 \text{ on (100) plane} = \frac{\text{Number of atoms on a face}}{\text{area of the face}}$$

$$= \frac{2}{a^2} = \frac{2}{(3.56 \times 10^{-8})^2} = 1.578 \times 10^{15}$$

(111) plane in a cubic cell represent an equilateral triangle of side $\sqrt{2a}$ as in Fig. Sp.l.l(a).



(a) (111) plane (b) (100) plane

Fig. Sp.1.1

Number of atoms/equilateral triangle of side $\sqrt{2a}$

$$= \frac{1}{2} \times 3 + \frac{1}{6} \times 3 = 2$$

$$\text{area of the triangle} = \frac{1}{2} \times \sqrt{2a} \,.\, \sqrt{2a} \sin 60$$

$$= \frac{\sqrt{3}}{2} a^2$$

$$\text{Number of atoms/cm on (111) plane} = \frac{2}{\frac{\sqrt{3}}{2} a^2}$$

$$= \frac{2 \times 2}{\sqrt{3} \times (3.56 \times 10^{-8})^2}$$

$= 1.822 \times 10^{15}$.

Problem 1.2

Determine the density of GaAs, given the lattice constant a = 5.65Å, molecular weight = 144.63 g/mole and Avogadro number is 6.02×10^{23} atoms/mole.

Solution

In one unit cell, there will be 4 atoms of Ga and 4 atoms of As or 4 molecules of GaAs.

$$\therefore \quad \text{Number of GaAs molecules/cm} = \frac{4}{a^3}$$

$$= \frac{4}{(5.65\times10^{-8})^3}$$

$$= 2.217 \times 10^{22} \text{ atoms/cm}^3$$

Density of GaAs' equals the weight of atoms in 1 cm^3 i.e., weight of 2.217×10^{22} atoms.

Weight of Avogadro number of molecules = molecular weight

$$\therefore \text{ density} = \frac{\text{molecular weight}}{\text{Avogadro number}} \times \text{number ot atoms / cm}^3$$

$$= \frac{144.63 \times 2.217 \times 10^{22}}{6.02 \times 10^{23}}$$

$$= 5.3 \text{ gm/cm}^3$$

Problem 1.3

Determine the energy of a photon having wave length a. λ = 10,000 Å and b. λ = 10 A. Express the result in Joules and eV.

Solution

$$1 \text{ Å} = 10^{-8} \text{ cm}$$

a. $\lambda = 10{,}000 \text{ Å} = 10^{-4}$ cm

$$\text{Energy of photon} = E = h\upsilon = \frac{hc}{\lambda}$$

$h = 6.626 \times 10^{-34}$ J/s $\quad C = 3 \times 10^{10}$ cm/sec

$$\therefore \quad E = \frac{6.626\times10^{-34}\times3\times10^{10}}{10^{-4}}$$

$$= 19.878 \times 10^{-20} \text{ J}$$

$$= \frac{19.878\times10^{-20}}{1.6\times10^{-19}} \text{ eV} = 1.242 \text{ eV}$$

b. $\lambda = 10 \text{ Å} = 10^{-7}$ cm

$$\text{Energy of photon } E = \frac{6.626\times10^{-34}\times3\times10^{10}}{10^{-7}}$$

$$= 19.878 \times 10^{-17} \text{ J}$$

$$= \frac{19.878\times10^{-17}}{1.6\times10^{-19}} \text{ eV}$$

$$= 1242.37 \text{ eV.}$$

Problem 1.4

The velocity of a free particle is 5×10^5 m/s. The mass of the particle is 10^{-30} kg. Determine a. the energy of particle b. the de-Broglie wave length.

Solution

$$v = 5 \times 10^5 \text{ m/s} = 5 \times 10^7 \text{ cm/s}$$

$$m = 10^{-30} \text{ kg} = 10^{-27} \text{ g}$$

a. Energy $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 10^{-27} \times (5 \times 10^7)^2$$

$$= 12.5 \times 10^{-13} \text{ J}$$

b. de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$\lambda = \frac{6.626 \times 10^{-34}}{10^{-27} \times 5 \times 10^7}$$

$$= 1.325 \times 10^{-14} \text{ cm.}$$

Problem 1.5

For a particle that has a mass of 2 g and energy 1.5 kT, determine de-Broglie wave length at 300 K. kT = 0.026 eV.

Solution

Given $E = 1.5\text{ kT}$

$$kT = 0.026 \text{ eV}$$

$$= 0.026 \times 1.6 \times 10^{-19} \text{ J}$$

$$E = 1.5 \times 0.026 \times 1.6 \times 10^{-19} \text{ J}$$

But $E = \frac{1}{2} mv^2$

or $v = \sqrt{\frac{2E}{m}}$

$$= \sqrt{\frac{2 \times 1.5 \times 0.026 \times 1.6 \times 10^{-19}}{2}}$$

$$= 7.9 \times 10^{-11} \text{ cm/s}$$

de-Broglie wave length, $\lambda = h/mv$

$$= \frac{6.626 \times 10^{-34}}{2 \times 7.9 \times 10^{-11}}$$

$$= 4.19 \times 10^{-24} \text{ cm.}$$

Problem l.6

Show that the probability that an electron with a mean free time t in a semiconductor remains unscattered for a time t is proportional to $e^{(-t/\hat{t})}$. Consider a semiconductor with different scattering processes independent of each other. Show that the resultant mean free time $\bar{t}$ is given by $\frac{1}{\bar{t}} = \frac{1}{\bar{t}_1} + \frac{1}{\bar{t}_2} + ... \frac{1}{\bar{t}_n}$.

Solution

Let N_o be the initial number of particles at t = 0 and the number of particles that are unscattered at time t be N(t). The rate of decrease of N(t) is proportional to N(t) itself. So,

$$-\frac{dN(t)}{dt} = \gamma N(t)$$

where γ is a constant of proportionality. γ has dimension of reciprocal time. So we can write $\gamma = \frac{1}{\bar{t}}$.

$$\text{i.e.,} \qquad \frac{dN(t)}{dt} = \frac{-N(t)}{\bar{t}}$$

The solution of the equation is N(t) = $N_0 e^{(-t/\bar{t})}$ and $\frac{N(t)}{N_0}$ is the probability p that a particle remain unscattered for time t.

$$p = e^{(-t/\bar{t})}$$

If we have n independent scattering process with probabilities p_1, p_2, p_3 . . . p_n with mean free time $\bar{t}_1, \bar{t}_2, \bar{t}_3 ... \bar{t}_{n,}$ the composite probability is given by

$$p = p_1 p_2 p_3 \ldots p_n$$

$$e^{(-t/\bar{t})} = e^{-t/\bar{t}_1} \times e^{-t/\bar{t}_2} \times e^{-t/\bar{t}_3} \times ... e^{-t/\bar{t}_n}$$

$$= e^{-t\left(\frac{1}{\bar{t}_1} + \frac{1}{\bar{t}_2} + ... \frac{1}{\bar{t}_n}\right)}$$

$$\therefore \quad \frac{1}{\bar{t}} = \frac{1}{\bar{t}_1} + \frac{1}{\bar{t}_2} + ... \frac{1}{\bar{t}_n}$$

Problem 1.7

Derive expression for the displacement of Fermi level in an intrinsic semiconductor from the middle of its band gap at equilibrium.

Solution

For intrinsic semiconductor under thermal equilibrium,

$$p_o = n_o$$

$$\text{i.e.} \qquad N_V\, e^{-(E_i - E_V)/kT} = N_C\, e^{-(E_C - E_i)/kT}$$

$$e^{-(E_V + E_C - 2E_i)/kT} = \frac{N_C}{N_V}$$

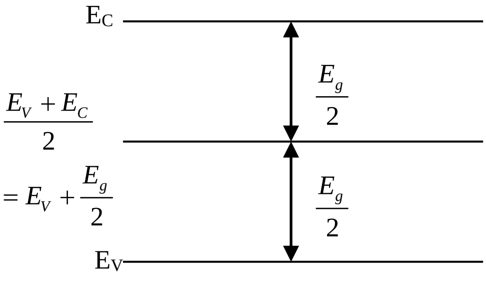


Fig. Sp.1.7

Taking natural logarithm

$$E_V + E_C - 2E_i = kT\ln\frac{N_C}{N_V}$$

$$\frac{E_V + E_C}{2} - E_i = \frac{kT}{2}\ln\frac{N_C}{N_V}$$

$$= \frac{kT}{2}\ln\left(\frac{m_n^*}{m_n^*}\right)^{\frac{3}{2}} \qquad \text{(by equations (1.40) and (l.46))}$$

$$= \frac{3}{4}kT\ln\left(\frac{m_n^*}{m_p^*}\right)$$

$$E_V + \frac{E_g}{2} - E_i = \frac{3}{4}kT\ln\left(\frac{m_n^*}{m_p^*}\right) \text{(see fig. Sp. 1.7)}$$

Remark: If $m_n^* = m_p^*; \frac{E_V + E_C}{2} - E_i = 0.$ Fermi level lies at the middle of the band gap $\left(E_V + \frac{E_g}{2}\right)$. Usually effective masses of electrons (m_n^*) and holes (m_p^*) are different in semiconductors. Because of this the Fermi level in intrinsic semiconductor will be slightly displaced from the middle of the band gap as shown by the above equation and as illustrated in the next problem.

Problem 1.8

Find the displacement of intrinsic Fermi level from the middle of the band gap for Ge, Si, and GaAs at 300 K. The effective masses are as given below.

	Ge	Si	GaAs
m_n^*	0.55m$_0$	1.18 m$_0$	0.068 m$_0$
m_p^*	0.37 m$_0$	0.81 m$_0$	0.56 m$_0$

Solution

$$\Delta E_i = E_V + \frac{E_g}{2} - E_i$$

$$= \frac{3}{4}kT\ln\left(\frac{m_n^*}{m_p^*}\right)$$

$$\Delta E_i \text{ for Ge} = \frac{3}{4} \times 0.026\ln\left(\frac{0.55}{0.37}\right) = 7.73 \times 10^{-3}\text{ eV}$$

$$\Delta E_i \text{ for Si} = \frac{3}{4} \times 0.026 \ln\left(\frac{1.18}{0.81}\right) = 7.73 \times 10^{-3} \text{ eV}$$

$$\Delta E_i \text{ for GaAs} = \frac{3}{4} \times 0.026 \ln\left(\frac{0.068}{0.56}\right) = -0.0411 \text{ eV}$$

Remark : E_i is below middle of band gap for Ge and Si and above it for GaAs.

Problem 1.9

For a silicon sample at 300 K $p_o = 4 \times 10^{12}$ cm^{-3} a. determine the electron density b. determine the acceptor density if donor density is 10^{12} cm^{-3}.

Solution

$$p_o = 4 \times 10^{12} \text{ cm}^{-3}$$

$$\therefore \text{no} = \frac{n_i^2}{p_o}$$

$$= \frac{\left(1.5\times10^{10}\right)^2}{4\times10^{10}} = 5.625 \times 10^7 \text{ cm}^{-3}$$

If all impurities are ionized,

$$p_o - n_o = N_A - N_D$$

$$\therefore N_A = p_o - n_o + N_D$$

$$N_D = 10^{12} \text{ cm}^{-3}$$

$$\therefore N_A = 4 \times 10^{12} - 5.625 \times 10^7 + 10^{12}$$

$$= 5 \times 10^{12} \text{ cm}^{-3}$$

Problem 1.10

The Fermi level in a silicon sample at 300 K is located at 0.3 eV below the bottom of the conduction band. The effective densities of states $N_C = 3.22 \times 10^{19}$ cm^{-3} and $N_V = 1.83 \times 10^{19}$ cm^{-3}. Determine (a) the electron and hole concentrations (b) the intrinsic carrier concentration at 300 K.

Solution

$$n_o = N_C \; e^{-(E_C - E_F)} / kT$$

$$= 3.22 \times 10^{19} \; e^{\frac{-0.3}{0.026}}$$

$$= 3.14 \times 10^{14} \text{ cm}^{-3}$$

$$E_F - E_V = E_g - (E_C - E_F) \text{ (see Fig. Sp. 1.10)}$$

$$= 1.1 - 0.3 = 0.8 \text{ eV}$$

$$p_o = N_V \; e^{-(E_F - E_V)} / kT$$

$$= 1.83 \times 10^{19} \; e^{\frac{-0.8}{0.026}}$$

$$= 7.93 \times 10^5 \text{ cm}^{-3}$$

$$n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$$

$$= \sqrt{3.22\times10^{19} \times 1.83\times10^{19}} e^{\frac{-1.1}{2\times0.026}} \; 1{\wedge}6$$

$$= 1.578 \times 10^{10}\ \text{cm}^{-3}$$

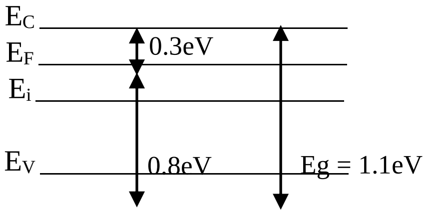


Fig. Sp.1.10

Problem 1.11

The Fermi level in a silicon sample at equilibrium is located 0.40 eV below the middle of band gap at 300 K.

a. Determine the probability of occupancy of a state located at the middle of band gap
b. Determine the probability of occupancy of acceptor states if they are located 0.04 eV above the top of the valence band.

Solution

a. Probability of occupancy of the middle of band gap

$$f(E_i) = \frac{1}{1+e^{(E_i-E_F)/kT}}$$

$$= \frac{1}{1+e^{\frac{0.4}{0.026}}}$$

$$= 2.08 \times 10^{-7}$$

b. From Fig. Sp. 1.11,

$$E_A - E_F = -[E_F - E_V - (E_A - E_V)]$$

$$= -[0.15 - 0.04]$$

$$= -0.11\ \text{eV}$$

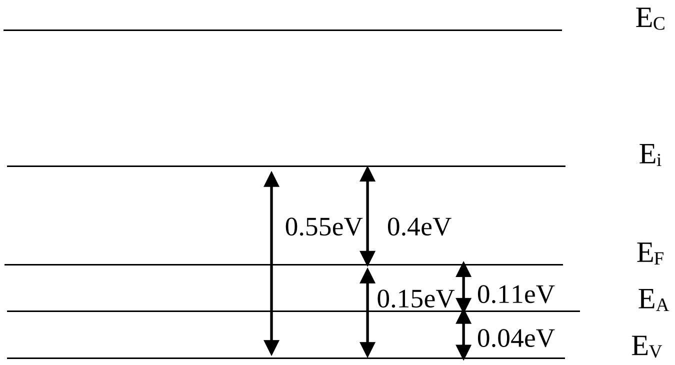


Fig. Sp.1.11

Probability of occupancy of acceptor state

$$f(E_A) = \frac{1}{1+e^{(E_A-E_F)/kT}}$$

$$= \frac{1}{1+e^{\frac{-0.11}{0.026}}} = 0.9856$$

Problem 1.12

The intrinsic carrier concentration of silicon at 300 K is 1.5×10^{10} cm^{-3}, $\mu_n = 1500$ cm^2/Vs and $\mu_p = 450$ cm^2/Vs. The silicon sample is doped with donors of concentration 5×10^{16} cm^3 and then compensated by acceptor doping. The compensated silicon sample has resistivity 1 Ω cm. Determine acceptor concentration, assuming that the mobilities remain unchanged.

Solution

$$N_D = 5 \times 10^{16} \text{ cm}^{-3} >> n_i$$

$$\therefore n_o = 5 \times 10^{16} \text{ cm}^{-3}$$

$$\rho = \frac{1}{qn_o\mu_n} = \frac{1}{1.6\times10^{-19}\times5\times10^{16}\times1500}$$

$$= 0.083\ \Omega\text{cm}$$

After compensation

Case 1: $N_D > N_A$

$$\rho = \frac{1}{qn_o\mu_n} 1\Omega\text{cm}$$

$$n_o^{'} = \frac{1}{1.6\times10^{-19}\times1500}$$

$$= 4.16 \times 10^{15}$$

$$n_o^{'} = N_D - N_A$$

$$\therefore N_A = N_D - n_o^{'}$$

$$= 5 \times 10^{16} - 4.16 \times 10^{15}$$

$$= 4.58 \times 10^{16} \text{ cm}^{-3}$$

Case 2: $N_A > N_D$

$$\rho = \frac{1}{qp_o\mu_p} 1\Omega\text{cm}$$

$$p_o' = \frac{1}{1.6\times10^{-19}\times450}$$

$$= 1.389 \times 10^{16}$$

$$p_o' = N_A - N_D$$

$$\therefore N_A = p_o' + N_D$$

$$= 1.389 \times 10^{16} + 5 \times 10^{16}$$

$$= 6.389 \times 10^{16}\ \text{cm}^{-3}$$

Problem 1.13

An n-type silicon bar 0.1 cm long and 100 μm^2 in cross sectional area has a majority carrier concentration of 5 x 10^{15} cm^{-3} and electron mobility is 1300 cm^2/Vs at 300 K.

What is the resistance of the bar?

Solution

$$n_o = 5 \times 10^{15}\ \text{cm}^{-3},\ A = 100\ \mu\text{m}^2 = 100 \times 10^{-8}\ \text{cm}^2$$

$$\rho = \frac{1}{\sigma} = \frac{1}{qn_o\mu_n}$$

$$= \frac{1}{1.6\times10^{-19}\times5\times10^{15}\times1300} = 0.962\ \Omega\ \text{cm}$$

$$R = \frac{\rho L}{A}$$

$$= \frac{0.962\times0.1}{100\times10^{-8}} = 96200\Omega cm$$

$$= 96.2\ \text{k}\Omega$$

Problem 1.14

An intrinsic Ge sample at room temperature has resistivity of 50 Ω cm. The sample is uniformly doped to the extent of 6 × 10^{13} Arsrnic atoms/cm^3 and 10^{14} Boron atoms per cm^3. Find the conduction current density if an electric field of 4 V/cm is applied across the sample. μ_n = 3800 cm^2 / Vs; μ_p = 1800 cm^2 / Vs.

Solution

$$\rho = \frac{1}{qn_i(\mu_n + \mu_p)}$$

$$n_i = \frac{1}{q\rho(\mu_n + \mu_p)}$$

$$= \frac{1}{1.6\times10^{-19}\times50(3800+1800)} = 2.23 \times 10^{13}\ \text{cm}^{-3}$$

$$N_D = 6 \times 10^{13}\text{cm}^{-3} \quad N_A = 10^{14}\ \text{cm}^{-3}$$

From charge neutrality equation, assuming that impurities are completely ionized

$p_o - n_o = N_A - N_D$

$= 10^{14} - 6 \times 10^{13}$

$= 4 \times 10^{13}\ \text{cm}^{-3}$

$n_o - p_o = n_i^2$

$\therefore\ p_o - \dfrac{n_i^2}{p_o} = N_A - N_D$

$p_0^2 - (N_A - N_D)\, p_0 - n_i^2 = 0$

$$p_0 = \frac{(N_A - N_D) \pm \sqrt{(N_A - N_D)^2 + 4n_i^2}}{2}$$

$$p_0 = \frac{4\times10^{13} \pm \sqrt{(4\times10^{13})^2 + 4\times(2.23\times10^{13})^2}}{2}$$

$= 5 \times 10^{13}\ \text{cm}^{-3}$ (negative value neglected)

$$n_0 = \frac{n_i^2}{p_o}$$

$$= \frac{\left(2.23\times10^{13}\right)^2}{5\times10^{13}}$$

$= 9.94 \times 10^{12}\ \text{cm}^{-3}$

$J = q\,(\mu_n n_o + \mu_p p_o)\, E_x$

$= 1.6 \times 10^{-19}\,(3800 \times 9.94 \times 10^{12} + 1800 \times 5 \times 10^{13}) \times 4$

$= 0.0817\ \text{A/cm}^2$

Problem 1.15

A sample of intrinsic semiconductor has resistance of 10 Ω, at 364 K and 100 Ω at 333 K, Assuming that this change is entirely due to the effect of variation of n_i with temperature, calculate the band gap E_{go} of the semiconductor.

Solution

At $T_1 = 364\ \text{K}$ $R = 10\ \Omega$

$T_2 = 333\ \text{K}$ $R = 100\ \Omega$

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}\, KT^{\frac{3}{2}} e^{-E_g/2kT}$$

$$\frac{R(T_1)}{R(T_2)} = \frac{\frac{\rho_{(T_1)}L}{A}}{\frac{\rho_{(T_2)}L}{A}} = \frac{\sigma_i(T_2)}{\sigma_i(T_1)} = \frac{n_i(T_2)}{n_i(T_1)}$$

$$\frac{R(T_1)}{R(T_2)} = \frac{10}{100} = \frac{n_i(T_2)}{n_i(T_1)} = \left(\frac{T_2}{T_1}\right)^{\frac{3}{2}} \frac{e^{-E_{go}/2kT_2}}{e^{-E_{go}/2kT_1}}$$

i.e., $$\frac{1}{10} = \left(\frac{T_2}{T_1}\right)^{\frac{3}{2}} e^{\frac{-E_{go}}{2k}\left[\frac{1}{T_2}-\frac{1}{T_1}\right]}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{3}{2}} \times \frac{1}{10} = e^{\frac{-E_{go}}{2k}\left[\frac{1}{T_2}-\frac{1}{T_1}\right]}$$

$$\left(\frac{364}{333}\right)^{\frac{3}{2}} \times \frac{1}{10} = e^{\frac{-E_{go}}{2k}\left[\frac{1}{333}-\frac{1}{364}\right]}$$

Taking natural logarithm

$$\ln 0.11428 = \frac{-E_{go}}{2k}(2.557 \times 10^{-4})$$

$$E_{go} = \frac{2.169 \times 2 \times 1.38066 \times 10^{-23}}{2.557 \times 10^{-4}}$$

$$= 2.34 \times 10^{-19}\text{ J}$$

$$= \frac{2.34 \times 10^{-19}}{1.6 \times 10^{-19}} eV = 1.46\text{eV}$$

Problem 1.16

Prove that the fractional change in conductivity of an intrinsic semiconductor is given by $\frac{d\sigma}{\sigma} = \frac{dT}{T}\left(\frac{3}{2} + \frac{E_g}{2kT}\right)$ assuming that the change in conductivity is only due to the variation of intrinsic carrier concentration with temperature.

Solution

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

$$= K_1 T^{\frac{3}{2}} e^{-E_g/2kT} \qquad \left(\begin{array}{l}\text{Substituting values of } N_C \text{ and } N_V \\ \text{from equations } (1.40) \text{ and } (1.46)\end{array}\right)$$

$$\sigma = 2qn_i(\mu_n + \mu_p)$$

$$\therefore \quad \sigma_{(T)} = 2q K_1 T^{\frac{3}{2}} e^{-E_g/2kT} (\mu_n + \mu_p) = K_2 T^{\frac{3}{2}} e^{-E_g/2kT}$$

where $K_2 = 2K_1 q(\mu_n + \mu_p)$

Taking natural logarithm

$$\ln \sigma = \ln K_2 + \frac{3}{2}\ln T - \frac{E_g}{2kT^2}$$

On differentiation with respect to T

$$\frac{d\sigma}{\sigma} = \frac{dT}{T}\left[\frac{3}{2} + \frac{E_g}{2kT}\right]$$

Problem 1.17

The conductivity of an intrinsic silicon sample at 300 K is 4.4×10^{-4} $(\Omega\text{ cm})^{-1}$. Determine the conductivity of the semi conductor if temperature rises to 315 K.

Solution

$$\frac{d\sigma}{\sigma} = \frac{dT}{T}\left[\frac{3}{2} + \frac{E_g}{2kT}\right]$$

$\sigma = 4.4 \times 10^{-4}\ (\Omega\text{ cm})^{-1}$

$dT = 15\text{ K}$

$kT = 0.026\text{ eV}$

$$T = 300\ \text{K}$$

$$\sigma_{\text{new}} = \sigma + d\sigma = \sigma\left[1+\frac{dT}{T}\left(\frac{3}{2}+\frac{E_g}{2kT}\right)\right]$$

$$= 4.4 \times 10^{-4}\left[1+\frac{15}{300}\left(\frac{3}{2}+\frac{1.1}{2\times 0.026}\right)\right]$$

$$= 9.38 \times 10^{-4}\ (\Omega\ \text{cm})^{-1}$$

Problem 1.18

The resistivity ρ_o of a Ge sample is measured at 300 K. The sample is then remelted and doped with 4.4×10^{16} arsenic atoms cm^{-3}. The new crystal has resistivity of 0.1 Ω cm and is n-type. Determine the type and concentration of impurity atoms in the original sample and value of ρ_o. Assume $\mu_n = 2\ \mu_p = 3000$ cm^2/Vs.

Solution

After doping we have,

$$\rho = \frac{1}{q\mu_n n_o} = 0.1\Omega cm$$

$$\therefore\quad n_0 = (1.6 \times 10^{-19} \times 3000 \times 0.1)^{-1}$$

$$= 2.083 \times 10^{16}\ \text{cm}^{-3}$$

which is less than the added donor concentration $N_D = 4.4 \times 10^{16}$ cm^{-3}.

The original sample was p-type.

$$N_D - N_A = n_o$$

$$\therefore\quad N_A = 4.4 \times 10^{16} - 2.083 \times 10^{16}$$

$$= 2.317 \times 10^{16}\ \text{cm}^{-3}$$

where N_A is the acceptor doping in the original sample.

Therefore, for the original sample $p_o = N_A = 2.318 \times 10^{16}$ cm^{-3}

Resistivity of original sample $p_o = \dfrac{1}{qp_0\mu_p}$

$$p_o = (1.6 \times 10^{-19} \times 2.318 \times 10^{16} \times 1500)^{-1}$$

$$= 0.179\ \Omega\ \text{cm}$$

Problem 1.19

The Fermi-level position in an n-type Ge film is 0.2 eV above the intrinsic Fermi-level. The thickness of the film is 0.5 μm. Calculate the sheet resistance of the film.

$n_i = 2.5 \times 10^{13}$ cm^{-3}, $\mu_n = 3500$ cm^2 /Vs, $\mu_p = 1500$ cm^2 /Vs, kT/q = 0.025 V.

Solution

$$n_o = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = 0.2\ \text{eV}$$

$$\frac{kT}{q} = 0.025\ \text{V} \qquad \therefore\ kT = 0.025\ \text{eV}$$

$$\therefore\ n_o = 2.5 \times 10^{13}\ e^{(0\text{-}2/0\text{-}025)}$$

$$= 7.45 \times 10^{16}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(2.5\times10^{13}\right)}{7.45\times10^{16}}$$

$$= 8.39 \times 10^{9}$$

$$\sigma = q(n_o\mu_n + p_0\mu_p)$$

$$= 1.6 \times 10^{-19}\ (7.45 \times 10^{16} \times 3500 + 8.39 \times 10^{9} \times 1500)$$

$$= 41.72\ (\Omega\ \text{cm})^{-1}$$

Resistance $R = \frac{\rho L}{A} = \rho.\frac{L}{w.t} = \frac{\rho}{t}.\frac{L}{w}$

$= Rs.\ \frac{L}{w}$ where R_S is sheet resistance

$$R_S = \frac{\rho}{t} = \frac{1}{\sigma t} = \frac{1}{41.72\times0.5\times10^{-4}}$$

$$= 479.39\ \Omega/\square$$

Problem 1.20

A GaAs sample is doped so that the electron and hole components of currents are equal in an applied electric field. Calculate the equilibrium electron-hole concentrations, the net doping and the sample resistivity at 300 K.

$\mu_n = 8500\ \text{cm}^2/\text{Vs}$, $\mu_p = 400\ \text{cm}^2\ /\text{Vs}$, $n_i = 1.79 \times 10^6\ \text{cm}^{-3}$

Solution

For a given electric field,

$J_{n\ drift} = J_{p\ drift}$ when $\sigma_n = \sigma_p$

i.e., $\mu_p\ p_o = \mu_n\ n_o$ or $p_o = \frac{\mu_n}{\mu_p}\times n_0$

For GaAs $n_o p_o = n_i^2$

$$n_o \times 21.25\ n_o = (1.79 \times 10^6)^2$$

$$\therefore\ n_o = 3.88 \times 10^5\ \text{cm}^{-3}$$

$$p_o = 21.25 \times 3.88 \times 10^5$$

$$= 8.25 \times 10^6\ \text{cm}^{-3}$$

$p_o > n_o$ $\therefore$ Net doping $= N_A - N_D = p_o - n_o$

Net doping $= (8.25 \times 10^6 - 3.88 \times 10^5)$

$= 7.86 \times 10^6\ \text{cm}^{-3}$

Sample resistivity $\rho = \frac{1}{q[\mu_n n_0 + \mu_p p_o]}$

$$= \frac{1}{2q\mu_n n_0}$$

$$= \frac{1}{2\times1.6\times10^{-19}\times8500\times3.88\times10^5}$$

$$= 9.48 \times 10^8\ \Omega\ \text{cm}.$$

Problem 1.21

A silicon sample is doped with 10^{15} phosphorous atoms/cm^3. At 300 K, calculate, the electron and hole concentrations and position of the Fermi level. Assume that the distribution of states in the conduction band is given by N(E)dE = $8 \times 10^{20}\sqrt{E}dE$ cm^{-3} where E is expressed in electron volts. Calculate the number of electrons between the energy interval 1.9 kT and 2.1 kT above the band edge E_C.

Solution

Let $n_i = 1.5 \times 10^{10}$ cm^{-3}

$N_D = 10^{15}$ cm^{-3} >> n_i

$\therefore$ $n_o = N_D = 10^{15}$ cm^{-3}

$n_o p_o = (1.5 \times 10^{10})^2$

$\therefore$ $p_o = 2.25 \times 10^5$ cm^{-3}

$$n_o = n_i e^{(E_F - E_i)/kT}$$

$$E_F = E_i + kT \ln \frac{n_0}{n_i} = E_i + 0.287 \text{ eV}$$

$$\therefore E_C - E_F = \frac{E_g}{2} - (E_F - E_i)$$

$$= 0.56 - 0.287 = 0.273 \text{ eV}$$

At 300 K, $E_C - E_F = 0.273$ eV

$$= \frac{0.273}{0.026} kT \qquad (\text{Q } 1 \text{ kT} = 0.026 \text{ eV})$$

$$= 10.5 \text{ kT} .$$

Assuming that f(E) is constant from (E_C + 1.9 kT) to (E_C + 2.1 kT),

$$f(E_C + 2kT) = \frac{1}{1 + e^{(E_C + 2kT - E_F)/kT}} = \frac{1}{1 + e^{(E_C - E_F + 2kT)/kT}}$$

$$= \frac{1}{1 + e^{(10.55 + 2)kT/kT}} = \frac{1}{1 + e^{12.5}} = 3.726 \times 10^{-6}$$

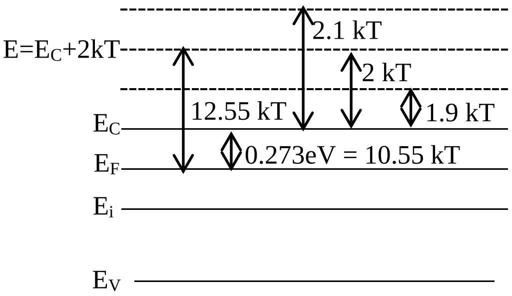


Fig. Sp. 1.21

To find the number of electrons (Δn) in the energy interval 1.9 kT to 2.1 kT above E_C, multiply the probability of occupancy at 2kT above E_C with total number of states this energy range. See also Fig. Sp.1.21.

$$\Delta n = f(E_C + 2kT) \int_{1.9kT}^{2.1kT} N(E)dE$$

$$= 3.726 \times 10^{-6} \times 8 \times 10^{20} \int_{19kT}^{2.1kT} \sqrt{E}dE$$

$$= 2.981 \times 10^{15} \times \frac{2}{3} \times \left[E^{\frac{3}{2}}\right]_{19kT}^{2.1kT} \qquad kT = 0.026 \text{ eV}$$

$$= 3.53 \times 10^{12} \text{ cm}^{-3}.$$

Problem 1.22

A semiconductor is known to have a band gap of 1.25 eV and intrinsic carrier concentraton of 1.6×10^{10} cm^{-3} at room temperature. Estimate N_C and N_V, if $m_n^* : m_p^* = 4:1$.

Solution

$$n_i^2 = N_C N_V \; e^{-(E_C - E_V)/kT}$$

$$N_C N_V = n_i^2 \; e^{-(E_C - E_V)/kT}$$

$$= (1.6 \times 10^{10})^2 \; e^{\frac{1.25}{0.025}}$$

$$= 1.327 \times 10^{42} \text{ cm}^{-6} \qquad \text{(A)}$$

$$\frac{N_C}{N_V} = \left(\frac{m_n^*}{m_p^*}\right)^{\frac{3}{2}} = (4)^{\frac{3}{2}}$$

$$\therefore \; N_C = 8N_V$$

Substituting in (A)

$$8\,N_V^2 = 1.327 \times 10^{42}$$

$$N_V = 4.07 \times 10^{20} \text{ cm}^{-3}$$

$$N_C = 3.26 \times 10^{21} \text{ cm}^{-3}$$

Problem 1.23

The current required to feed the hole injection at x = 0 is given by $I_p(x=0) = qAD_p\Delta_p/L_p$. Show that this current can be obtained by integrating the charge stored in the steady-state hoie distribution $\delta_p(x)$ and then dividing by the average hole lifetime τ_p. Explain why this approach gives $I_p(x = 0)$.

Solution

$$Q_p = qA \times \int_0^\infty \delta p(x)dx$$

$$\therefore Q_p = qA\int_0^\infty \Delta p e^{-x/L_p} dx$$

$$= qAL_p\Delta_p$$

$$= \frac{Q_p}{\tau_p} = \frac{qAL_p\Delta p}{\tau_p}$$

$$D_p\tau_p = L_p^2$$

i.e., $$\frac{D_p}{L_p} = \frac{L_p}{\tau_p}$$

$$\therefore\ I_p = qAD_p.\ \frac{\Delta_p}{\tau_p}$$

The charge distribution Q_p disappears due to recombination in every τ_p seconds. Therefore, the steady-state distribution is retained by injection in every τ_p seconds.

$\therefore$ hole current injected at x = 0 is $\frac{Q_p}{\tau_p}$

Problem 1.24

In figure Sp.1.24, the steady-state excess hole concentration at x = 0 is $\Delta_p = 10^{15}$ cm^{-3}. The semi infinite Ge bar has a cross-sectional area of 10^{-2} cm^2. The hole diffusion length L_p is 10^{-3} cm and the hole lifetime is 10^{-4} s.

a. What is the steady-state stored charge Q_p in the exponential excess hole distribution?

b. What is the hole current $I_p(x = 0)$ feeding this steady-state distribution.

c. What is the slope of the distribution in cm^{-4} at x = 0.

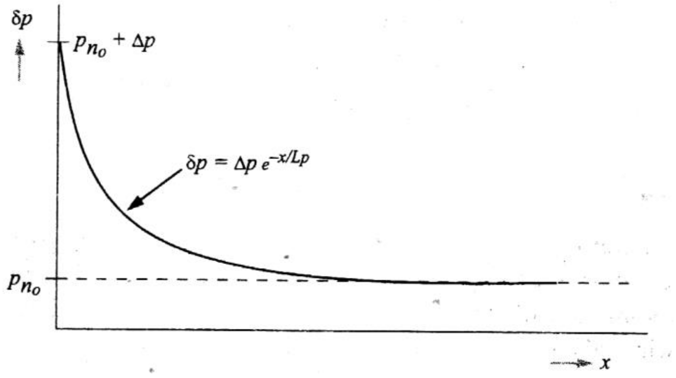


Fig. Sp. 1.24

Solution

a. $Q_p = qA\,L_p\,\Delta_p$

$= 1.6 \times 10^{-19} \times 10^{-2} \times 10^{-3} \times 10^{15}$

$= 1.6 \times 10^{-9}$ C

b. $I_p = \frac{Q_p}{\tau_p}$

$= \frac{1.6\times10^{-9}}{10^{-4}} = 1.6 \times 10^{-5}$ A

c. $$\left.\frac{d\delta p}{dx}\right|_{x=0} = \frac{I_p}{qAD_p} = \frac{-1.6\times10^{-5}}{1.6\times10^{-19}\ 10^{-2}\times10^{-2}}$$

$= -10^{18}$ cm^{-4}.

Problem 1.25

A p-type semiconductor sample with a resistivity of 40 Ωcm at 300 K is uniformly illuminated with light that generates 10^{13} EHP cm^{-3} s^{-1}. In steady-state, calculate the change in resistivity of the sample caused by light. If light is switched off at t = 0, calculate the time required for the excess conductivity to drop to one tenth of its value at t=0. Assume $\tau_n = 10^{-6}$ s. $\mu_n = 3900$ cm^2/Vs, $\mu_p = 1900$ cm^2/Vs, $n_i = 2.4 \times 10^{13} cm^{-3}$

Solution

$$\delta_p = \delta_n = G_{op}\tau_n$$

$$= 10^{13} \times 10^{-6} = 10^7 \text{ cm}^{-3}$$

$$\text{Photo conductivity } \sigma_{op} = q(\mu_n + \mu_p)\, \delta p$$

$$= 1.6 \times 10^{-19} (3900 + 1900)\, 10^7$$

$$= 9.28 \times 10^{-9} (\Omega \text{ cm})^{-1}$$

$$\text{Original-conductivity} = \frac{1}{\rho} = \frac{1}{40} = 2.5 \times 10^{-2} (\Omega \text{ cm})^{-1}$$

which is large compared to Δσ.

Therefore, change in σ is negligible. Δσ drops to 10% when δp and δn drops to 10% of its initial value.

$$\frac{\delta n(t)}{\Delta n} = e^{-t/\tau_n} = \frac{10}{100} = 0.1$$

$$e^{-t/T_n} = \frac{1}{10}$$

$$t = \tau_n \ln 10$$

$$= 10^{-6} \times 2.302$$

$$= 2.3 \times 10^{-6} \text{ s.}$$

Problem 1.26

An n-type silicon sample with $n_o >> n_i$ and $\mu_n = 2\mu_p$ is subjected to a steady illumination such that the electron concentration doubles from its thermal equilibrium value. By what factor will the specimen's conductivity increase?

Solution

Let n_o be the equilibrium electron concentration in the sample. Since the sample is n-type $n_o >> p_o$. Therefore equilibrium conductivity

$$\sigma_o = qn_o\mu_n = qn_o \,.\, 2\mu_p = 2qn_o\mu_p$$

Let n and p be the new electron and hole concentrations. As the electron concentration doubles

$$n = 2n_o \qquad \text{i.e., } \delta n = n_o = \delta p$$

$$\therefore \qquad p = p_o + \delta p \cong \delta p = n_o$$

Therefore conductivity of specimen with illumination

$$\sigma = qn\mu_n + qp\mu_p$$

$$= q(2n_o)2\mu_p + qn_o\mu_p$$

$$= 4qn_o\mu_p + qn_o\mu_p$$

$$= 5qn_o\mu_p$$

$$\frac{\sigma}{\sigma_o} = \frac{5qn_o\mu_p}{2qn_o\mu_p} = 2.5$$

Therefore the conductivity of the specimen will increase by a factor of 2.5.

Problem 1.27

The current I in a forward biased p-n junction in Fig. Sp. 1.27 is entirely due to diffusion of holes from x = 0 to x = L. The injected hole concentration distribution in the n-region is linear as shown in figure with $p(0) = 10^{12}$ /cm^3 and $L = 10^{-3}$ cm.

Determine (a) the current density in the diode assuming that $D_p = 12$ cm^2/s (b) the velocity of holes in the n-region at x = 0

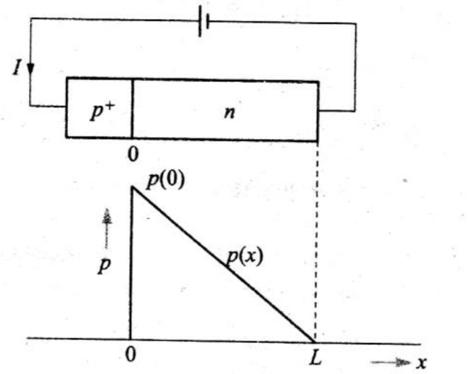


Fig. Sp. 1.27

Solution

$$\frac{dp(x)}{dx} = \frac{p(0)-p(L)}{0-L}$$

$$= \frac{10^{12}-0}{0-10^{-3}} = -10^{15} cm^{-4}$$

$$J = -qD_p \frac{dp(x)}{dx} = -1.6 \times 10^{-19} \times 12 \times -10^{15}$$

$$= 1.92 \times 10^{-3} \text{ A/cm}^2 = 1.92 \text{ mA/cm}^2$$

$$J(0) = p(0)qv$$

$$\therefore \text{ velocity } v = \frac{1.92\times10^{-3}}{1.6\times10^{-10}\times10^{12}} = 1.2\times10^{4}\text{ cm/s}$$

Problem 1.28

Interpret the energy band diagram shown in Fig. Sp.1.28.

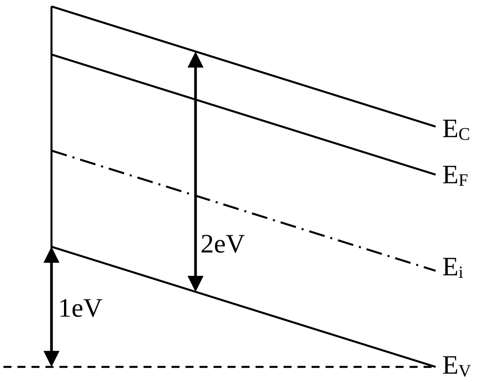


Fig. Sp.1.28

Solution

a. Band gap = 2 eV,

b. n-type semiconductor,

c. non equilibrium, $\frac{dE_F}{dx} \neq 0$,

d. External electric field in the negative x-direction,

e. $\delta n = \delta p = 0$ as $F_n = F_p = E_F$,

f. Specimen is uniformly doped (Q E_C - E_F is constant),

g. Potential difference between the ends of the semiconductor

$$V = \frac{\Delta E}{(-q)} = \frac{1eV}{-q} = -1V.$$

h. No excess carriers present.

Problem 1.29

Interpret the energy band diagram shown in Fig. Sp.1.29.

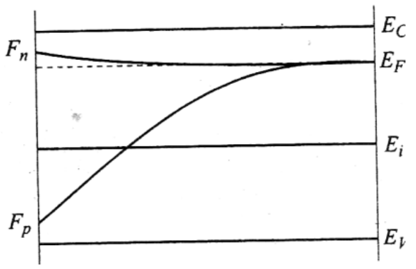


Fig. Sp.1.29

Solution

a. The specimen is not in equilibrium $\left(\frac{dE_F}{dx} \neq 0\right)$.

b. The energy band diagram represents a uniformly doped n-type semiconductor with excess carriers injected from x = 0.

c. As the energy bands are flat no electric field exist in the semiconductor.

d. Excess carrier injection is low-level as the shift in majority carrier Fermi-level (F_n) is negligible.

e. Seperation of quasi Fermi-levels decreases from left to right and join together at the end. This indicates that injected excess carriers diffuse into the bar, undergoes. recombination and decays to zero at the end of the bar.

Problem 1.30

A Silicon sample is doped with 10^{15} donors cm^{-3}. Calculate the excess electron and hole concentrations $\delta n = \delta p$ required to increase the sample conductivity by 15%. What carrier generation rate is required to maintain this excess concentration. Assume $\mu_p = 0.3\ \mu_n$, $\tau_p = 10^{-6}$ s, T = 300 K.

Solution

$$n_o = 10^{15}\ cm^{-3}$$

Since specimen is n-type

$$\sigma = q\mu_n n_o = 1.6 \times 10^{-19} \times 10^{15}\ \mu_n$$

$$= 1.6 \times 10^{-4}\ \mu_n$$

For sample conductivity to increase by 15%

$$\Delta\sigma = q(\mu_n + \mu_p)\ \delta p = 0.15\ \sigma$$

$$1.6 \times 10^{-19}\ (\mu_n + 0.3\mu_n)\ \delta p = 0.15 \times 1.6 \times 10^{-4}\ \mu_n$$

$$\delta p = 1.15 \times 10^{14}\ cm^{-3} = \delta n$$

$$\text{Carrier generation rate}\quad G = \frac{\delta p}{\tau_p}$$

$$= \frac{1.15\times10^{14}}{10^{-6}} = 1.15 \times 10^{20}\ cm^{-3}\ s\text{-}1.$$

Problem 1.31

An n-type bar of GaAs has a length / and an area of cross-section A. The bar is illuminated with light that generates G_{op} electron hole pairs $cm^{-3}\ s^{-1}$ uniformly. Assuming the hole mobility to be negligible, show that the steady-state photo current in the bar can be written as $I_{op} = qAlG_{op} \frac{\tau_p}{\tau_t}$ where τ_t is the average transit time of electrons through the bar and τ_p is the minority carrier lifetime.

Solution

$$\delta n = \delta p = G_{op}\ \tau_p$$

$$E = \frac{v_d}{\mu_n}$$

Since $\mu_n >> \mu_p$, $\sigma_{op} = q\mu_n\ \delta n$

Photo current $I_{op} = \sigma_{op} E = qA\mu_n \delta n E = qAG_{op} \tau_p v_d$

but $$v_d = \frac{l}{\tau t}$$

$$\therefore\ I_{op} = qAlG_{op}\frac{\tau_p}{\tau_t}$$

Problem 1.32

A Ge sample is uniformly doped with 5 × 10^{16} atoms/cm^3 of In. Assume all impurity atoms are ionized and take $n_i = 2 \times 10^{13}$ cm^{-3} at 290 K.

a. Calculate the electron and hole concentrations in the sample.
b. Assume that the intrinsic concentration in Ge increases by 6% per ^{o}K rise in temperature, estimate the temperature (T) at which the sample becomes intrinsic.

Solution

a. Indium is acceptor impurity in Germanium

so $N_A = 5 \times 10^{16}$ cm^{-3} >> n_i

$$\therefore\quad p_o = 5 \times 10^{16}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2\times10^{13}\right)^2}{5\times10^{16}}$$

$$= 8 \times 10^9\ \text{cm}^{-3}$$

b. At intrinsic temperature $N_A \cong 5n_i$

$$n_i = \frac{N_A}{5} = \frac{5\times10^{16}}{5} = 10^{16}\ \text{cm}^{-3}$$

It is given that

$$\frac{1}{n_i}\frac{dn_i}{dT} = 0.06$$

$$\frac{dn_i}{n_i} = 0.06\ dT$$

Integrating, $\ln n_i = 0.06T + C$; where C is constant of integration

$$\therefore\quad n_i = C\ e^{006T}$$

$$n_i\ (T) = C\ e^{0.06T} = 10^{16} \qquad \text{(A)}$$

$$n_i\ (290) = C\ e^{(0.06 \times 290)} = 2 \times 10^{13} \qquad \text{(B)}$$

Equations (A) ÷ (B)

$$\frac{C\ e^{0.06T}}{C\ e^{17.4}} = \frac{10^{16}}{2\times10^{13}}$$

$$e^{(0.06T-17.4)} = 500$$

$$0.06T - 17.4 = \ln 500$$

$$\therefore\ T = \frac{6.215+17.4}{0.06} = 393.6K.$$

▪ Points to Remember

- Heisenberg uncertainty principle

$$\Delta P \Delta x \geq \hbar \;; \qquad \Delta E\, \Delta t \geq h$$

- de Broglie wavelength $\lambda = \frac{h}{mv}$
- Time independant Schrodinger wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\Phi = 0$$

- Fermi Dirac distribution function

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

- Effective mass

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dK^2}}$$

- Thermal generation rate $g_{(T)} = \alpha_r\, n_i^2$
- Recombination rate $r = \alpha_r\, np$
- Density of states in conduction band

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

- Density of states in valence band

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_o = N_C e^{-(E_C - E_F)/kT = n_i e(E_F - E_i)/kT}$$

$$p_o = N_V e^{-(E_F - E_V)/kT = n_i e(E_i - E_F)/kT}$$

- Mass action law,

$$n_o\, p_o = n_i^2$$

$$n_i(T) = \sqrt{N_C N_V}\, e^{-E_g/2kT} = KT^{3/2} e^{-E_{go}/2kT}$$

- For T > 50 K

$n_i(T) = 1.76 \times l0^{16}\, T^{3/2}\, e^{-4550/T}\ \text{cm}^{-3}$ for Ge
$n_i(T) = 3.88 \times l0^{16}\, T^{3/2}\, e^{-7000/T}\ \text{cm}^{-3}$ for Si

- $J_{n\,drift} = -nqv_x = \sigma_n E_x$
- $J_{p\,drift} = pqv_x = \sigma_p E_x$
- Conductivity $\sigma_n = qn\mu_n$; $\sigma_p = qp\mu_p$

Mobility $\mu_n = \frac{q\bar{t}}{m_n^*} = \frac{-v_x}{E_x}$

$\mu_p = \frac{q\bar{t}}{m_p^*} = \frac{v_x}{E_x}$

- $\dfrac{dE_F}{dx} = 0$ at thermal equilibrium
- By Hall effect

  $p_o = \dfrac{I_x B_z}{qtV_{AB}}$

  $\rho = \dfrac{V_{CD} / I_x}{L / wt}$

  $R_H = \dfrac{1}{qp_o}$ or $\dfrac{-1}{qn_o}$

  $\mu = \dfrac{R_H}{\rho}$
- $\tau_n = \dfrac{1}{\alpha_r p_o}$

  $\tau_p = \dfrac{1}{\alpha_r n_o}$
- For direct recombination

  $\delta n = \delta p = g_{op}\, \tau_p$
- Under non-equilibrium

  $np = n_i^2\, e^{(F_n - F_p)/kT}$
- Diffusion currents

  $J_{p\ diff} = -qD_p.\dfrac{dp}{dx}$

  $J_{n\ diff} = qD_n.\dfrac{dn}{dx}$
- Einstein relation

  $$\frac{Dn}{\mu_n} = \frac{Dp}{\mu_p} = \frac{kT}{q}$$
- Continuity equations

  $$\frac{\partial p}{\partial t}(x,t) = \frac{-1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

  $$\frac{\partial n}{\partial t}(x,t) = \frac{-1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$
- Diffusion length is the average distance a charge carrier diffuse before it recombincs.
- Minority carrier lifetime is the average time a charge carrier survive without recombination after it is generated.
- Built-in field is the field that exist in a semiconductor or semiconductor device at equilibrium.

▪ Exercise Problems

(l) (a) The position of an electron is determined with an error less than 0.5 Å. What is the minimum uncertainty in its momentum?

(b) Energy of an electron is measured with an uncertainty of 0.5 eV. What is the minimum uncertainty in the time over which the measurement was made?

Ans: (a) 2.109×10^{-26} gcm/s (b) 1.316×10^{-15} s.

(2) Find the nearest neighbour distance in the following lattices

(a) Body centered cubic

(b) Face centered cubic and

(c) A diamond lattice

Ans: (a) $\frac{\sqrt{3}a}{2}$ (b) $\frac{a}{\sqrt{2}}$ (c) $\frac{\sqrt{3}a}{4}$

(3) For a particle that ha.s a mass of 2 g and energy 1.5 kT, determine de-Broglie wave length at 300 K. kT = 1.24 eV.

Ans: 6.0699×10^{-25}cm.

(4) The antenna of an AM radiostation transmitter radiates 10 kW of power at 1250 KHz. Calculate

(a) Energy of each radiated photon

(b) Number of photons radiated per second

Ans: (a) 8.283×10^{-28} J. (b) 1.207×10^{31}.

(5) At 300 K, what percentage of the valence electrons in a cm^3 of silicon are located in the conduction band? Repeat for 500 K.

Ans: 7.51×10^{-12}%, 1.805×10^{-7}%

(6) The average energy of a free electron is given by E = $\frac{3kT}{2}$ where k is Boltzmann constant, T is temperature in degree Kelvin. Determine

(a) Velocity of electron

(b) The de Brogue wave length in m and Å

(c) Momentum at 300 K

Ans: (a) 3.694×10^3 cm/s (b) 1.969×10^{-12} m, 0.01969 Å (c) 33.64×10^{-25} gcm/s.

(7) Find the resistivity of a silicon sample at 300 K if it is doped with a pentavalent impurity such that there is one impurity atom for 10^7 silicon atoms. μ_n = 1350 cm^2/Vs, μ_p = 480 cm^2/Vs.

Ans: 0.9255 Ω cm.

(8) A Ge sample is doped with 10^{14} As atoms/cm^3 and 7×10^{13} Boron atoms/cm^3 at 300 K. The intrinsic resistivity at this temperature equals 60 Ω cm. Determine the total conduction current density, if an electric field of 2 V/cm is applied. Assume μ_n = 3900 cm^2/Vs and μ_p = 1900 cm^2/Vs.

Ans: 54.5 mA/cm^2.

(9) Show that, the ratio of maximum resistivity to intrinsic resistivity of a semiconductor is given by

$$\frac{\rho_{\max}}{\rho_i} = \frac{\mu_n + \mu_p}{2\sqrt{\mu_n \mu_p}}$$

(10) Calculate the intrinsic resistivity of Ge at 300 K. If a donor type impurity is added to the extent of 1 atom per 10^8 Ge atoms, calculate the resistivity, μ_n =3900 cm^2/Vs, μ_p = 1900 cm^2/Vs, $n_i = 2.5 \times 10^{13}$ cm^{-3}.

Ans: 43.05 Ω cm, 3.6208 Ω cm.

(11) A silicon sample of length 2 cm and area of cross-section 2 mm^2 at 300 K is doped with 10^{17} atoms/cm^3 of As and 9×10^{16} cm^{-3} of Boron. Compute the specimen's resistance and compare the contribution of electron and holes to the total conductivity. Assume μ_n = 1350 cm^2/Vs and μ_p = 480 cm^2/Vs.

Ans: 0.9247Ω; $\frac{\sigma_n}{\sigma} \cong 1 \frac{\sigma_n}{\sigma_p} = 1.25 \times 10^{12}$

(12) A Si sample isjdoped with 10^{15} Boron atoms and a certain number of shallow donors. The Fermi-level is 0.33 eV above E_i at 300 K. What is the donor concentration?
Take n_i as 1.5×10^{10} cm^{-3}.

Ans: 5.878×10^{15} cm^{-3}.

(13) In a semiconductor at room temperature (300 K) the intrinsic carrier concentration and resistivity are $1.5 \times 10^{16}/m^3$ and 2×10^3 Ω m respectively. It is converted to an extrinsic semiconductor with a doping concentration of $10^{20}/m^3$. For extrinsic semiconductor calcualte

1. Minority carrier concentration,
2. Resistivity,
3. Shift in Fermi-level due to doping and
4. Minority carrier concentration when its temperature is raised to a value at which intrinsic carrier concentration doubles. Assume $\mu_n = \mu_p$, kT = 26 m eV at room temperature.

Ans: (a) 2.25×10^{12} m^{-3} (b) 6.49×10^{-3} Ω m (c) 0.2289 eV (d) 9×10^{12} m^{-3}.

(14) A Ge sample is properly contacted and oriented in a 5 kG magnetic field as in Fig. 1.40. The current is 4 mA. The sample dimensions are w = 0.25 mm, t = 50 μm and L = 2.5 mm. The following measurements are made. V_{AB} = -2.5 mV and V_{CD} = 170 mV. Find the type and concentration of majority carriers and its mobility.

Ans: n-type, $n_0 = 9.98 \times 10^{16}$ cm^{-3}, 2943.39 cm^2/Vs.

(15) Boron is diffused into an intrinsic Si sample giving the acceptor distribution shown in Fig. Ex.1.15. Sketch the equilirbiuin energy band diagram and show the direction of the electric field.

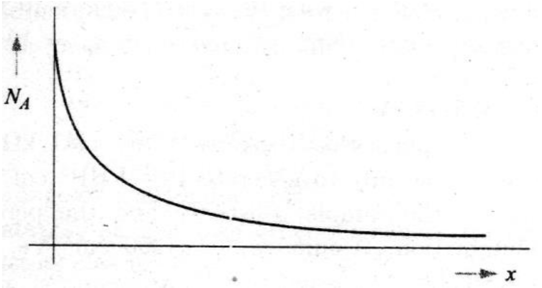


Fig. Ex.1.15

(16) An n-type silicon bar is doped uniformly by phosphorus atoms to a concentration of 4.5 × 10^{15} cm^{-3}. The bar has cross-section of 1 mm^2 and length of 10 cm. It is illuminated uniformly for regions x < 0 as shown in Fig. Ex. 1.16.

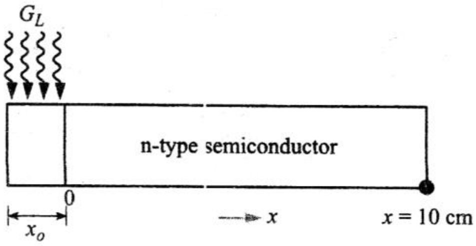


Fig. Ex.1.16

Assume optical generation rate 10^{21} EHP cm^{-3} s^{-1} for this case. The hole lifetime and electron lifetime are equal and is equal to 1 μs . Evaluate the hole and electron diffusion currents and electron drift current at x = 34.6 μm. Given D_p = 12 cm^2/s D_n = 30 cm^2/s q = 1.6 × 10^{-19} C, kT/q = 26 mV.

Ans: $I_{p\ diff}$ = 2.043 mA, $I_{n\ diff}$ = -4.665 mA. $I_{n\ drift}$ = 2.622 mA

(17) A silicon sample is doped uniformly with 10^{15} As atoms cm^{-3} and has $\tau_p = 1$ μs

(a) Determine the photo generation rate that will produce 2 × 10^{13} excess EHP^s cm^{-3} in the steady-state.

(b) Calculate the conductivity of the sample and the p'osition of the electron and hole quasi Fermi levels in the steady-state at 300 K.

Assume μ_n = 1350 cm^2 /Vs, μ_p = 480 cm^2/Vs and n_i = 1.5 × 10^{10} cm^{-3}.

Ans: (a) 2 × l0^{19} EHP/cm^3s (b) 0.22 $(\Omega cm)^{-1}$, $F_n - E_i$ = 0.289eV $E_i - F_p$ = 0.1798 eV

(18) A homogeneous semiconductor bar is illuminated uniformly by a penetrating light that generates EHP^s at a constant rate G_L cm^{-3} s^{-1}. Assuming low-level injection,

(a) Calculate the excess carrier concentration as a function of time if light is switched on at t = 0

(b) Determine the steady-state values of electron and hole concentrations and show that the photo conductivity ($\Delta\sigma$) of the sample is given by $q(\mu_n + \mu_p) G_L\tau_p$.

(19) A Germanium sample doped with 10^{17} cm^{-3} donors at 300 K is optically excited such that g_{op} = 10^{20} EHP/(cm^3 - s) and $\tau_n = \tau_p$ = 10μs. Find the separation between quasi Fermi-levels.

Ans: $F_n - F_p$ = 0.3118 eV.

(20) A sample of n-type silicon has dark resistivity of 1 kΩcm at 300 K. The sample is illuminated uniformly to generate 10^{21} EHP^s cm^{-3} s^{-1}. The hole lifetime is 1 μs. Calculate the sample resistivity and the percentage change in resistivity after illumination. Assume μ_n = 1350 cm^2/Vs. μ_p = 480 cm^2/Vs and n_i = 1.5 × 10^{10} cm^{-3}.

Ans: 1.0034 k Ω cm, 0.3411%

(21) A semiconductor (n-type) bar is injected at one end by minority carrier holes and an electric field of 100 V/cm is uniformly applied along the length of the bar that move the holes a distance of 0.1 cm in 3μs. Determine

(a) The drift velocity of holes

(b) The diffusion constant of holes at T = 300 K

Ans: (a) 3.33 × 10^4 cm/s (b) 8.67 cm^2/s.

(22) Given a Ge sample that has at 300 K $n_i = 2.5 \times 10^{13}$ cm^{-3}, $\mu_n = 3900$ cm^2/Vs and $\mu_p = 1900$ cm^2/Vs. Determine all the possible values of hole and electron densities that cause the conductivity of a semiconductor to be equal to the intrinsic conductivity.

(23) A hole current of 10^{-5} A/cm^2 is injected into the side (x = 0) of a long n-silicon. Assuming that holes flow only by diffusion and that at very large values of x, the distribution of excess holes decays to zero, Determine

(a) The steady-state excess hole density at x = 0

(b) Hole current density at x = 100 μ_m. Given $\mu_p = 480$ cm^2/Vs $\mu_n = 1350$ cm^2/Vs and the lifetime of holes is 2.5 μ_s

Ans: (a) 2.794×10^{10} cm^{-3} (b) 1.667×10^{-6} A/cm^2.

(24) Draw the equilibrium energy band diagram of an n-type silicon at 300 K with following doping profile.

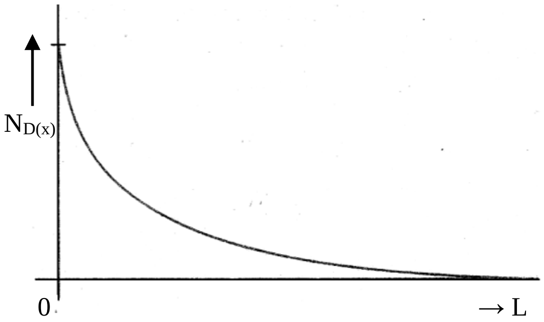


Fig. Ex.1.24

(25) In a p-type semiconductor the Fermi-level lies 0.4 eV above the valence band. If the concentration of acceptor atoms is trippled find the new position of Fermi-level.

▪ Review Questions

(1) What is resistivity range of semiconductors?

(2) What are the advantages of silicon over other semiconductor materials?

(3) How are semiconductors classified?

(4) Compare classical mechanics and quantum mechanics. Why is quantum mechanics used in the study of semiconductors?

(5) Explain Heisenberg uncertainty principle.

(6) What are the postulates of quantum mechanics?

(7) Write one dimensional time independent Schrodinger wave equation. List few of its applications.

(8) Explain tunneling mechanism with the help of quantum mechanics.

(9) Explain Fermi-Dirac, Maxwell-Boltzmann and Bose-Einstein statistics.

(10) How do Fermi-level position vary with doping in a semiconductor?

(11) Define photon and phonon.

(12) What are the different types of bonds that exist in solids?

(13) Distinguish between single crystalline and poly crystalline materials.

(14) Define the following terms related to crystal structure.

- (a) Unit cell
- (b) Primitive unit cell,
- (c) Co-ordination number,
- (d) Tetrahedral radius,
- (e) Lattice constant and
- (f) Packing efficiency.

(15) What are different cubic crystal structures?
(16) How are crystal planes and directions designated?
(17) Explain the formation of energy bands in carbon.
(18) Distinguish between the energy band diagrams of metals, semiconductors and insulators.
(19) Distinguish between direct and indirect bandgap semiconductors.
(20) Explain effective mass.
(21) Explain the conduction process by holes.
(22) Draw the energy band diagram of

- (a) Intrinsic semiconductor,
- (b) n-type semiconductor and
- (c) p-type semiconductor at 0 K and at 300 K. Show the donor/acceptor levels also.

(23) Explain Fermi-Dirac distribution function. Plot it as a function of energy (E) for different temperatures.
(24) Explain graphical estimation of carrier concentration in a semiconductor.
(25) Derive the relation $n_o p_o = n_i^2$
(26) Derive $n_o = n_i \, e^{(E_F - E_i)/kT}$ $p_o = n_i \, e^{(E_i - E_F)/kT}$
(27) Explain temperature dependence of intrinsic carrier concentration with mathematical model and figure.
(28) List values of n_i for Si, Ge, GaAs at 300 K.
(29) Explain temperature dependence of majority carrier concentration in a semiconductor.
(30) What is the charge of n-type semiconductor? Explain.
(31) What is the use of charge neutrality equation?
(32) Show that L_p is the average distance a charge carrier diffuse before it recombines.
(33) Derive expression for conductivity of a semiconductor.
(34) Why is the conductivity of a silicon sample minimum when it is slightly p-type?
(35) Derive the expression for minimum conductivity of a semiconductor.
(36) Find expression for σ_i/σ_{min} of semiconductor.
(37) What is meant by compensation?
(38) What is thermal relaxation time?
(39) What are the scattering mechanisms in a semiconductor?
(40) Explain temperature and doping dependence of mobility.
(41) Show that the gradient in Fermi-level is zero under thermal equilibrium.
(42) Explain Hall Effect. Explain the procedure to measure majority carrier concentration and mobility of a semiconductor specimen.
(43) Explain the variation of drift velocity with electric field for Si and GaAs.
(44) Explain the transient decay of excess carriers in a semiconductor with direct recombination. What is minority carrier lifetime?
(45) Define quasi Fermi-level. When do they exist?

(46) What are the different recombination mechanisms?
(47) What is Auger recombination?
(48) What are the origins of recombination centres?
(49) Explain diffusion process. Derive expression for diffusion current.
(50) How do the energy band diagram, change with application of electric field?
(51) State and derive Einstein relation.
(52) Write continuity equation. What are its applications?
(53) Derive continuity equation.
(54) Derive steady-state diffusion equations.
(55) How is minority carrier mobility and diffusion constant measured? Explain with diagrams.
(56) You have a Ge device and Si device with comparable dopings. Which one can be operated at higher temperature? Why?
(57) Show that life time is the average time a charge carrier survives without recombination after it is generated.